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A CLASS BOOK OF PHYSICS

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A CLASS BOOK
OF
PHYSICS

BY

SIR RICHARD GREGORY

AND

H. E. HADLEY, B.Sc.

ASSOCIATE OF THE ROYAL COLLEGE OF SCIENCE, LONDON
PRINCIPAL OF THE SCHOOL OF SCIENCE, KIDDERMINSTER

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FROM THE PREFACE TO THE FIRST EDITION

THERE is a diversity of opinion as to the value of text-books in the teaching of Science, and of practice as to their use. Prof. Armstrong has urged that "each child should write its own text-book and be taught to regard it as a holy possession"; and in a few schools this exalted doctrine is accepted in theory if not in fact. The principle is sound enough when applied by the private tutor (and the authors have no desire to belittle it) but it cannot be adapted satisfactorily to the work-a-day conditions of science teaching in schools where a large number of pupils of varying capacity and limited resource are receiving instruction in classes. In most cases, the time available for a science course will not permit the go-as-you-please pace postulated by some educational reformers as essential to good work, even if it be assumed that each pupil not only realises the necessity of working out his own intellectual salvation but is capable also of constructing his own road to it. How few pupils there are who possess the motive and purpose required for successful scientific study without assistance from a text-book is known only to the practical teacher.

Text-books may not be essential in the early work in science, but after pupils have acquired some familiarity with the scientific method it is desirable to fix ideas by more systematic study. Without practical experience there is no real scientific knowledge; but, on the other hand, unless laboratory work is accompanied by descriptive reading or lectures, it usually ends in a nebulous state of mind equally dispiriting to the teacher and pupil. The study of science is best encouraged by a right combination of experiment, discussion, and reading, and the

only limit to the amount of either is that of time. The exigencies of the school time-table do not permit the pursuit of an indefinite course of practical work or the leisurely consultation of authoritative treatises upon the subjects under consideration. For this reason books are demanded which present more or less concisely the essential principles of a branch of science and weave the scattered threads of thought into a fabric of definite pattern and reasonable dimensions.

It is not pretended that this volume is other than a text-book designed to facilitate the work of the teacher and concentrate the attention of the pupil: its purpose is not so much to inspire as to instruct. Between the prolix popular work, the treatise for reference, and the students' manual there are sharp lines of distinction; and though each has a place in scientific literature their respective merits must be judged by different standards. A text-book should be concise in description and precise in instructions for practical work: it should provide not only substance and guidance for study, but also exercises to test the increase of effective mental equipment and to cultivate the art of clear expression. In class teaching it is not sufficient to prescribe topics for reading or experiment: there must be some means of determining whether the work has been performed. A well kept record of laboratory experiments is doubtless an excellent index of progress made, but unless it is supplemented by exercises intended to test the ability to apply the results and conclusions arrived at to the solution of related problems, and the interpretation of wider experience, it may prove to be a vain thing. Such a record is a measure of the kinetic energy of a pupil's laboratory performances but not of the extent to which the energy has been transformed into potential power. It is good discipline and helpful teaching, therefore, by oral or written questions to make a periodic valuation of the capacity of students to comprehend the full meaning of the work done.

Considerations of time and space determine the scope of a science course and of a text-book; and in neither case should

sins of omission be judged so severely as those of commission. In most British schools the chief work in science consists of the subjects of fundamental physical measurements and heat included in Parts I., II., and III. of the complete volume. Practical acquaintance with these aspects of Physics is a necessary preliminary or accompaniment to the successful study of physical or chemical science ; and this must be the excuse for the apparently excessive amount of space devoted to them. Light, Sound, Magnetism and Electricity are regarded now as special subjects to be taken up selectively after a foundation has been laid in physical principles. Local circumstances and the requirements of examining bodies decide which of these subjects shall be studied, but in any case it is hoped that a satisfactory first course of systematic work will be found in the following pages.

In almost every case the exercises at the end of the chapters are from papers set at various examinations. It will be noticed that these questions are frequently of the nature of problems to be solved—either practically or otherwise—and cannot be answered by the mere repetition of the substance of the text preceding them.

R. A. GREGORY.
H. E. HADLEY.

PREFACE TO SECOND EDITION

When the Indian edition of our *Class-book of Physics* was first published in 1912, the opportunity was taken to make a few changes and additions which seemed to be required in certain sections of the book. The chief differences between the English and the Indian editions were, however, in Parts VI., VII. and VIII., devoted to magnetism and electricity. These Parts were greatly expanded for the Indian edition, and typical examination papers set at some of the chief Universities of India were added.

Since that time very remarkable developments have been made in physical science, and it became necessary almost to rewrite Parts VI.-VIII. of the English edition in order to make them represent more closely modern conceptions of magnetic and electrical phenomena and their applications. These revised Parts have now been introduced into the Indian edition, and it will be noticed that they include such subjects as electrons, structure of atoms, radio telegraphy and telephony, and many others which have almost become part of everyday life and thought.

The system of School Leaving Certificate Examinations now largely determines the scope of the work done in Secondary Schools in Great Britain as well as in India. The exercises at the ends of chapters are, therefore, mostly selected from papers set at such examinations. In order to bring the book into touch with requirements in India, a large number of questions of this standard has been added at the end of this book in groups of subjects ; and also questions from Intermediate papers, as well as several typical papers set at Intermediate examinations of Indian Universities. Answers are given to questions in which numerical results are required ; and where any points raised are not sufficiently covered by the text, supplementary explanations are provided with the answers. It is hoped that the new chapters, and the additional questions and answers, will increase the value of the book to students preparing for School Leaving Certificate Examinations and for Intermediate Examinations.

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CHAPTER I.

MEASUREMENT OF LENGTH, ANGLE, AND TIME.

Units.—The science of physics is based largely upon exact measurement; and to render such measurements intelligible they must be stated in terms of convenient or conventional standards. Thus, to express the magnitude of any measurable quantity by a number, it is necessary to decide upon a **unit**, such as a unit of length, surface, volume, weight, or time, as the case may be, with which the measurement obtained may be compared. What are known as fundamental quantities are length, mass and time, and from units based upon these other physical units are derived.

Measurement of Length.

The British and Metric systems of length.—In the British Empire the standard of length adopted is the length between two marks on a certain bronze bar deposited with the Board of Trade, the bar being at a certain fixed temperature when the measurement is made. This length is quite arbitrary and is called a **yard** (yd.). The yard is subdivided into three equal parts, each of which is a **foot** (ft.). The foot is divided in its turn into twelve equal parts, called **inches** (in.).

In France and many other countries, and for scientific work generally, what are known as metric measures are used. The standard length in this system is the **metre,** or the distance at

a particular temperature between the ends of a certain platinum rod deposited in the national archives at Sèvres. This standard is equal in length to 39·37079 inches. The metre is subdivided into ten equal parts, each of which is called a **decimetre**, the tenth part of the decimetre is called a **centimetre**, and the tenth part of the centimetre is known as a **millimetre**. Thus we get

10 millimetres (mm.) = 1 centimetre (cm.).

10 centimetres } = 1 decimetre (dm.).
100 millimetres }

10 decimetres }
100 centimetres } = 1 **metre** (m.).
1,000 millimetres }

The multiples of the metre are named **deka-**, **hekto-**, and **kilometres.** Their value is seen from the following table:

10 metres = 1 dekametre.
100 metres = 1 hektometre.
1,000 metres = 1 **kilometre.**

(*N.B.*—Dekametre and Hektometre are spelt Decametre and Hectometre frequently.)

The kilometre is equal to about five-eighths of a mile.

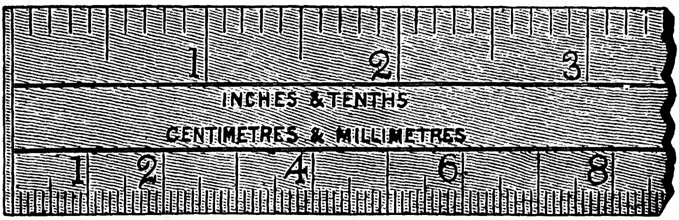

FIG. 1.—British and metric scales.

Exact relations between measures in the British and Metric systems of units are shown in the following tables, and also in Fig. 1:

METRIC TO BRITISH.

1 centimetre	= 0·394 inch.
1 metre	= 39·370 inches
	= 1·094 yards.
1 kilometre	= 0·621 mile.

BRITISH TO METRIC.

1 inch	= 2·54 centimetres.
1 yard	= 0·914 metre.
1 mile	= 1609·00 metres
	= 1·609 kilometres.

METHODS OF MEASURING LENGTH.

Measurement of straight lines.—As an example we may consider the measurement of the dimensions of a rectangular wooden block by means of an ordinary wooden scale divided into inches and tenths or into centimetres and millimetres. The following precautions must be observed:

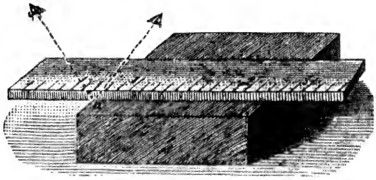

FIG. 2.—Wrong method of using a scale.

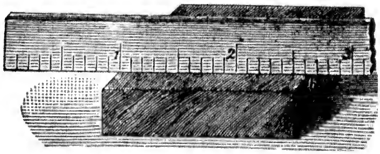

FIG. 3.—Correct way to use a scale.

(*a*) *The scale must be held so that the divisions are actually in contact with the line to be measured* (Fig. 3). This precaution is necessary in order to avoid errors due to *parallax*, which can be understood by reference to Fig. 2, where the scale reading of the point *a* evidently depends upon the position of the observer's eye.

(*b*) *The zero end of the scale must not be used*, since the end is frequently more or less worn away: some definite division, other than the zero, of the scale should be used (Fig. 3).

(*c*) Since the position of the point under observation may not coincide with any one division of the scale, *it is necessary to estimate fractions of a scale division*; thus the scale reading of the point *b* (Fig. 3) is between 2·6 and 2·7, and by regarding each division as divided into 10 equal parts, it is evident that the reading is expressed more accurately by the number 2·63.

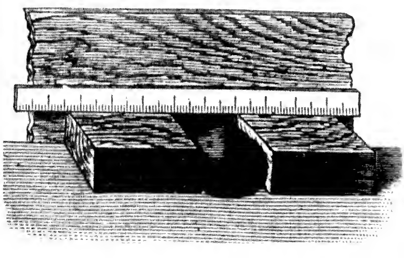

FIG. 4.—Method of measuring the diameter of a sphere.

Fig. 4 indicates a method of using a rigid scale for the measurement of the diameter of a sphere.

Measurement of curved lines.—All curved lines can be regarded as made up of a very large number of straight lines. If

an infinite number of such small straight lines could be taken, there would be no difference between the sum of their lengths and the length of the curved line (Fig. 5). The procedure is as follows:

EXPT. 1.—**By dividers.** Open the dividers until the points are about 5 mm. apart; make a fine pencil-mark on the curve and place one

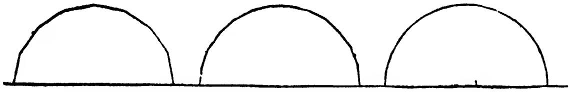

FIG. 5.—Curved lines produced by many short lines.

point of the dividers on the pencil-mark, place the second leg on the curve, raise the first leg and rotate the dividers on the second leg until it is on the curve and beyond the second leg; repeat this process, while counting the number of lengths measured by the dividers, until the end of the curve is reached. Any portion of the curve, less than 5 mm., which remains, must be measured separately by readjusting the dividers. The length of the curve is given by the product of the distance between the divider points and the number of lengths measured by the divider.

EXPT. 2.—**By cotton thread.** Cut one end of a piece of cotton thread cleanly with scissors, and place the end in contact with a pencil-mark on the curve (Fig. 6). Make the thread coincide as nearly as you can with a small part of the curve, and place the nail of the first finger of your right hand upon it. Now release your left-hand finger and carefully place it at the point where your right-hand finger is held; then, using your right hand, go on to make some more of the thread exactly coincide with another small length of curve. Repeat this until you have completed the whole curve. Measure the length of thread with a mm. scale.

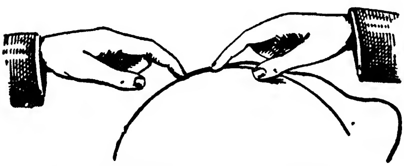

FIG. 6.—Measurement of a curved line.

EXPT. 3.—**By a strip of paper.** Wrap a strip of paper closely round a wooden cylinder (Fig. 7), and make a small hole with a pin at a place where the paper overlaps. Unroll the paper and measure the distance between the two holes. This gives the distance round the cylinder, that is, its circumference.

EXPT. 4.—**Circumference and diameter of a circle.** Cut out from a sheet of thin cardboard two discs of 4 cm. and 6 cm. radius. Make

a pencil mark near to the edge of each disc ; place one of the discs in a vertical position with its pencil-mark touching an observed division of a millimetre scale ; roll the disc along the scale until the mark again touches the scale. The difference between the two scale readings gives the length of the **circumference** of the disc. Determine the circumference of the other disc in the same manner.

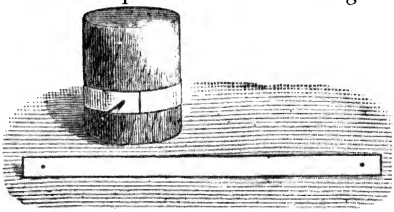

FIG. 7.—Measurement of the circumference of a cylinder.

Tabulate your results, thus :

Diameter.	Circumference.	$\frac{\text{Circumference}}{\text{Diameter.}}$
1.		
2		

The ratio, in the last column, should be a *constant* quantity : it is usually denoted by the Greek letter π. Hence

length of circumference $= \pi \times$ **diameter** $= 2\pi \times$ **radius.**

The vernier.—This device enables lengths to be measured accurately to a given fraction of the shortest division on the scale used. The method was devised by Paul Vernier in 1630: it consists in the addition of a second scale, the divisions of which bear a simple relationship to those of the standard scale with which it is used. The simplest vernier scale is one which enables lengths to be read to $\frac{1}{10}$ of a scale division ; and, in this case, the vernier scale is constructed either by dividing 9 divisions of the standard scale into 10 equal parts, or by dividing 11 divisions of the standard scale into 10 equal parts. The former, which is more usually adopted, is termed a *No.* 1 *vernier*, and the latter a *No.* 2 *vernier*.

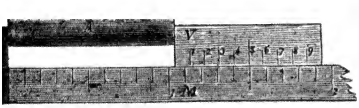

FIG. 8.—Use of a No. 1 vernier.

Fig. 8 represents the method of using a No. 1 vernier for measuring the length of an object A: M is an inch scale divided

into tenths, and V is the vernier scale. The length of A is evidently between 1 inch and 1·1 inch; and the vernier scale enables the amount by which the length exceeds 1 inch to be measured accurately. It will be noticed that the divisions of the two scales *coincide at one point only*, which is, approximately, the 4th of the vernier scale. Since 1 division of scale V is equal to $\frac{9}{10}$ of one division of scale M, *i.e.* to 0·09 inch, the 3rd division of V is 0·01 inch in advance of the division 1·3 on scale M, and

the 2nd division of V is 0·02 inch in advance of the division 1·2 on scale M,

the 1st division of V is 0·03 inch in advance of the division 1·1 on scale M,

the zero division of V is 0·04 inch in advance of the division 1·0 on scale M.

The last quantity, 0·04 inch, is the fraction of a division which had to be measured. Hence, the length of A is 1·04 inches.

An alternative method of reasoning is as follows:

The length of A + the length of 4 vernier divisions = 1·4 inches.

Hence, the length of A = 1·4 − (4 × 0·09)
= 1·04 inches.

It is evident that the following rule may be adopted in using such a vernier scale: *Note where divisions on the two scales coincide; the number attached to the division of the vernier scale which coincides gives the numeral in the second place of decimals.*

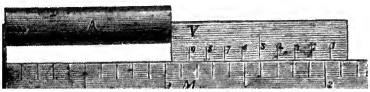

FIG. 9.—Use of a No. 2 vernier.

It may be desired to use a vernier reading to a smaller fraction than 1/10 of a scale division; in this case, 19 scale divisions may be divided into 20 equal parts, giving a vernier-scale reading to 1/20 of a scale division. Generally speaking, in order to read to $1/n$ of a scale division, $(n-1)$ divisions must be divided into n parts.

Fig. 9 represents a No. 2 vernier, in which each division of the vernier scale is equal to 1·1 divisions of the standard scale M. It will be noticed that the divisions on the scale V are numbered from right to left. If the 4th division on scale V coincides with a division on scale M, then

the length of A + the length of 6 vernier divisions = 1·7 inches.

Hence, the length of A = 1·7 − (6 × 0·11)
= 1·04 inches.

The same rule, therefore, as given above when using a No. 1 vernier, may be adopted in the case of a No. 2 vernier, providing that the scale divisions are numbered in the reverse direction.

The slide calipers.—Fig. 10 represents a simple form of slide calipers. The calipers consist of a thin steel rod with a jaw A

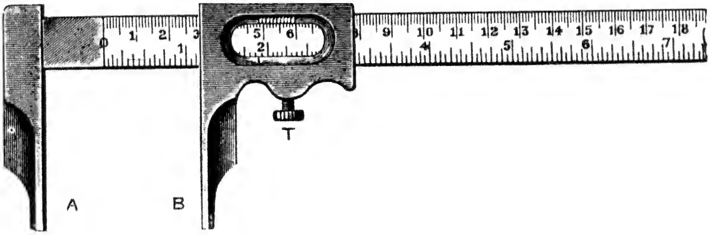

FIG. 10.—Slide calipers.

fixed at one end; B is a movable jaw with a vernier scale V which traverses a scale etched on the steel rod. The movable jaw can be fixed by means of the screw T. When the faces of the jaws are in contact, the zero division of the vernier scale should coincide with the zero division of the fixed scale. The dimensions of an object are measured by noting how far the zero division of the vernier scale has moved along the fixed scale when the jaws have been separated until the object is *just* touched by the faces of the two jaws. Before taking a measurement, it is necessary to note whether the fixed scale is divided into millimetres or into parts of an inch, and to what fraction of a scale division the vernier is intended to read. The instrument shown in Fig. 10 has both metric and British scales.

The screw-gauge.—The screw-gauge (Fig. 11) furnishes a very accurate means for measuring the dimensions of small objects. It consists of a fixed frame F, attached to which is a hollow cylinder C. A screw thread is cut on the inside surface of C.

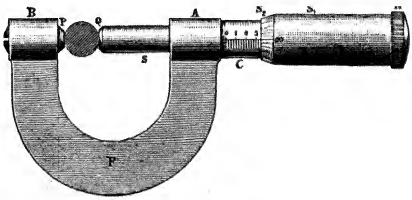

FIG. 11.—Screw-gauge.

The shaft S is the continuation of a screw which travels along the

threads cut within C; and a sleeve S_1 is attached to the head H of the screw. The edge S_2 of the sleeve is divided into a definite number, usually 50 or 100, of equal parts. The end of the shaft S is a truly planed surface Q, and a similar surface P is obtained on the end of a fixed screw *f* carried by the other limb B of the fixed frame. The screw *f* is adjusted, once for all, so that, when the edge of S_2 coincides with the zero division of the scale on C and when the zero division of the scale S_2 coincides with the base line of the scale on C, the two plane faces P and Q are in contact.

Before taking a measurement, it is necessary to observe whether the scale on C is divided into millimetres or into tenths of an inch. The *pitch* of the screw S—*i.e.* the distance through which Q advances or recedes by one complete rotation of H—must then be determined. This is obtained by observing whether one division, or only *half* a division, of the scale on C is uncovered when H is rotated backwards by one complete revolution. Finally, the scale-value of one division of the scale S_2 is required.

As a general rule, the pitch of the screw S is 0·5 mm., and S_2 is divided into 50 equal parts; hence 1 division of

$$S_2 = \tfrac{1}{50} \times 0{\cdot}5 = 0{\cdot}01 \text{ mm.}$$

The object to be measured is placed between the faces P and Q, and the milled head H is rotated until the object is *lightly* gripped between the faces. The readings of the scales on C and on the sleeve at S_2 enable the size of the object to be measured.

The spherometer.—The principle of this instrument closely resembles that of the micrometer screw-gauge. The instrument consists of a tripod, the legs of which are of equal length and are adjusted relatively to each other so that the three points occupy the corners of an equilateral triangle. A fine screw, which works through the centre of the tripod, terminates above in a milled head and a large circular disc, the edge of which is divided into 100 equal parts. A vertical scale, usually divided into millimetres, is fixed to one arm of the tripod, and with its divisions close to the edge of the disc.

Before using the instrument, it is necessary to determine the *pitch* of the screw: this may be equal to 1 mm. or to 0·5 mm. This is done by reading the position of the disc's edge on the vertical scale, and then rotating the disc through an observed

number of complete turns; the difference in the scale-reading divided by the number of turns gives the pitch of the screw. If the pitch of the screw is 0·5 mm., and if the disc is divided into 100 equal parts, then 1 division of the disc is equal to 0·005 mm.

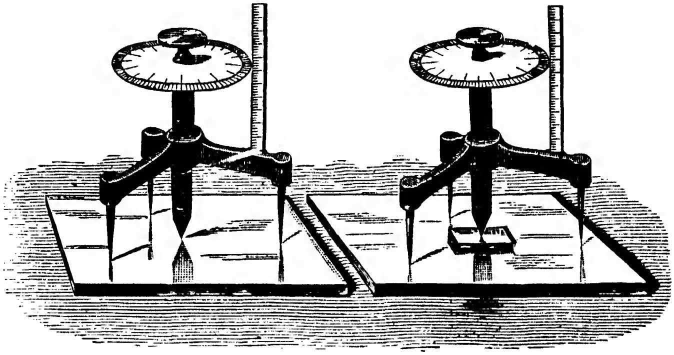

FIG. 12.—Use of a spherometer.

In measuring the thickness of an object, the following procedure is adopted: Place the instrument on a truly horizontal surface, *e.g.* a sheet of plate glass, and rotate the screw downwards until its point *just* touches the surface (Fig. 12); this is determined most accurately by placing the thumb and first finger against opposite sides of one leg of the tripod, and endeavouring to make the instrument rotate round the centre leg; if the latter projects downwards too far the instrument readily rotates, but if contact is not complete there is an unmistakable sense of resistance to rotation. Having made this adjustment, take the reading of the scale and disc. Now raise the screw considerably, place the object underneath the screw point, and rotate the screw downwards until contact is just made again. The difference between the two sets of readings gives the thickness of the object.

The **wedge.**—The wedge gauge is useful in order to determine the internal diameter of a tube with a circular bore. A

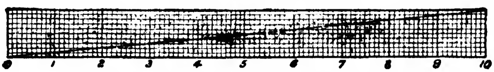

FIG. 13.—Construction of a measuring wedge.

simple form may be made by cutting, from squared paper or thin sheet metal, a right-angled triangle with a base 10 cm. long

and a perpendicular 1 cm. long, as indicated in Fig. 13. If the acute angle is pressed into the tube until it occupies a diameter of the bore, the diameter is given by $\frac{1}{10}$ of the length of that part of the base which has been introduced within the tube.

Measurement of Angles.

Unit of angular measurement.—The general plan adopted in measuring angles is to divide a circle into 360 equal parts, and to call each part a **degree** (1°). Thus, a movable hand pivoted at the centre of a circle has traced out an angle of one degree when it has gone round $\frac{1}{360}$th part of a complete revolution. When it has performed one quarter of its journey round, it has made an angle of ninety degrees, or a **right angle**, as it is called.

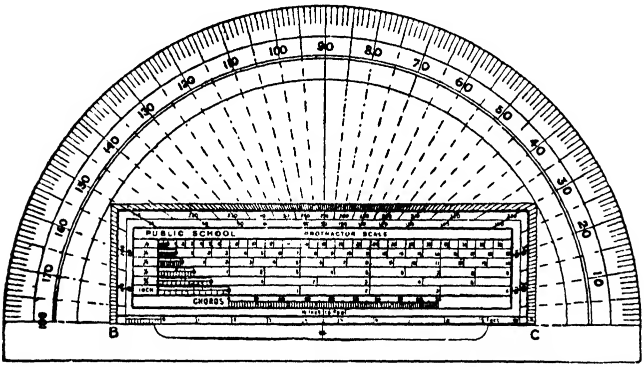

Fig. 14.—Semi-circular and rectangular forms of protractor, used for the measurement of angles.

The minute hand of a watch or clock moves through 360 degrees in an hour, or ninety degrees in every quarter of an hour, and this is true whatever the size of the timepiece. This illustrates the important fact that the size of an angle is quite independent of the length of the lines between which it is contained. All circles contain 360 degrees. All right angles contain ninety degrees, so there are four right angles to every circle. In accurate measurement, parts of a degree are required, and the sub-divisions used are that one degree (1°) equals sixty minutes, and one minute (1′) equal sixty seconds (60″).

The magnitude of an angle can be found by means of a protractor, two forms of which are shown in Fig. 14. The simplest form is a semi-circle divided into degrees, but a more common form is oblong in shape. The marks upon the edge of a protractor of this kind are obtained from the corresponding divisions on a semi-circle in the manner represented in the illustration.

Measurement of Time.

The earth's rotation.—The apparent daily motion of the sun and stars across the sky is a direct consequence of the earth's rotation on its axis. The sun appears regularly to go through certain periodic changes of position. It rises, travels higher and higher into the sky, reaches its highest position, sinks lower and lower, and finally sets. When the sun is at its highest altitude on any day it is over the meridian of the place of observation, and lies in a true north and south direction. The interval of time between the sun's highest position on any one day and its corresponding position on the next succeeding day is an **apparent solar day.**

Mean solar day.—The length of days measured by the sun varies throughout the year, hence no single one of these days will do for a convenient standard of time. But if the lengths of all the days in the year be added together, or the length of a year measured by the sun be divided by the number of days in the year, we obtain an interval of time which is always the same. Such a day, which is of course an imaginary one, is called a **mean solar day.** Sometimes the mean solar day is longer than the solar day, sometimes it is shorter, and occasionally both days are exactly the same length. Solar time is known as **apparent time,** and clock time as **mean time.**

Sidereal day.—As in the case of the sun, so it is with most of the stars; they rise, cross the meridian, and set. But whereas with the sun the interval between two successive meridian passages at the same place varies throughout the year, it is found that the time which elapses between two succeeding transits of a star at any season of the year is always the same. This interval constitutes **a star or sidereal day.**

Period of rotation of the earth.—As the apparent motions of stars across the sky are produced by the rotation of the earth,

it is evident that the exact time of rotation can be determined by finding the interval which elapses between two successive returns of any particular star to the same point of the sky. A star may, indeed, be regarded as a fixed reference mark under which the earth turns; so that by observation of it we are able to determine the time taken by the earth to spin round once. The interval between two successive transits of the same star, that is, a sidereal day, is the time of such rotation. No matter which star is selected for observation the interval is the same, thus showing that the earth is a rigid body, and that all parts of its surface have the same angular velocity.

Units of time.—The sidereal day, like the mean solar day, is subdivided into hours, minutes, and seconds, but as the latter is four minutes longer than the former, the units are not of the same value. We may take either the **mean solar second** as the unit of time or the **sidereal second.** In the former case, the unit is founded on the average length of the solar day, and in the latter upon the length of the invariable star day, or the time of rotation of the earth upon its axis. But in either case the second, that is, the unit of time, is the 86,400th part of the day used.

In physical measurements the unit of time adopted is the mean solar second, that is, it is the 86,400th part of the average time required by the earth to make one complete rotation on its axis relatively to the sun considered as a fixed point of reference.

Instruments for measuring time.—We need only concern ourselves with the modern contrivances for measuring time, viz., clocks and watches. It will be sufficient to regard these as instruments for measuring intervals of time in terms of the mean solar day to which attention has been directed. In a clock the rate is regulated usually by means of the pendulum, the properties of which can be best understood by an experiment.

The simple pendulum.—A **simple pendulum** may be defined as a **heavy particle suspended by a weightless thread.** An approximation to this ideal is obtained by suspending a small metal sphere by a very thin thread. In the arrangement represented in Fig. 15, stout cotton is *threaded* along the axis of a cork, which serves

as the carrier for the pendulum. The *bob* consists of a truly turned solid brass sphere through which a very narrow hole is bored along a diameter; this hole is bored out to larger size for a short distance from one end. In fitting up the pendulum the cotton is threaded through the bob, held with the wide end of the bore downwards; the cotton is knotted sufficiently for the knot to pass *just* within the wide end of the bore. The use of a cork, as suggested, allows the more accurate determination of the point of support; and the length of the pendulum is varied readily by pulling more or less of the cotton through the cork. The diagram indicates how the length of the pendulum may be measured by means of a metre scale and a wooden cube. The true length of the pendulum is approximately the distance from the point of support to the *centre* of the bob; and this length is best obtained by measuring the distance to the bottom of the bob and subtracting from this the radius of the bob. In the following experiment a cheap stop-watch is desirable.

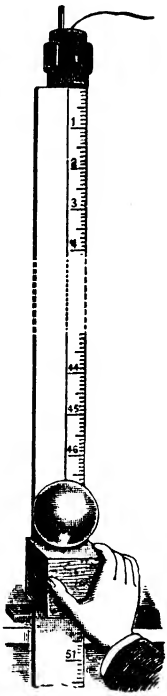

FIG. 15.—A simple pendulum.

EXPT. 5.—**Length and rate of swing of a pendulum.** Support immediately behind the thread of the pendulum a piece of cardboard on which a vertical pencil line is drawn. Sit down in front of the pendulum, and, *using one eye only*, adjust the position of the cardboard so that the thread exactly covers the pencil line. Keep the eye in this same position during the following observation. Set the pendulum swinging through a small angle not exceeding 15°. A single swing from side to side is termed usually a **vibration**, and a swing-swang, or complete movement to and fro, is an **oscillation**. The extent of a swing is termed the **amplitude** of vibration. Note the time indicated by the stop-watch, and start the watch just as the thread is passing in front of the pencil line. Count the number of subsequent passages of the thread until at least 50 vibrations have been completed, and stop the watch at an instant when the pendulum is passing the pencil line. Read the watch, and calculate the time interval between two consecutive passages of the pendulum in front of the pencil line. Repeat the experiment; and if this result differs from the first determination by more than 0·01 second, take a third observation. Measure the length of the pendulum. Alter the length of the pendulum.

and determine the time of vibration again in the same manner. Repeat this for different lengths, varying from 20 cm. to 120 cm. Tabulate the results thus :

Length.	Time of vibration.	$\sqrt{\text{Length}}$.	$\frac{\sqrt{\text{Length}}}{\text{Time of vibration}}$.

Plot on squared paper the *length* and the *time of vibration*, taking the latter as ordinates. Similarly plot the $\sqrt{length}$ and the *time of vibration*, or *length* and *time*^{2}. From the curves obtained deduce the relationship between the time of vibration and the length of the pendulum.

The following readings were obtained with a simple pendulum constructed in the manner described above :

Length.	Time of vibration.	Length.	Time of vibration.
20 cm.	0·45 sec.	88 cm.	0·94 sec.
30 "	0·55 "	95 "	0·98 "
42 "	0·65 "	102 "	1·01 "
55 "	0·74 "	115 "	1·07 "
70 "	0·835 "	130 "	1·14 "

Fig. 16 represents how these readings may be plotted on squared paper. Two **axes**, OX and OY, are drawn at right angles to each other, with the point O near to the left-hand bottom corner of the paper. The line OX is called the axis of **abscissae**, and OY the axis of **ordinates**. For the purpose of the present experiment, the horizontal scale along OX is taken to represent the *length* of the pendulum, and the scale along OY is taken to represent the time of vibration.

In the diagram, each division of OX represents 2 cm., and each division of OY represents 0·02 sec. In general, the value attached to each scale division should be chosen so that the lengths of the two scales utilised for plotting the observations are nearly equal, and so that the lengths are as great as the paper will allow.

Bearing in mind the simplicity of the apparatus, the liable error of observation, and the possibility of irregularity in the squared paper, it cannot be expected that all the plotted readings

may coincide with the curve (or straight line) which represents a theoretically accurate result. Having plotted the points which indicate the observations taken, a line should be drawn which, as nearly as possible, passes through each point.

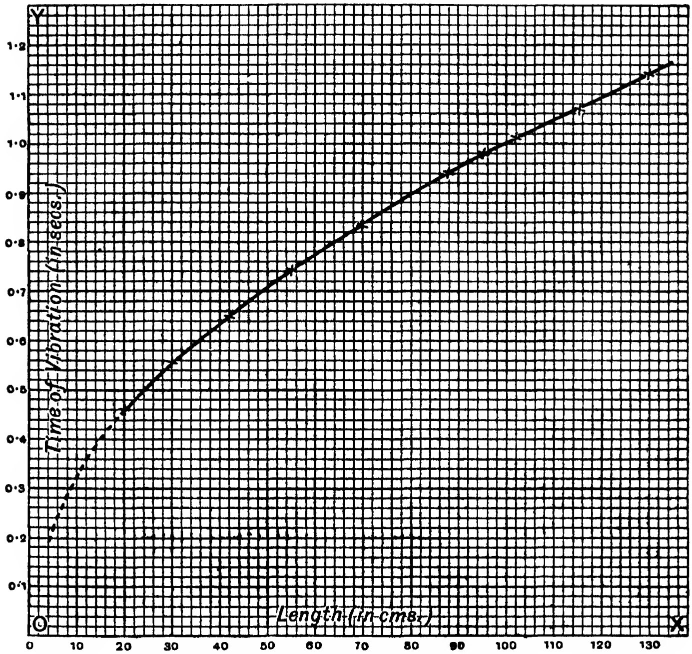

Fig. 16.—Graphical representation of the relation between the length and time of vibration of a simple pendulum.

Expt. 6.—**Amplitude of swing.** Draw the pendulum ball aside, say ten inches from the vertical, and find the times required to complete twenty swings. Then draw it only three inches aside and again find the time of twenty vibrations. Hence determine whether the time is independent of the amplitude.

Expt. 7.—**Mass of pendulum bob.** Determine the time required for a certain number of swings when balls of the same size and shape, but different masses, are suspended from the same length of thread It will be found that within certain limits the mass of the bob does not affect the rate of swing.

EXERCISES ON CHAPTER I.

1. Convert 1 kilometre to yards, feet, and inches.

2. Mont Blanc is 15,780 ft. high. Express this in metres (to 1 place of decimals).

3. How many pieces of string, each 2·8 metres long, can be cut from a length of 6780 cm.? What length would be left over?

4. If the height of a column of mercury is 760 mm., what is it in inches?

5. What fraction is (*a*) 1 mm. of 1 inch, (*b*) 1 decimetre of 1 foot, (*c*) 1 centimetre of 1 inch?

6. Measure the length of this page in inches and centimetres, and use the results to find the number of centimetres equal to one inch

7. What is meant by a unit of length, and why is it necessary to have such a unit?

8. Measure the length of three straight lines in inches and in centimetres, and calculate the mean value of one inch in centimetres.

9. Draw a circle and measure the lengths of the circumference and the diameter. How many times does the circumference contain the diameter?

10. Measure the diameter and circumference of a cylinder, and calculate the ratio of the diameter to the circumference.

11. Construct a vernier to read to one-tenth of a scale division.

12. Construct a graduated wedge to measure the diameter of a small tube.

13. Draw a triangle and measure the value of each of its angles in degrees. Find the sum of the three angles.

14. Draw two lines crossing one another at any angle. Measure the four angles thus obtained and find their sum.

15. What is the difference between an apparent solar day and a mean solar day?

16. What is the unit of time and how is it related to the period of the earth's rotation?

17. Describe a simple pendulum, and state the relationship between its length and the rate at which it swings.

18. What is sidereal and what is mean time?

19. What do you understand by 'amplitude' and time of swing or oscillation as applied to a pendulum? What relation exists between time of oscillation and (*a*) the amplitude, (*b*) the length of the pendulum?

If the time of oscillation of a pendulum 100 cm. long is t sec., state the time of oscillation when (1) the amplitude is doubled, (2) the weight of the bob is doubled. What must be the length of pendulum whose time of oscillation is $t/2$ sec.?

CHAPTER II.

MEASUREMENT OF AREA.

Area.—In order to express an **area** (or, *extent of surface*) it is not sufficient to measure one length only; two lengths must be considered, viz. length and width. We may select any **unit of surface,** *e.g.* a sheet of foolscap paper; and the area of a surface, such as a table-top, might be measured by determining how many similar sheets were necessary in order to cover completely the top of the table: the number of such units required would be given by multiplying the number of sheets in each row by the number of rows. In practice, instead of using such an arbitrary unit, we use units derived from the standard units of length, viz. the **square foot** or the **square centimetre,** according to whether the British or metric system is used.

EXPT. 8.—**British and Metric Measures of Area.** Draw a square decimetre and divide it into square centimetres. Prove, by counting, that the area of the square is equal to its length multiplied by its height. Now draw two or three oblongs and determine their areas by means of this rule. Determine the areas of the square and oblongs both in square inches and square centimetres, and use your results to find the number of square centimetres in one square inch, thus:

Area of a given rectangle in square inches.	Area of same rectangle in square centimetres.	$\frac{\text{Square centimetres}}{\text{Square inches}}$.

Verify your results by calculation from the relation 1 inch = 2·54 cm.

Errors of observation.—The student should bear in mind that the unavoidable errors in measuring a length are augmented

considerably when two such observed lengths are multiplied together, and that the *product* of the lengths may be greatly in error. Thus, in using a millimetre scale for measuring a straight line 1 inch long, it may be judged to be 2·54 cm. long; but the last digit is *estimated* only, and a second reading of 2·55 cm. might appear to be equally trustworthy. The *liable* error of observation would be 1 in 250, or 0·4 %. The area of a square inch, calculated from such readings, would be either 6·4516 sq. cm. or 6·5025 sq. cm., and the liable error would be about 5 in 650, or 0·75 %. Hence the liable error is increased very much, and it is evident that *any figures beyond the third significant digit are totally untrustworthy, and therefore they should not be written when stating the result of an experiment.*

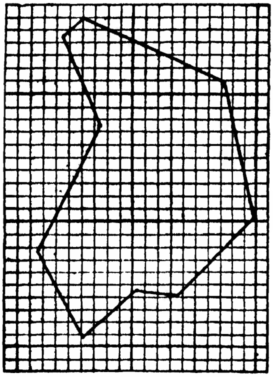

Fig. 17—Determination of the area of an irregular figure.

Measurement of irregular areas.—The following exercise will explain the method of using squared paper:

Expt. 9.—**Areas by squared paper.** Trace out an irregular figure, as shown in Fig. 17, on squared paper. Observe carefully the area of each small square: paper divided into $\frac{1}{100}$ sq. inch is used most frequently. Count the number of complete small squares enclosed within the figure; if more than half of a square is inside the boundary count it as one square, but neglect it if less than half is inside the figure.

If the student is familiar with the use of a balance, the result may be verified by cutting out in thin cardboard (i) the same area, and (ii) a 2-inch square from the same material. These two areas are weighed, and the areas will be found proportional to their weights (see p. 37).

Mensuration of Geometrical Figures.

Expt. 10.—**The parallelogram.** Cut out two cardboard parallelograms ABCD, EFGH (Fig. 18) and draw a line from D perpendicular to BC, and from G perpendicular to EF. Cut off the two triangles DCL, GMF, and place them so as to convert each parallelogram into a rectangle.

Evidently the area of each complete figure is the same whether the triangle is in one position or the other. In other words, a parallelogram has the same area as a rectangle on the same base and having the same **altitude**, or perpendicular height.

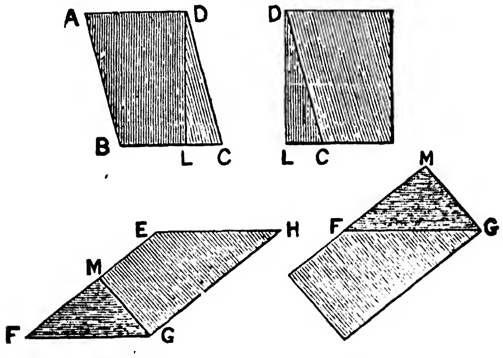

FIG. 18.—Every parallelogram can be considered as a rectangle.

The area of a rectangle (p. 17) is equal to the base multiplied by the height ; hence,

area of a parallelogram = base × perpendicular height.

This statement may be tested by the following exercise :

EXPT. 11.—Draw any parallelogram, such as ABCD (Fig. 19), on squared paper. Count the number of squares, and state its area in sq. cm.

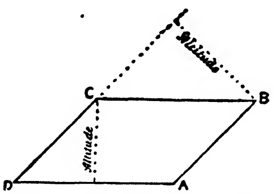

FIG. 19.—Measurement of the area of a parallelogram.

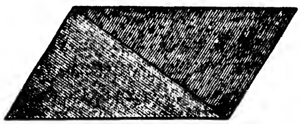

FIG. 20.—A parallelogram can be cut into two equal triangles.

Verify this result by calculating the area from measurements of the base and the perpendicular height, taking (i) DA as base, (ii) CD as base.

EXPT. 12.—**The triangle.** Draw a parallelogram on paper or thin card, and then cut it in two from corner to corner. You have now

two triangles, and by laying one on the other it will be found that they fit or are equal in area (Fig. 20). Repeat the exercise with a parallelogram of different form.

It has been seen that

$$\text{Area of parallelogram} = \text{base} \times \text{altitude}.$$

But a triangle is half a parallelogram.

Therefore, $$\textbf{area of triangle} = \frac{\textbf{base} \times \textbf{altitude}}{\textbf{2}}.$$

EXPT. 13.—Draw any triangle ABC on squared paper, and determine its area by counting squares.

Verify this result by calculating the area from measurements of the base and the perpendicular height, taking (i) BC as base, (ii) AC as base, (iii) AB as base.

EXPT. 14.—**Any four-sided figure.** Draw any irregular four-sided figure on squared paper, and determine its area by counting squares.

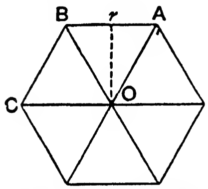

FIG. 21.—Area of a hexagon.

Divide the figure into two triangles by joining either pair of opposite corners, and calculate from its dimensions the area of each triangle. The sum of these areas should equal the total area as determined by squared paper.

EXPT. 15.—**The hexagon.** Divide up a regular hexagon into six equal triangles (Fig. 21). Find the area of one of these triangles, viz. OAB; it is equal to $Or \times \frac{1}{2}AB$; similarly, the area of OBC is equal to $Or \times \frac{1}{2}BC$. Hence, the area of the hexagon is equal to

$$Or \times \frac{\textbf{sum of the bases}}{\textbf{2}};$$

or, $$\textbf{area of hexagon} = Or \times \frac{\textbf{length of perimeter}}{\textbf{2}}.$$

This rule applies to any regular many-sided figure. When the number of sides is infinitely great, the figure becomes a *circle*, in which Or is the *radius*, and the perimeter is the *circumference* (Expt. 4).

Hence, $$\textbf{area of a circle} = \textbf{radius} \times \frac{\textbf{circumference}}{\textbf{2}}.$$

This may be demonstrated in the following manner:

EXPT. 16.—**The circle.** Cut out a circular disc of cardboard about four inches in diameter. Divide it into small triangles as in Fig. 22. The area of the circle could be found by determining the areas of all these triangles.

Cut out the triangles and arrange them as in Fig. 23.
Find the area of this figure, regarding it as a parallelogram.

BD is the length of the radius of the circle, and AB is half the length of the circumference. Therefore, since the area of

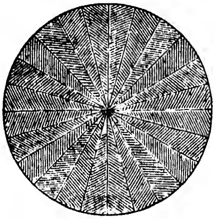

FIG. 22.—Circle divided into triangles.

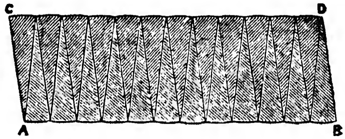

FIG. 23.—Figure formed by triangles cut from a circular disc.

the figure equals that of the original circle, **the area of a circle is equal to the radius multiplied by half the circumference.** It should be understood that this rule is proved only approximately by the method adopted.

Since the circumference $= 2\pi \times$ radius, then

$$\textbf{area of circle} = radius \times \frac{2\pi \times radius}{2}$$
$$= radius \times (\pi \times radius)$$
$$= \pi \times (\textbf{radius})^2.$$

EXPT. 17.—**Square on radius.** Draw a circle of about 5 cm. radius on squared paper, and describe a square on a radius. Determine the area of the circle by counting squares in one half of the circle and multiplying the total by 2. Similarly determine the area of the square described on the radius.

Repeat the measurements for a circle of larger radius. Tabulate the results thus :

Area of circle.	Area of (radius)^2.	$\frac{\text{Area of circle}}{\text{Area of (radius)}^2}$.

The ratio, in the last column, should be a constant quantity, and be identical with that obtained in Expt. 4.

AREAS OF SURFACES OF SOLIDS.

Pyramid.—Let A (Fig. 24) be the apex, and BCDEF... the base, of any pyramid. It is required to find the total area of the sides ABC, ACD, ADE, ... etc. Draw AM perpendicular to BC, and calculate the area of the triangle ABC (from the product $\frac{1}{2}$AM × BC). Proceed similarly with each of the triangular faces, and finally add together the areas thus obtained.

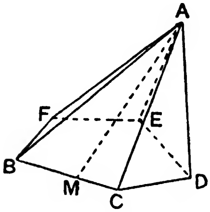

FIG. 24.—Lateral surface of a pyramid.

The line AM is called the *slant height* of the face ABC.

In the case of a *regular* pyramid all the triangles forming the lateral surface are equal in all respects; and, therefore, all the perpendiculars similar to AM are equal. Hence, the total surface is equal to $\frac{1}{2}$AM(BC + CD + ...); or

the lateral surface of a regular pyramid = $\frac{1}{2}$ × slant height × perimeter of base.

Curved surface of a cone.—Let DE (Fig. 25) be a small element of the circumference of the base BDEC of a cone with apex A. Let M be the middle point of DE. If DE is very small, it may be considered as approximately straight, and ADE is then a plane isosceles triangle with AM as its vertical height. The area of the triangle ADE is coincident, when DE is very small, with the curved surface of the cone between the lines AD and AE and the arc DE.

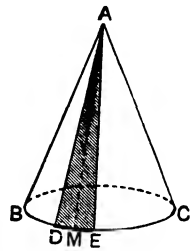

FIG. 25.—Curved surface of a cone.

Suppose the whole of the circular base to be split up into elements like DE, and the surface into triangles corresponding to ADE; then the *height* of each triangle will be equal to AM (which is the *slant height* of the cone). Therefore,

the sum of these triangles = $\frac{1}{2}$AM × sum of the bases.

When each of the bases is very small, the sum of the bases is equal to the total perimeter of the base of the cone. Hence,

the lateral surface of the cone = $\frac{1}{2}$AM × *perimeter of the base of the cone*
= $\frac{1}{2}$ **slant height × circumference of base.**

Sphere.—Fig. 26 represents a sphere surrounded by a cylinder, of which both the diameter and the height are equal to the

diameter of the sphere. It can be proved that the surface of the sphere is equal to the curved surface of the cylinder. The dotted lines indicate two horizontal cross-sections; and it can be shown that the area of the zone, of which *ab* is a section, cut off from the surface of the sphere, is equal to the area of the

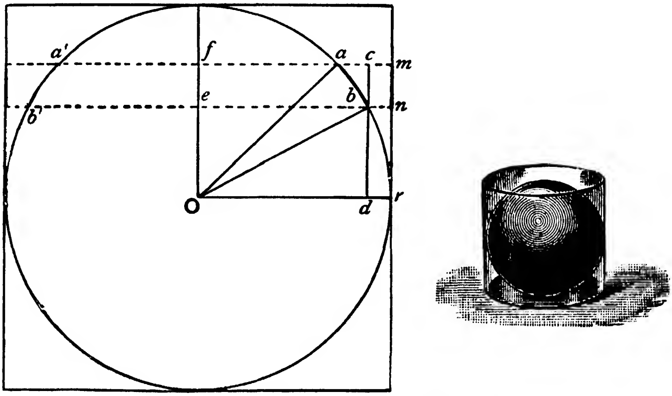

FIG. 26.—Sphere and circumscribing cylinder.

corresponding band *mn* cut off from the cylinder's surface. If the cross-sections are *very near* together, then

ab is approximately a straight line,

therefore, *ab* is approximately coincident with the tangent at *b*,

hence, the angle O*ba* is a right angle;

$$\left.\begin{array}{ll}\text{therefore,} & \text{the angle } Obe = \text{angle } abc. \\ \text{But,} & \text{angle } acb = \text{a right angle} = \text{angle } Oeb;\end{array}\right\}$$

wherefore, the triangles *abc* and O*eb* are equiangular, and

$$\frac{Ob}{eb} = \frac{ab}{bc},$$

or

$$Ob \,.\, bc = ab \,.\, eb.$$

Since, in the limit when the sections are *very* close, $fa = eb$,

$$\begin{aligned}\textit{area of zone of sphere} &= ab \,.\, (2\pi \,.\, eb)\\ &= bc \,.\, (2\pi \,.\, Ob)\\ &= mn \,.\, (2\pi \,.\, en)\\ &= \textit{area of band of cylinder.}\end{aligned}$$

This result holds good for all such cross-sections which may be drawn. Therefore, **the whole area of the sphere's surface is equal to that of the curved surface of the circumscribing cylinder.**

If r is the radius of the sphere, the area of the curved surface of the cylinder is equal to $2\pi r \times 2r$, or $4\pi r^2$. Hence,

the area of the surface of a sphere of radius $r = 4\pi r^2$.

That the surface area of a sphere is four times the area of a circle of the same diameter as the sphere can also be demonstrated by weighing hemispherical shells of sheet brass and circular discs of sheet brass of the same thickness and of the same diameter as the sphere.* If the hemispheres are fairly large, say three inches in diameter, two of them together weigh, within a fair degree of accuracy, four times as much as a disc. Here is an actual result:

Mass of disc, - - - - -	22·1 grams.
Mass of two hemispheres, - - -	88·8 grams.

EXERCISES ON CHAPTER II.

1. Calculate the areas of the following rectangles: 1·2 metres by 75 cm.; 1 25 ft. by 10 inches; 2·04 metres by 4·4 metres.

2. Find the length of the other side in the following rectangles: Area 3·54 sq. metres, length 59 cm.; 225 sq ft, length 5 yd.

3. Calculate the surface area of (i) a 4 cm. cube, and (ii) four separate cubic centimetres.

4. Describe a triangle with sides 5, 6, and 7 cm. long. Measure the angles with a protractor, and also calculate (without using squared paper) the area of the triangle.

5. The parallel sides of a trapezium are 54 ft. and 36 ft., and the perpendicular distance between them is 10 ft. Find the area.

6. If the metre is equivalent to 39·371 inches, and corresponds to the ten-millionth part of the distance from the pole of the earth to the equator, find the circumference of the earth in miles.

7. A well is 100 ft. deep. How many coils of rope, rolled in one layer, will be required to reach to the bottom, the roller on which they are wrapped being 8 inches in diameter?

8. How many trees at a consecutive distance of 8 yd. can be planted round the edge of a circular field of radius 100 yd.?

9. If the pressure of steam in a boiler is 100 lb. per sq. in., find the total pressure on a circular valve 3 inches diameter.

10. A circular bowling-green, of which the diameter is 125 ft., is surrounded on the outside by a gravel path $7\frac{1}{2}$ ft. wide. Find the area of the path in square yards.

11. Find the whole surface of a square pyramid, a side of the base being 12 ft., and the slant height 25 ft.

* Brass hemispheres and discs can be obtained from Messrs. Griffin & Sons, Ltd., Kingsway, London.

12. The diameter of the base of a cone is 1 ft., and the slant height is 8 in. Find the whole surface of the solid.

13. How many sq. in. of tin foil are required to cover a sphere 18 inches in diameter?

14. A round tower is 50 ft. in diameter and 120 ft. high. It is surmounted by a conical top 30 ft. high. Find the whole exterior surface.

15. Construct a square with side 2 in. in length upon paper divided into millimetre squares. Find from this figure the number of (*a*) square millimetres, (*b*) square centimetres in one square inch.

16. Construct a square with side 3 cm. upon paper divided into $\frac{1}{10}$ in. squares. Use the figure to calculate what fraction 1 sq. cm. is of 1 sq. in.

17. Construct a triangle upon squared paper. Determine its area by means of the formula area $= \frac{1}{2}(\text{base} \times \text{altitude})$, and also by counting the squares.

18. Draw upon squared paper two different parallelograms upon the same base and with the same altitude. Find the area of each by counting the squares.

CHAPTER III.

MEASUREMENT OF VOLUME.

Measurement of volume.—Each edge of a cubic foot (Fig. 27) is measured as a *length*. Each of the faces of the cube has an *area*, which can be obtained by multiplying together the lengths of two of the edges which meet at a corner. The size of the cube, or the amount of room it takes up, or the space it occupies, is called its **volume.**

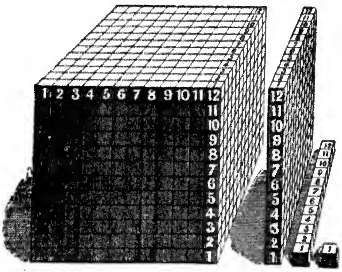

FIG. 27.—To explain why 1728 cubic inches make 1 cubic foot.

A cubic inch, such as is represented upon a small scale on the right hand of Fig. 27, has each edge 1 inch in length and each face 1 sq. inch in area. Twelve of these cubes placed in a row have a total length of 1 foot; and 12 of these rods laid one upon the other will build up a slab containing $12 \times 12 = 144$ cubic inches. Consider this layer of 144 cubic inches, or little cubes each edge of which is an inch, and each face of which is a square inch. Evidently there are twelve such layers in the whole cubic foot.

Consequently, in the whole cube we have $144 \times 12 = 1728$ little cubes the edges of which are one inch long and the faces of which are each one square inch. Or, one cubic foot contains 1728 cubic inches.

By reasoning in the same way the number of cubic feet required

to build up a cubic yard can be found. We may write, therefore,

1728 ($= 12 \times 12 \times 12$) cubic inches make 1 cubic foot.
27 ($= 3 \times 3 \times 3$) ,, feet ,, 1 ,, yard.

Units of volume and capacity.—The unit of volume in the metric system is also derived from the unit of length. A block built up with cubes representing cubic centimetres is shown in Fig. 28. This cube measures 10 centimetres each way, and its volume is therefore a cubic decimetre. There are 10 centimetres in a decimetre, so the edge of the decimetre cube is 10 centimetres in length; the area of one of its faces is $10 \times 10 = 100$ square centimetres; and its volume is $10 \times 10 \times 10 = 1000$ cubic centimetres.

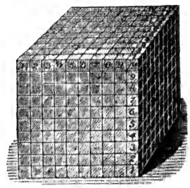

FIG. 28.—Representation on a reduced scale of a cubic decimetre divided into cubic centimetres.

If a *hollow* cube be made 1 decimetre long, 1 decimetre broad, and 1 decimetre deep, it will hold 1000 cubic centimetres of liquid. This capacity is called a **litre**. All liquids are measured in litres in countries where the metric system is adopted. Thus in France, wine, milk, and such liquids, are sold by litres instead of by pints. A litre is equal to about one and three-quarters English pints.

In the British system an arbitrary unit, the **gallon**, is the standard unit of capacity and volume. It is defined as **the volume occupied by 10 lb. of pure water at a temperature of 62° F.** Since water, like most other substances, expands when heated, it is necessary to state the temperature in defining the unit.

A **cubic foot** is equal in volume to about $6\frac{1}{4}$ gallons. One cubic foot of cold water weighs about 1000 oz., or 62·5 lb.

It is important to remember that

1 cubic yard* = 3^3 cubic feet, and
1 cubic foot = 12^3 cubic inches.

1 cubic metre = 10^3 cubic decimetres (or *litres*)
= 10^6 cubic centimetres;
1 cubic decimetre (or litre) = 10^3 cubic centimetres; and
1 cubic centimetre = 10^3 cubic millimetres.

* It is usual to write a small ^{3} on the top right-hand side of a number when the cube of a number has to be used; so that 8^3 means the cube of 8, or $8 \times 8 \times 8$. In a similar manner $4 \times 4 \times 4 \times 4$ is written 4^4, and $12 \times 12 \times 12 \times 12 \times 12$ is abbreviated to 12^5.

EXPT. 18.—**British and Metric volumes.** Measure the length, breadth, and thickness of a rectangular block of wood both in inches and in centimetres. From these measurements calculate the volume of the block, both in cubic inches and in cubic centimetres. Use the results to determine how many cubic centimetres are equivalent to 1 cubic inch.

Verify your result by calculation from the relation 1 inch = 2·54 centimetres.

DETERMINATION OF VOLUMES BY GEOMETRICAL RULES.

Cylinder.—The preceding paragraphs have shown that the volume of a rectangular solid can be determined by multiplying the area of the base by the height. Fig. 29 represents a cylinder cut into horizontal slabs, each 1 cm. thick. The number of cubic centimetres (*c.c.*) in the bottom slab is given by the number of sq. cm. in the area of the base; and the total number of c.c. in the whole cylinder is obtained by multiplying the number of c.c. in the bottom slab by the number of slabs. Hence, if the radius of the base of the cylinder be r, and if the height be h, then area of base $= \pi r^2$, and the **volume of the cylinder** $= \pi r^2 h$.

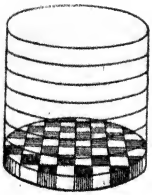
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FIG. 29.—Volume of a cylinder.

Parallelepiped.—Fig. 30 represents how two triangular blocks or prisms may be placed together, to form either a cube or a parallelepiped. The volume, the base, and the height are the same in either case. Hence, since the volume of a cube is equal to (area of base × height),

volume of parallelepiped = area of base × height.

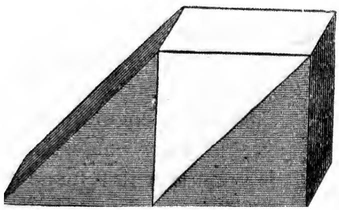


FIG. 30.—Cube and parallelepiped.

Pyramid.—A cube may be considered to be made up of six pyramids as indicated in Fig. 31. Suppose a cube of the same size be cut into six slabs as in Fig. 31. Then the volume of one of the pyramids is equal to the volume of one of the slabs, for each is one-sixth the volume of the cube. The volume of a slab is equal to the base multiplied by the height of the layer, which is one-third the height of a pyramid. Therefore, **the volume of a pyramid on a square base is equal to the base multiplied by one-third the height.** This rule applies to every pyramid. In other words,

a pyramid could be flattened down to a rectangular block having the same base and one-third the height.

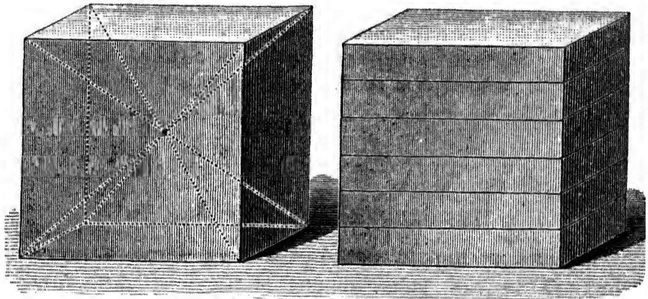


FIG. 31.—A cube formed by six pyramids and a cube formed by six slabs.

EXPT. 19.—**Volume of pyramid.** Find by means of the above rule the volume of a pyramid on a square base.

Cone.—A cone may be regarded as a pyramid with an infinite number of sides. The rule for obtaining its volume is, therefore, the same as for the pyramid. That is,

$$\begin{aligned}\textbf{volume of cone} &= \tfrac{1}{3} \times \textbf{area of base} \times \textbf{height}\\ &= \tfrac{1}{3} \times \pi r^2 \times h\\ &= \tfrac{1}{3}\pi r^2 h.\end{aligned}$$

EXPT. 20.—**Volume of cone.** Measure the height of a cone and also the diameter of the base, from which the radius can be determined. Using the numbers thus found and the rule just given, calculate the volume of the cone.

FIG. 32.—Sphere formed by numerous small pyramids. A few of the pyramids have been taken out.

Sphere.—The surface of a sphere can be divided into a large number of small triangles, each of which may be considered to form the base of a pyramid having a height equal to the radius of the sphere. Fig. 32 shows such a sphere with a few of the pyramids taken out. All the bases added together equal in area the surface of the sphere, and all the

pyramids added together equal the volume of the sphere. Therefore the volume of the sphere can be found by multiplying the surface by one-third the height of the pyramids, that is, by one-third the radius. As has been previously shown:

Surface of a sphere $= 4\pi r^2$ (p. 24).

But, **volume of a sphere** $=$ surface $\times \frac{1}{3}r$

" " $= 4\pi r^2 \times \frac{r}{3}$

" " $= \frac{4}{3}\pi r^3$

Use of Graduated Vessels.

Vessels used as metric measures of capacity.—A number of vessels used in the measurement of capacity are shown in Fig. 33. The **flask** marked 500 c.c. has a mark upon its neck, and when filled up to this mark the number shows how many cubic centimetres of liquid are in it. The tall jar (or **cylinder**) has a mark at every 10 cubic centimetres up to 1000 c.c.; the number of cubic centimetres of a liquid may thus be found by pouring the liquid into the jar and reading the scale division which is on the same level with the surface. Since the volume indicated by each scale division varies according to the diameter of the cylinder, it is necessary, before using any cylinder, to determine by means of the numbers attached to the scale the capacity represented by consecutive scale divisions.

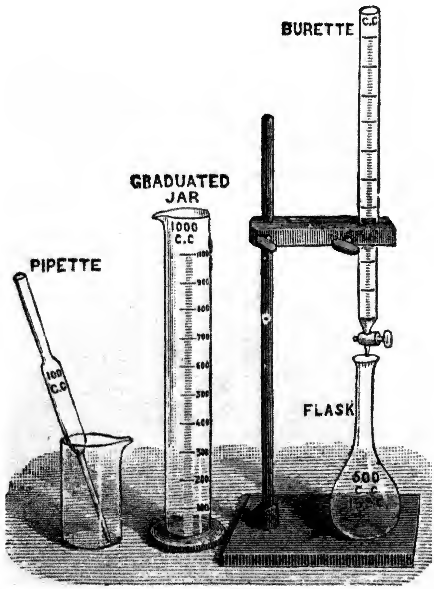


Fig. 33.—Graduated vessels for measurements of volume.

The graduated tube is a **burette**, used for measuring out exact quantities of liquid. At the bottom is a tap or clip for allowing liquid to flow out of the burette. Supposing that the burette is filled to the mark 10 c.c., and that 35 c.c. of liquid are required from it, the tap would be opened gradually, and when the liquid had fallen to the mark 45 c.c. it would be closed quickly. Burettes are graduated always from the top *downwards*. When a burette is first filled with the liquid, the tap or clip should be opened and the liquid allowed to run out until all air has been expelled from the exit tube: the instrument is then ready for use.

The narrow tube supported in a beaker is a **pipette**, by means of which small quantities of liquid may be conveyed from one vessel to another.

Fig. 34 indicates the correct method of reading the position of the liquid surface (or **meniscus**) in a measuring vessel. The surface is curved downwards (except when mercury is used), and it appears to have an upper and lower margin: the scale reading of the middle point of the lower margin should be obtained always. This is observed most satisfactorily if the meniscus is illuminated by holding a piece of white paper horizontally close to the glass vessel and two or three inches below the meniscus. The eye must be on a level with the meniscus: the sloping dotted lines show how incorrect readings are obtained if this precaution is not taken.

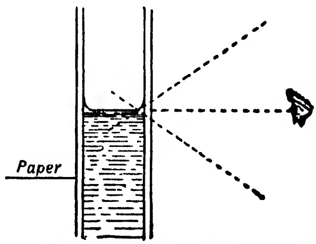


FIG. 34.—Observing the height of a liquid surface in a tube.

EXPT. 20A.—**Graduation of a cylinder.** Clean and dry the inside of a small glass cylinder (*e.g.* 100 c.c. capacity). Gum a narrow strip of mm. squared paper up the outside of the cylinder. Introduce into it, from a burette, 10 c.c. of distilled water. Mark on the paper strip the position of the meniscus. Add another 10 c.c., and mark the position. Repeat this until the cylinder is full. By means of the squared paper, the scale intervals thus obtained may be subdivided so as to read to smaller volumes than 10 c.c.

Measurement of volume by displacement.—Graduated vessels provide a convenient means of determining the volume of a solid by observing the volume of water it displaces.

EXPT. 21.—**Cube.** Find the volume of a metal cube by observing the volume of water which it displaces. Select a measuring cylinder into which the cube will just pass without friction, and pour in a considerable volume of water. Read the position of the meniscus. Gently lower, by means of thin cotton, the cube into the cylinder until it is immersed *completely*, and again read the position of the meniscus. The difference between the two readings indicates the volume of the cube.

Verify this result by calculating the volume of the cube from its dimensions. Enter your results thus:

First reading of meniscus.	Second reading of meniscus.	Volume, by displacement.	Average length of edge of cube.	Volume, by calculation.

EXPT. 22.—**Various geometrical solids.** By the method of Expt. 21, find the volume of (*a*) a cylinder, (*b*) a pyramid, (*c*) a cone, and (*d*) a sphere.

In each case verify your result by calculation from dimensions. If the solid is of a material which floats in water, it may be forced below the surface by means of a thin hat-pin.

EXPT. 23.—**Large solids.** If no measuring vessel of sufficient width to admit the solid is available, the following procedure may be adopted: Attach a narrow piece of gummed paper vertically to the side of a beaker; and, at a level which will be well above the top of the immersed solid, make a fine horizontal mark by cutting through the paper with a penknife. Dry the beaker, and fill the beaker up to the mark with water measured from a burette. Again dry the beaker, place the solid inside, and fill the beaker up to the mark as before. The volume of the solid is represented by the difference in the volumes of water required.

EXPT. 24.—**Lead shot.** Find the average volume of lead shot by the following method: About half-fill a burette with water, and read the meniscus. Select a given number (*e.g.* 20 or 30) of the shot, and introduce them into the burette—which should be held in a slanting position. Again read the meniscus. From the volume of the displaced water calculate the average volume of one shot.

EXERCISES ON CHAPTER III.

1. Find the number of litres in one cubic foot.

2. Find the number of gallons in 50 litres, also in one kilolitre.

3. The internal dimensions of a tank are: Length 10·5 m., width 2·25 m., depth 2·75 m. Find the number of litres of water in it when the tank is full.

4. The internal dimensions of a rectangular tank are 4 ft. 4 in., 2 ft. 8 in., and 1 ft. $1\frac{1}{2}$ in. Find its volume in cubic feet, the number of gallons it will hold when full, and the weight of the water.

5. Find the volume of a metal cylinder of which the diameter is 12 cm. and the height 20 cm.

6. How many cubic feet of water will be discharged from a pipe in 24 hours, if the diameter of the pipe is $3\frac{3}{20}$ inches, and if the velocity of the water is 2 feet per second?

7. Find the whole surface and the volume of a square pyramid, of which the side of the base is 10 ft. and the height 19·36 ft.

8. The interior of a building is in the form of a cylinder of 40 ft. radius and 20 ft. in height. A cone surmounts it, having a radius of 40 ft. and height 10 ft. How many cubic feet of air will the building contain?

9. A hollow spherical shell has an external diameter of 14 in., and the thickness of the shell is 1 inch. How many cubic inches of metal does it contain?

10. A circular disc of lead, 3 in. thick and 12 in. in diameter, is converted into small shot, each of radius 0·05 inch. How many shot does the disc make?

11. A sphere just slips into a cubical box, one edge of which measures 5 cm. How much space is left unoccupied?

12. A cone, a hemisphere, and a cylinder stand on the same base, and are of the same height. Find the ratio of their volumes.

13. An ink-bottle consists of a cube of glass, out of which has been cut an exactly hemispherical hole. What measurements and what calculations would be necessary to determine the volume of the glass?

How could the previous result be verified or checked, if you had a beaker of water, a gummed paper strip, and a burette or a measuring cylinder?

14. A porcelain weight of one pound breaks into two unequal fragments. How could you without using a spring balance or ordinary balance determine the weight of each fragment?

15. Measure a pint or fraction of a pint in cubic centimetres by means of a graduated vessel, and so find the number of c.c. equal to one pint.

16. A sphere just fits into a cylindrical box and is level with the top of the box 2 inches in diameter. Calculate the volume which the box will hold, and also the volume of the sphere. Find what fraction the volume of the sphere is of the internal volume of the box.

CHAPTER IV.

MASS, WEIGHT, AND DENSITY.

Mass. A precise definition of mass must be deferred to a later stage (p. 100). We here limit ourselves to the statement that though mass is not weight, **masses can be compared by weighing.** When, therefore, two bodies balance one another in a pair of scales, the quantities of substance or matter in them are equal. Equal masses thus measured may, however, have very unequal volumes; as, for instance, when a small piece of lead in one pan of a pair of scales balances a large pile of cotton wool in the other.

Mass is not weight.—When the mass of a pound is dropped from the hand it falls to the ground. When the same mass is hung upon the end of a coil of wire, the coil is made longer by the downward pull of the mass fixed to its end. In the instrument called a **spring balance**, the amount by which a steel spring is lengthened, as the result of such downward pull of masses attached to its end, is used to measure their **weights.** When a delicate balance of this kind is used, the *weight* of a small piece of iron hung on to the balance may be made to appear greater by holding a strong magnet beneath it. But, though the weight may seem to have increased, the quantity of matter is, of course, the same whether the magnet is under the iron or not.

It is known by common experience that unsupported things fall to the ground; a fact which is expressed also by saying that they are pulled downwards by the earth. Even when they are supported, as in the case of things upon a table, the earth attracts them just as much, only the table prevents them from falling, as they would do if there were no table there. Exactly why there is this tendency downwards need not at present be

described. The point from which the attraction may be regarded as being exerted is the centre of the earth, and **the weight of a body may be considered as a measure of the attractive influence of the earth upon it.**

The weight of a given mass may vary from place to place.—Bearing the definition of weight in mind, it will be clear that since a mass is farther away from the earth's centre when it is up in a balloon than when at the sea-level, the weight of this mass ought to be more at the sea-level, for it is there nearer the earth's centre than when up in a balloon. This is found to be the case, but actually to demonstrate the difference, the weight must be measured by a spring balance.

Similarly, because the earth is not a perfect sphere, but is flattened in north polar regions, points at the surface of the earth in the region of the tropics are at a greater distance from the centre than points further north. Consequently, on this account, and also because the earth's rotation causes bodies near the equator to have a greater tendency to be thrown off the earth than when they are in higher latitudes, a constant mass suspended from a spring balance increases in weight when carried toward the poles. As, however, equal masses balance one another when weighed in an ordinary pair of scales, the weight of an object may for all practical purposes be regarded at a given place as a measure of its mass.

Measurement of mass.—Just as in measuring lengths it is necessary to have a standard with which to compare, so in measuring mass there must also be a standard, or unit. In this country the standard of mass is the amount of matter in a piece of platinum which is deposited with the Board of Trade. This lump of platinum is called the **imperial standard pound,** and from it all other imperial measures of mass are ascertained.

A mass of 1 lb. avoirdupois is kept at a weights and measures office in every city, so as to test the lb. "weights" used by tradesmen, and to see whether they really have the mass of 1 lb. or are too light.

The kilogram and gram.—The standard of mass adopted in France, and in other countries where the metric system is used, is called the **kilogram.** The kilogram is the amount of matter in

a lump of platinum which is kept in safety at Sèvres. It is heavier than the British pound; indeed, it is equal to about two and one-fifth of these pounds. It is interesting to know how the mass of a kilogram was derived. The mass of the piece of platinum was made equal to that of one thousand cubic centimetres of water, that is, of a litre of water, at a particular temperature. The names used for the divisions, etc., of the kilogram are as follows:

Metric Measurement of Masses.

10 milligrams	= 1 centigram.	10 grams	= 1 dekagram.
10 centigrams	= 1 decigram.	10 dekagrams	= 1 hektogram.
10 decigrams	= 1 **gram.**	10 hektograms	= 1 **kilogram.**

The relative values of some of the British and metric measures of mass are:

Metric to British.		British to Metric.	
1 gram	= 15·4323 grains.	1 oz. Av. =	28·35 grams.
1 kilogram =	2·2046 lb. Av.	1 lb. Av. =	453·6 grams.

The Balance and its Use.

The principle of the balance.—The principle of the balance may be explained by means of a few observations with the simple arrangement of apparatus shown in Fig. 35. The apparatus consists of a boxwood half-metre scale AB; through the middle division a hole is bored (nearer to the divided edge, and not at the centre). The scale is pivoted, with its divided edge *upwards*, by means of a piece of stout iron wire bent at right angles and clamped between the boss and the upright of a retort-stand; a straight knitting-needle may be used instead of the bent iron wire. Weights are hooked on to loops of cotton thread by means of thin copper wire bent so as to form a hook and twisted round the shank of the weight.

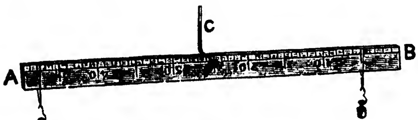


Fig. 35.—Principle of the balance.

Expt. 25.—**Adjustment.** If the scale is not in equilibrium in a horizontal position, suspend a *small* mass from the lighter side, adjusting the position of the mass so that the scale is balanced

correctly; or, a *rider* cut from sheet lead will serve the same purpose. This mass, or rider, must remain in the same position on the scale throughout the experiment.

EXPT. 26.—**Equal distances.** Hang a known mass (taken from a box of *weights*) at any convenient distance on one side of the pivot, or **fulcrum**, as it is termed, and balance it with a mass of the same amount on the other side. *The distance of the masses from the fulcrum will be found the same in each case.*

EXPT. 27.—**Unequal distances.** Suspend a mass of 50 grams from a point mid-way between the fulcrum and the left-hand end of the scale, and retain it in this position during the experiment. Suspend some other mass, *e.g.* 100 grams, from the right-hand side of the scale, adjust its position until the scale is in equilibrium, and note its distance from the fulcrum. Repeat this several times, using different masses in each case, and suspending more than one mass from the cotton loop if necessary. Show that, in each case,

$$\frac{\textit{Mass on}}{\textit{one side}} \times \frac{\textit{Distance}}{\textit{from fulcrum}} = \frac{\textit{Mass on}}{\textit{other side}} \times \frac{\textit{Distance}}{\textit{from fulcrum.}}$$

From this it is evident that **the masses are equal only when the points of support are equidistant from the fulcrum.** The ordinary **balance** may be described as a beam supported at its middle point and with pans supported at points equidistant from the fulcrum, and used for the purpose of counterpoising known masses with an unknown mass.

It must be remembered carefully that in taking a so-called *weighing* of a body by means of a balance, the known mass which has the same weight as the unknown mass only is found; exactly the same result would be obtained if the balance were used at any other locality on the earth's surface, though it is known that the weight varies according to the locality. The *weight* of a mass can only be determined by means of a spring balance; in this appliance the extension of the spring is an exact measure of the weight of the mass attached to the spring.

The balance.—The balance is really another form of the supported lath in Expt. 26. All the parts are very carefully made, and every means is taken to have very delicate supports and accurate adjustments. Fig. 36 shows a simple form of balance which is suitable for easy experiments, such as are described in this book, which can all be done accurately by its means. Instead of the wooden lath used in Expt. 26

there is a brass beam supported at its middle line by a knife-edge of hard steel, which, when the balance is in use, rests on a true surface of similar steel. The hooks to which the pans

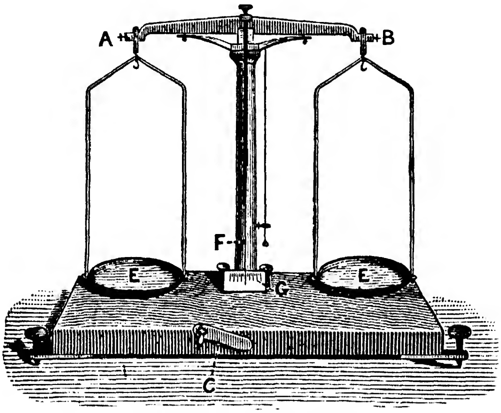


FIG. 36.—A physical balance.

are attached are provided similarly with a V-shaped depression of hard steel, which also, when the balance is in use, rests upon knife-edges on the upper parts of the beam. To the middle of the beam is attached a pointer, the end of which moves over an ivory scale fixed at the bottom of the upright which carries the beam. When not in use the beam and hooks are lifted off the knife-edges by moving the handle C.

EXPT. 28.—**Parts of a balance.** Uncover the balance and identify the different parts by reference to Fig. 36. Raise the beam, AB, of the balance, off the supports by turning the handle C. Notice whether the pointer F swings equally on both sides of the middle of the scale G; if it does the balance is ready for use; but if not, let down the beam and turn the small screw at A or B, then try again. Repeat this adjustment until the swings to right and left are equal.

Set of metric weights.—Sets of metric weights are marked usually in grams and milligrams, and a complete set will include the following:

(i) (Brass weights), 100 gm., 50 gm., 20 gm., 20 gm., 10 gm., 5 gm., 2 gm., 2 gm., 1 gm.

(ii) (Aluminium weights),

500 mgm. (= 0·5 gm.), 50 mgm. (= 0·05 gm.),
200 mgm. (= 0·2 gm.), 20 mgm. (= 0·02 gm.),
200 mgm. (= 0·2 gm.), 20 mgm. (= 0·02 gm.),
100 mgm. (= 0·1 gm.), 10 mgm. (= 0·01 gm.).

By means of this set any weight which is a multiple of 10 mgm. and does not exceed 211 gm. may be obtained. The box contains a pair of **forceps**, which must be used always when weights are removed from, or replaced in, the box.

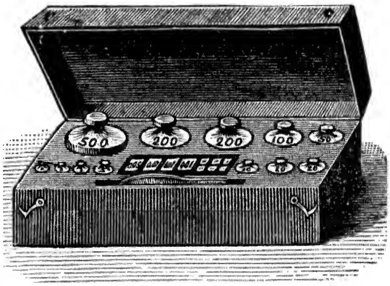


FIG. 37.—Metric weights.

The weights are used in the following manner: the object to be weighed is placed in the left pan, and such weights as are estimated as sufficient to counterbalance the object are placed in the right pan.

Suppose that 20 + 10 gm. are used, and that, on *slightly* releasing the beam, the pointer moves towards the object: these weights are evidently too great. The 5 gm. weight is substituted for the 10 gm. weight; if this is now too small, a 2 gm. weight is added; and, if this is still too small, the second 2 gm. weight is added. The subsequent steps are as follows:

20 + 5 + 2 + 2, too great.
20 + 5 + 2 + 1, too small.
20 + 5 + 2 + 1 + 0·5, too small.
20 + 5 + 2 + 1 + 0·5 + 0·2, too great.
20 + 5 + 2 + 1 + 0·5 + 0·1, too great.
20 + 5 + 2 + 1 + 0·5 + 0·05, approximately correct.

The weight is thus found to be 28·55 gm.

Special precautions in weighing.—1. See, by means of the plummet, that the balance is level.

2. See that the stirrups are not displaced; see also that the pans are dry and clean.

3. Lower the arrestment to see whether the pointer swings equally on both sides of the middle point of the scale. If necessary, adjust the balance by means of the screw-nuts at either end of the beam.

4. Do not stop the swinging of the balance with a jerk, but stop it gently when the pointer is nearly at its central position.

5. Place the body to be weighed on the left-hand, and the weights on the right-hand, pan.

6. Lower the arrestment before adding or removing any weight.

7. Manipulate the arrestment with the left hand, and convey weights with the right hand. On no account touch weights with the hand, but always use the forceps.

8. Do not weigh a body when hot: the heat causes air currents, which affect the weighing.

9. Close the balance case when observing the swinging of the pointer, and keep the case closed when the balance is not in use.

10. Always replace each weight in its proper compartment in the box.

Sensibility of a balance.—In determining the sufficiency of the weights on the balance pan it is unnecessary to wait until the beam ceases to swing; it is sufficient to observe the extreme positions on the scale to which the pointer travels, and from these the final position of rest can be calculated. The scale divisions should be numbered, as shown in Fig. 38. Suppose that consecutive turning points are 8 and 14; the true position of rest will not be far from $\frac{8+14}{2} = 11$.*

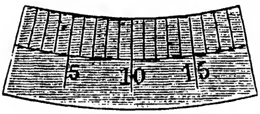


FIG. 38.—Scale divisions of a balance.

The **sensibility** of a balance may be defined as *the change in the position of rest of the pointer due to a given change of weights*; it is expressed usually as the change of weights necessary to alter the position of rest through one scale division. The sensibility of the simple balances generally used in elementary experiments varies from 2 to 4 milligrams per scale division; it varies slightly, according to the load on the balance, becoming less with heavy loads; and it is usually a maximum with a load of 20-30 grams.

* Strictly speaking, we ought to allow for the fact that the extent of swing is gradually becoming less, owing to friction and air resistance. We should take, therefore, two consecutive readings on *one* side and the intervening reading on the other side. Thus, suppose the readings are 8 (left), 14 (right), 8·4 (left); the average of the 1st and 3rd gives the reading on the left which would have been obtained if the vigour of the swing at the moment of taking the right turning point remained unaltered. Hence, the true turning point on the left is $\frac{8+8{\cdot}4}{2} = 8{\cdot}2$; and the true position of rest is $\frac{8{\cdot}2+14}{2} = 11{\cdot}1$.

Example. Load, 20 gm. in each pan.

$$\text{Resting point} = \frac{8+14}{2} = 11.$$

Resting point, with 20·01 gm. on right pan, $= \frac{5\cdot5+9}{2} = 7\cdot25$.

Change of resting point, due to 10 mgm.,

$$= 11 - 7\cdot25 = 3\cdot75 \text{ divisions};$$

$$\therefore \text{ sensibility} = \frac{10}{3\cdot75} = 2\cdot7 \text{ mgm. per scale division.}$$

EXPT. 29.—**Sensibility with and without load.** Find the sensibility of a balance, with (i) no load, (ii) a load of 20 grams in each pan, (iii) a load of 50 grams in each pan.

When recording the swings, close the front of the balance case, and allow the pointer to swing to and fro several times before taking the readings.

Weighing by vibrations.—In weighing an object it is not necessary to modify the known weights until the resting point is the same as that obtained when the balance is unloaded, providing that the sensibility of the balance is determined previously.

Example. Sensibility of balance = 3 mgm. per scale div.

Resting point, with no load, = 11·5

Resting point, with object on left, and 21·55 gm. on right, = 9.

The weight 21·55 gm. is evidently *too great* by an amount equivalent to 11·5 − 9 = 2·5 div.

But, 1 div. is equivalent to 3 mgm.,

or, 2·5 div. are „ „ 7·5 „

Hence, true weight = 21·55 − 0·0075 = 21·5425 gm.

If the sensibility is not known with sufficient certainty, the following procedure may be adopted:

Example. Resting point, with no load, = 11·5.

Resting point, with 21·55 gm. on left, = 9.

„ „ 21·54 „ „ = 12·4.

Hence, 10 mgm. are equivalent to 3·4 scale div.

The weight 21·54 gm. is evidently *too small* by an amount equivalent to (12·4 − 11·5) = 0·9 scale div.

But 0·9 scale div. is equivalent to $\left(10 \times \frac{0\cdot9}{3\cdot4}\right) = 2\cdot6$ mgm.;

$$\therefore \text{ true weight} = 21\cdot54 + 0\cdot0026 = 21\cdot5426 \text{ gm.}$$

EXPT. 30.—**Vibration method.** Find, by the method of vibrations, the weight, in grams, of a 1 oz. weight. Repeat the experiment, using a ½ oz. weight and a 2 oz. weight.

EXPT. 31.—**Graduation of a measuring flask.** Select a glass flask of such a size that when 50 c.c. (or 100 c.c.) of cold water are poured into it the meniscus is within the neck of the flask. Carefully dry the flask and weigh it. Add 50 gm. to the weights already on one pan of the balance, and pour cold water into the flask until the weights are *nearly* counterpoised. Add more water, drop by drop, from a narrow tube until the weights are counterpoised exactly. If by chance too much water is added, the excess may be removed by dipping momentarily a narrow strip of blotting paper into the water. When the adjustment is complete, remove the flask and make a horizontal file-mark with a wetted triangular file on the neck, so as to indicate exactly the position occupied by the lower edge of the meniscus. The flask has now been graduated to *contain* exactly 50 c.c.

If the flask were now emptied it would be found that rather less than 50 c.c. of water were transferred from the flask, the remainder of the water being inside the flask and wetting its surface. If the flask is to be graduated so as to *pour out* exactly 50 c.c. of water, the inner surface should be wetted previously to being counterpoised. The procedure should be as follows: fill the flask with water, pour out the contents, and hold the flask inverted for a definite period, *e.g.* 5 seconds. Place the flask on the balance pan and weigh it. Add 50 grams to this weight, and pour water into the flask until the weights are counterpoised. Mark the position of the meniscus. If now the flask is emptied and held inverted for the same period it may be assumed that exactly 50 c.c. of water have been transferred.

Weighing by substitution.—A balance may be, occasionally, totally out of adjustment; thus, the arms may be of unequal length, or the pans of unequal weight. In such a case the following procedure may be adopted: Place some shot or sand in a box or dish, on the left-hand pan, using sufficient to have a greater weight than that of the object to be weighed. Place known weights (w_1) on the right-hand pan until the balance is counterpoised, and obtain the resting point. Remove the weights, place the object on the right-hand pan, and add weights (w_2) until the balance is counterpoised again. Obtain the resting point, and calculate the correction to w_2 necessary to make the two resting points coincide. The weight of the object is then equal to $(w_1 - w_2)$ grams.

EXPT. 32.—**Substitution method.** Find, by the method of substitution, the weight in grams of a 1 oz. weight.

Calibration.

It is found frequently that graduated vessels, such as the pipette, burette and cylinder, are not sufficiently correct for use in accurate experimental work. The process of determining the error, if any, in the graduations is termed **calibration.** The following experiments illustrate the principle of an approximate method of calibrating the above types of measuring vessel, and provide also exercises in weighing.

Expt. 32a.—**Calibration of a pipette.** Use distilled water which has been standing in the room for several hours. Note its temperature. Carefully clean the inside of a 10 c.c. pipette.* Clean and dry a small wide-necked flask. Cover the neck with a small watch-glass, and weigh the flask with the cover. Fill the pipette with the distilled water, and adjust the meniscus to the graduation mark. Transfer the water to the flask by holding the point of the pipette against the inside of the neck; when empty, blow down it once. Cover the flask with the watch-glass, and again weigh it. The increase in weight expressed in grams gives in c.c. the capacity of the pipette; when the temperature of the water is not higher than 15° C., the assumption that 1 gm. of water occupies 1 c.c. introduces an error of less than 1 in 1000.

If a considerable error is found in the graduation, fix a narrow strip of gummed paper along the upper stem of the pipette, and find by trial the position which the meniscus must occupy in order that the pipette may deliver exactly 10 c.c. Mark this position permanently by means of a scratch made with a file.

Expt. 32b.—**Calibration of a burette.** Clean the inside of the burette, and fix it vertically in a stand. Fill it with distilled water at the temperature of the room. Run some of the water out at the tap or jet, so as to remove all air bubbles. Carefully run more water out until the meniscus coincides with the zero of the scale; and remove any drop adhering to the end of the jet. Weigh a flask together with a small watch-glass to serve as a cover. Run water from the burette into the flask until the meniscus is at the 10 c.c. mark; and touch the inside of the flask with the end of the jet. Replace the watch-glass, and again weigh the flask. Run water into the flask until the meniscus is at the 20 c.c. mark, and again weigh. Continue this process of

* The best method of removing grease, and other matter, from a glass vessel is to rinse it thoroughly with strong sulphuric acid with a little potassium bichromate added. If possible, leave this liquid in the vessel for an hour; then rinse several times with tap water, and finally with distilled water.

weighing successive quantities, of 10 c.c. each, until the 50 c.c. mark is reached. The total increase in weight (in gms.) of the flask at each weighing gives the approximate true volume delivered between the zero and each of the graduated marks used.

Plot on squared paper the total apparent volume of water taken from the burette, and the total increase in weight of the flask.

EXPT. 32C.—**Calibration of a measuring cylinder.** Clean and dry a measuring cylinder. Fix vertically in a stand a burette which has been previously calibrated, and fill it with water. Transfer an observed volume of water from the burette into the cylinder: 10–50 c.c. may be used, according to the capacity of the cylinder. Note the reading of the cylinder scale. Transfer a second volume, as before, and again read the cylinder scale. Repeat this process until the cylinder is full. Plot on squared paper the total volume introduced and the scale reading of the cylinder.

Cross-section of a narrow tube.—The following experiment represents the method usually adopted for finding the average cross-section of a narrow tube, and for finding the variation in cross-section of a considerable length of the tube.

EXPT. 32D.—**Calibration of a capillary tube.** Select a length of thermometer-tubing with circular bore (1–2 mm. diameter) and about 1 metre long. If necessary, chemically clean and dry the inside of the tube. Make a file-mark about 1 cm. from one end. Attach to one end a long piece of narrow rubber tubing; and, by this means, suck up into the tube a column (about 10 cm. long) of *pure* mercury. Note the temperature of the mercury. Lay the tube horizontally on an accurately divided metre-scale, so that the file-mark coincides with the zero of the scale. By tilting the tube adjust the position of the mercury thread so that the centre of the thread is approximately over the 5 cm. mark of the scale, and carefully measure the length of the thread. Move the thread along the tube until the centre is over the 15 cm. mark and again measure the length. Repeat this measurement with the centre of the thread over the 25 cm., 35 cm., etc. divisions of the scale, until the end of the tube is reached. Weigh accurately a porcelain crucible. Transfer the mercury thread into the crucible and again weigh. Having given that the density of mercury at the temperature of the room is approximately 13·56 gm. per c.c., calculate the average cross-section of the tube for each position of the thread in the tube.

Plot on squared paper the positions of the centre of the thread and the average cross-section in each position.

Example of Calibration of a Capillary Tube.

Temp. of mercury, 16° C.
Weight of crucible, 7·484 gm.
" " +mercury, 9·246 gm.
" mercury alone, 1·762 gm.

Cross-section of tube = Weight ÷ (Length × Density)
= 1·762 ÷ (Length × 13·56)
= 0·130 ÷ Length.

Position of centre of thread.	Length of thread.	Cross-section.
5	10·00 cm.	0·0130 cm.
15	9·95 "	0·01306 "
25	9·77 "	0·0132 "
35	9·90 "	0·01313 "
45	10·06 "	0·01292 "
55	10·02 "	0·01297 "
65	9·92 "	0·0131 "
75	9·92 "	0·0131 "
85	9·90 "	0·01313 "

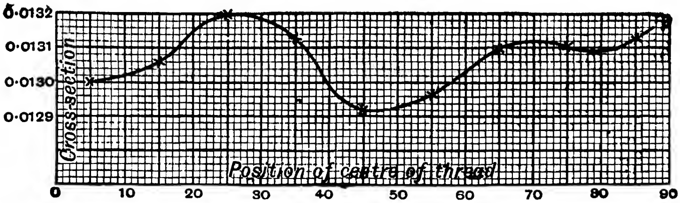


FIG. 38A.—Calibration curve of a capillary tube.

The principle illustrated by the foregoing experiment is used in the calibration of a thermometer tube. A thread of mercury is detached from the mercury in the bulb of the thermometer, and its length is measured, in scale-divisions, at different parts of the stem. If the bore of the tube has a uniform section throughout its length, the thread of mercury will obviously extend along the same number of scale-divisions in any part of the stem. Usually, however, the mercury thread is found to have slightly different lengths in different parts of the stem, thus introducing a source of error in the readings of the thermometer.

In the case of mercurial thermometers required for the accurate determination of temperature, the instruments are calibrated in much the same way as is illustrated by Expt. 32D, and a correction curve is constructed to show the error introduced at various parts of the stem.

The spring balance.—The spring balance has been referred to already as an appliance for determining the *weight* of a body—the weight of a body being the force with which it is attracted towards the earth's centre. A spring balance (Fig. 39) consists of a spring with a hook attached to the lower end. An index is attached to the spring and travels over a graduated scale. The principle of the spring balance may be learnt by means of observations taken with a **spring dynamometer**, to which is attached an arbitrary scale (*i.e.* a scale *not* graduated in grams weight).

FIG. 39.—A spring balance.

An efficient form of dynamometer may be obtained by suspending a spiral spring, made from thin piano wire, in a vertical position; a millimetre scale fixed to the side of the spring serves to measure the elongations. A scale pan and index are attached to the lower end of the spring. A satisfactory spring may be made by winding the thin wire in a close and uniform spiral on a round metal rod held in the chuck of a lathe; the chuck should be rotated slowly by hand, and the wire fed out under uniform and considerable tension.

FIG. 40.—Rintoul's spring dynamometer.

EXPT. 33.—**Graduating a dynamometer.** Fix a dynamometer (Fig. 40), with a millimetre scale attached in a vertical position, and attach to it a scale pan. Take the reading of the index. Add a known weight, say 10 or 20 gm., to the pan, and again read the index. Add more weights, 10 to 20 gm. at a time, and take the index reading for each change of weight. Continue the observations until the spring is extended to nearly twice its original length. Tabulate the readings in columns, under the headings *Weight* and *Index Reading*. Now gradually remove the weights, 10 or 20 gm. at a time, and take the index reading at each stage.

Plot these readings on squared paper, taking the scale of *weights* as abscissae and the scale of *index-readings* as ordinates (Fig. 41). From the form of the *graph* thus obtained, state your inference as to

the relationship between the weight applied and the resulting extension of the spring.

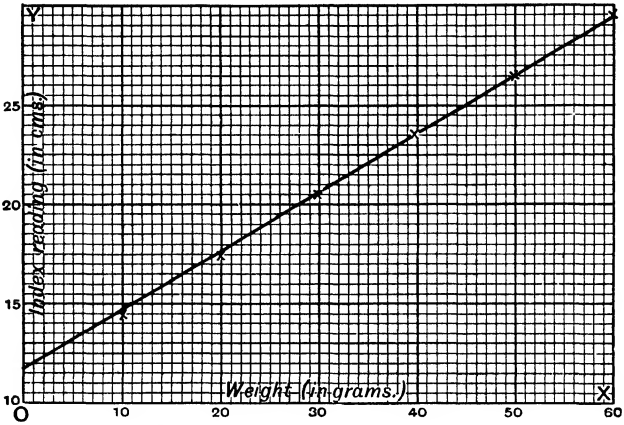


FIG. 41.—Extension of a spring dynamometer with different masses.

The following readings were obtained by means of a spiral (11 cm. long and 1 cm. diameter) :

Weight (in gm.).	Index Reading	
	(With increasing weight.)	(With diminishing weight.)
0	11·75	11·75
10	14·60	14·68
20	17·50	17·50
30	20·50	20·58
40	23·52	23·52
50	26·50	26·50
60	29·50	

The results of the foregoing experiment show that, in the case of a spring, **the amount of elongation is proportional to the load.** This relationship holds good generally for the stretching of wires, rods, or similar test-pieces, provided that the limits of elasticity are not exceeded; that is, they should return to the original length when the load is removed. Using the word **strain** to signify the deformation or distortion produced by a load, and **stress** for the internal forces, which are equal to the distorting

force, tending to bring the body back to its original form, the relationship is expressed in the statement: **The strain is proportional to the stress (Hooke's Law).** This law applies to any deformation of an elastic body, such as bending or twisting, as well as to stretching.

EXPT. 34.—**Use of graph.** Attach an object of unknown weight to the dynamometer or spring. Note the index-reading, and deduce the weight of the object by means of the graph obtained in Expt. 33.

Verify the result by weighing the object on a balance.

The rubber dynamometer.—Fig. 42 represents a dynamometer which consists of a rubber cord suspended by a hook from the top edge of a scale. A pan, attached to the lower end of the rubber, serves to carry weights. Two needles, A and B, are fixed horizontally through the rubber, and adjusted so that their points traverse the fine divisions of the scale.

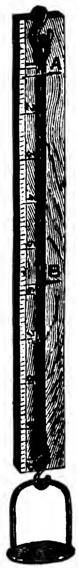


FIG. 42.—Rubber dynamometer.

EXPT. 35.—**Rubber cord and spiral spring.** Clamp a ½-metre wooden scale in a vertical position, and suspend the rubber cord, with pan attached, from the upper end. Take the scale-reading of the two needles, and from these obtain the length of rubber between the needles. Place a 20-gram weight on the pan, and again take the reading of each needle. Take a series of readings, adding 20 grams each time to the weight, until the rubber is half as long again as it was without any weight in the pan. Now remove the weights carefully, 20 grams at a time, and read the corresponding length of rubber with each weight. When altering the weights, steady the pan with the left hand. Tabulate your readings in column, and plot on squared paper. State any inference which may be drawn from the curve obtained, when it is compared with the curve obtained when using a spiral spring dynamometer.

Density.

The meaning of density.—Different solids of the same size or volume may have different masses. Suppose, for instance, determinations are made of the mass of a cubic centimetre of wood, lead, cork, and marble, one after the other. The lead will be found to have the greatest mass, or to be heaviest, the marble will come next, and then will follow the wood and cork in this order.

By filling two bottles of the same size with different liquids, it can also be shown that equal volumes of different liquids may have different masses; and when different gases are compared in the same way, equal volumes of these, too, are found to have different masses.

A pound of feathers or cotton-wool has exactly the same mass as a pound of lead, though both the feathers and cotton-wool take up much more room, or have a larger volume, than the piece of lead. The matter in the lead may be regarded as packed more closely than in the cotton-wool, which accounts for it taking up less room, or it may be said that lead is **denser** than either cotton-wool or feathers.

If the relative size of a thing is small while its mass is great, then it is called a **dense** thing, or is said to have a **high density**. If, on the other hand, the relative size of a thing is great and its mass small, it is said to have a **low density**. Lead is a substance with a high density, because a small piece of it has a large mass Pith and cork, on the contrary, have a low density, because a large lump of either of them has a small mass.

A cubic centimetre of marble has a mass of 2·5 gm.; the same volume of iron has a mass of 7·5 gm.; of gold, 19·3 gm.; of mercury, 13·6 gm.; and each of these numbers represents the density of the particular substance to which it refers, but the units of volume and mass must be expressed. In fact, **density is the mass of a unit volume of a substance.** It follows from this definition that if the volume of a body is multiplied by its density, its mass is obtained.

$$\textbf{Volume} \times \textbf{density} = \textbf{mass},$$

$$\textbf{or, } \text{density} = \frac{\text{mass}}{\text{volume}}.$$

In using this relation between the volume and mass, care must be taken to use the proper units. In all scientific work it is customary to adopt the cubic centimetre and gram as the units of volume and mass respectively.

The ratio of the mass of *any volume* of a substance to the mass of the same volume of water is termed the **relative density** of the substance, or, as it is called frequently, the **specific gravity** (p. 66).

MEASUREMENT OF DENSITY.

EXPT. 36.—**Regular solids.** Select a cube, a cylinder, a cone, and a sphere, of different materials. Calculate the volume of each solid from its dimensions, and also weigh each of them. Calculate the density of each material. Tabulate your results thus :

Material and shape.	Dimensions.	Volume.	Mass	Density.
Brass (cylinder)	Radius = Length =	$(\pi r^2 \times l) =$		

In expressing the density of any material it is essential to state the units in which the measurements are taken. Thus, the mass of 1 c.c. of water, at 4° C., is 1 gram, and that of 1 cubic foot of water is 62·5 lb. Hence, the density of water is **1 gm. per c.c.** (in the metric system) or **62·5 lb. per c. ft.** (in the British system). It is *not* sufficient to say that the density of water is either *1*, or that it is *62·5*; either statement is incomplete, and it is essential to state the units which have been used.

On the other hand, when it is desired to express the *relative* density (p. 66) of a material—*i.e.* the number of times that the material is denser, or less dense, than a standard material—a simple numerical result is correct. Thus, the *density* of copper is 8·8 *gm. per c.c.*, but the *relative density* of copper is 8·8.

EXPT. 37.—**Irregular solids.** Select a glass stopper, and carefully examine it to see that it is free from internal cavities. Weigh it, and determine its volume by displacement of water. Calculate its density.

Carry out similar observations with other irregular solids.

EXPT. 38.—**Liquids.** Weigh a clean, dry beaker. Partly fill a pipette (20 c.c. or 25 c.c. capacity) with the liquid ; rinse it round the

inside of the pipette, and throw the liquid away. Completely fill the pipette with the liquid up to the mark on the stem, and transfer the contents to the beaker. Weigh the beaker and the liquid contained in it. From these weighings, and from the known volume of the pipette, calculate the density of the liquid.

EXERCISES ON CHAPTER IV.

1. The density of lead is 11·34 gm. per c.c. What is its density in lb. per cubic foot?

2. Find the length of a lead rod 1 cm. in diameter, and weighing 1 kilogram (density of lead 11·34 gm. per c.c.).

3. A block of mahogany 4 in. long, $1\frac{1}{2}$ in. broad, and $\frac{7}{16}$ in. thick weighs 30·35 gm. Find its density in gm. per c.c.

4. Find the diameter of a cylindrical kilogram weight made of brass (density, 8·4 gm. per c.c.), its height being 7·5 cm.

5. The radii of two spheres are 2 cm. and 3 cm., and their masses are 200 gm. and 250 gm. respectively. Compare their densities.

6. How many grams of glycerin of density 1·26 gm. per c.c. can be put into a bottle which will hold 100 gm. of sulphuric acid (density 1·84 gm. per c.c.)?

7. A length of glass tubing, having a narrow circular bore, weighs 14·65 gm. A thread of mercury, 10·5 cm. long, is drawn into the tube, and the tube is now found to weigh 19·13 gm. If the density of mercury is 13·6 gm. per c.c., calculate the diameter of the bore of the tube.

8. Define mass, volume, and density, and state the relation that exists between them.

Suppose you were given two irregular pieces of metal, one of which was gold and the other gilded brass. How would you find out, by a physical method, which piece was gold?

9. A number of nails are driven into a rough piece of wood, one cubic centimetre of which weighs 0·5 gm. It is required to find the weight of the nails without pulling them out. How could this be done by experiment?

10. By what experiments can you find (*a*) whether the arms of a balance are of the same length; (*b*) whether a balance is sensitive; (*c*) the weight of substances by means of a balance having arms of unequal length?

11. If you were provided with a 1 lb. weight and a lath balanced on a pivot, how could you determine the weight of a small bag of nails?

12. Describe the units of length, volume, and mass on the British and on the metric systems.

13. The density of a body is 7·8 on the scientific system; what is it in pounds per cubic foot? (1 lb. = 453 grams; 1 inch = 2·54 cm.)

14. A coil of brass wire consists of 55 turns of average diameter 14 inches, and the diameter of the wire is 0·017 inch. Find the weight of the coil in grams if the density of the material is 8·58 grams per cubic centimetre. (1 inch = 2·54 cm.; $\pi = 3 \cdot 14$.)

PART II.

HYDROSTATICS AND MECHANICS.

CHAPTER V.

MATTER: ITS THREE STATES AND GENERAL PROPERTIES.

In the preceding chapters the fundamental measurements of length, area, volume, time, and mass have been explained. A knowledge of these principles and methods of measurement is essential in the study of physics; but before proceeding to the various parts of the science in which this knowledge is applied, it is desirable to describe the states and some of the general characteristics of matter in order that exact meanings may be attached to terms expressing particular conditions and properties.

The three states of matter.—At the present moment, the question *what is matter?* is almost unanswerable. We know much about its intimate structure and properties, but still remain ignorant of its exact nature. A definition of the term is desirable; and, of the many which have been suggested, it is perhaps sufficient to state that **matter is that which can occupy space.**

Matter is recognised generally in three different states or conditions, viz.: **solid, liquid,** and **gaseous.** The distinctive properties of these states of matter may be expressed in the following terms:

(i) *A solid body does not alter its size or shape readily*: it maintains its original volume and shape unless subjected to considerable force.

(ii) A *liquid readily alters its shape and adapts itself to the shape of the vessel containing it*; but it maintains its original volume unless subjected to extreme force.

(iii) A *gas readily adapts itself both to the shape and size of the vessel containing it*; a small volume of gas introduced into a large empty vessel expands and distributes itself uniformly throughout the enclosed space.

Several substances which appear to be solid are found, on closer examination, to exhibit properties characteristic of liquids. Thus, in the three series (i) *steel*, *lead*, and *jelly*, (ii) *sealing-wax*, *pitch*, *treacle*, *water*, and *ether*, (iii) *air* and *hydrogen*, the first three are solids, the following five are liquids, and the last two are gases. A rod of cold sealing-wax supported from its ends gradually becomes bent by its own weight. A lump of cold pitch gradually spreads over a horizontal surface, and gradually will run down an inclined surface; also a fragment of metal will sink slowly through a slab of pitch. On the other hand, *jelly* exhibits the properties of a solid since a small solid object does not sink when placed on the surface of jelly, nor does jelly spread over a surface supporting it.

Change of state.—The same matter can exist in either of the three states: this statement is true for nearly all substances; and it may be exemplified by the following experiments:

EXPT. 39.—**Ice.** Procure a lump of ice and notice that it has a particular shape of its own, which, so long as the day is sufficiently cold, remains fixed.

EXPT. 40.—**Water.** With a sharp brad-awl, or the point of a knife, break the ice into pieces, and put a convenient quantity of them into a beaker. Place the beaker in a warm room, or apply heat from a laboratory burner or spirit lamp. The ice disappears, and its place is taken by water. Notice the characteristics of the water. It has no definite shape, for by tilting the beaker the water can be made to flow.

EXPT. 41.—**Steam.** Replace the beaker over the burner and go on warming it. Soon the water boils and is converted into vapour, which spreads itself throughout the air in the room, and seems to disappear. The vapour, as it escapes out of the vessel, can be made visible only by blowing cold air on it, when it becomes white and visible owing to its condensation into small drops of water.

The **state**, or **physical condition**, of a substance depends largely upon its *temperature*, that is, its condition of hotness or coldness: thus, all metals can be melted by raising the temperature sufficiently, and at a still higher temperature they may be converted into gases. On the other hand, by diminishing the temperature, a gas may be liquefied and, in most cases, even solidified. Iodine and camphor are typical examples of a few substances which, when heated, appear to pass suddenly from the solid condition to that of a gas, without assuming the intermediate liquid state: substances which do this are said to **sublime.**

EXPT. 42.—**Sublimation.** Warm a round-bottomed flask by holding it above the flame of a Bunsen burner. When it is too warm to bear the finger upon the bottom, introduce a crystal of iodine, and notice it is converted at once into a beautiful violet vapour.

Further subdivision of the states of matter.—A solid body requires a great force, or **stress**, to alter its size or shape; if its size or shape is altered, we say that the solid undergoes a **strain.**

A **perfect solid** is one which, when a stress is applied, undergoes a certain strain which remains constant so long as the stress is maintained, but the solid reassumes its original condition as soon as the stress is removed: a bent steel spring is an example of a perfect solid.

A **rigid solid** is one which does not change its shape in the least when acted upon by a stress. No *perfectly rigid* solid is known, although several solids suggest this property of perfect rigidity: thus, a brick apparently is not strained when several other bricks are supported on it.

A **soft solid** is one in which considerable change of shape results from a small stress applied to it. Jelly is an extreme case of a soft solid.

In the case of liquids, some flow very slowly and others flow rapidly; then we speak of the **viscosity**, or resistance to flow, as being greater or less. Thus, the viscosity of pitch is very high; that of treacle is less; it is still less in water, and least of all in ether. According as the viscosity is high or low, so the liquid is termed either **viscous** or **mobile.**

Hence, the states of matter may be classified thus :

1. **Solids** $\begin{cases}\textit{Rigid.}\\ \textit{Soft.}\end{cases}$ 2. **Liquids** $\begin{cases}\textit{Viscous.}\\ \textit{Mobile.}\end{cases}$ 3. **Gases.**

Constancy of mass in different states.—When a solid is converted into a liquid, or a liquid into a vapour, no change of mass is experienced. This has been found to be true in all cases, and the following experiments will illustrate the fact :

EXPT. 43.—**Water to steam.** Boil water gently in a distilling flask, as in Fig. 43, and catch the condensed steam, taking care that none escapes, in another flask. The water thus collected will be found to have the same mass as that boiled away.

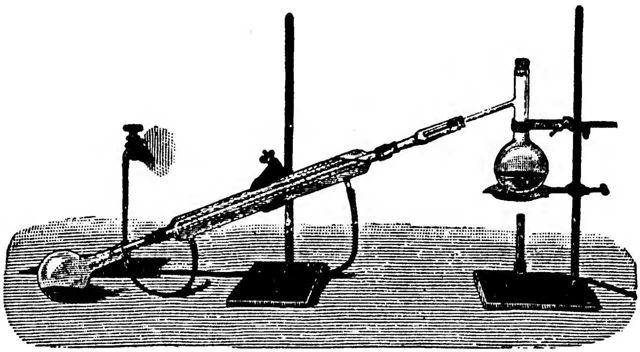


FIG. 43.—Distillation of water.

EXPT. 44.—**Ice to water.** Place a piece of ice in a flask suspended from one arm of a balance. Counterpoise the flask with the ice in it ; then melt the ice by warming the flask, and show that the mass is unaltered.

EXPT. 45.—**Solution.** Put some warm water in a flask and some salt in a piece of paper. Counterpoise the flask of water and the paper of salt together, and then dissolve the salt in the water. The total mass remains unaltered.

Though we may change the form or condition of any portion of matter, we are not acquainted with any process by which it can be destroyed : in other words, matter is **indestructible.** Thus, when a candle burns, it ceases to exist as tallow or wax ; but, if we collect carefully the products of combustion (one is a liquid, and the other a gas), we find that these two things together weigh *more* than the part of the candle which has disappeared.

Molecular constitution of matter.—To account for many simple phenomena, *e.g.* the intimate mixing of coal-gas with the air of a room into which the gas is allowed to escape, or the dissolving of salt in water, it is assumed that all forms of matter are not uniform throughout, but that they consist of very minute particles, called **molecules.** It is assumed that the molecules of any substance are separated more or less from each other, and that in a mixture of two or more liquids, or in a solution of a solid in a liquid, the molecules of one substance occupy the interspaces between the molecules of the other.

Simple phenomena indicate that the molecules are in *continual and rapid movement.* The gradual evaporation of a liquid exposed to the air can be explained only by assuming that the molecules of the liquid are in more or less rapid motion, and that they escape at a definite rate from the bulk of the liquid and pass away into the surrounding space.

In a gas contained within a glass flask, we have good reason to regard the molecules as moving rapidly and continuously, frequently colliding with each other, and bombarding the walls of the flask; indeed, the latter process causes the *pressure* which the gas exerts upon the surface enclosing it. We may imagine, too, that the effect of raising the temperature of the gas is to increase the rapidity of movement of the molecules and, therefore, the pressure which the gas exerts. If the containing walls are elastic, the increased pressure brings about an increase in the space occupied by the gas; and this suggests the *expansion* (p. 163) which the gas undergoes when its temperature is raised. The same phenomenon of expansion due to a rise of temperature is observed in the case both of liquids and of solids; we have reason, therefore, to suppose that the molecules of a solid, as well as those of a liquid and of a gas, are in a condition of movement.

The difference between the molecular conditions of the three states of matter is found in the distance apart of the molecules. In the case of a gas this distance apart is great as compared with that in the case of a liquid; thus, when steam is condensed to water it shrinks to $\frac{1}{1600}$ of its original volume. To a smaller extent, the distances separating the molecules of a liquid are large in comparison with the distances in the case of a solid.

The size of the molecule of any substance is excessively small, and far smaller than can be observed by the most powerful microscope. The average diameter of a molecule is perhaps $\frac{1}{2500000}$ millimetre; and this is about six hundred times smaller than the smallest particle which can be rendered visible.

PROPERTIES OF MATTER.

The various forms of matter are found to exhibit some, or all, of the following properties: **divisibility, porosity, compressibility, elasticity, tenacity, malleability, hardness, cohesion.**

Divisibility.—A lump of sugar may be broken into several smaller lumps, and so possesses the property of **divisibility.** Is there no limit to this divisibility? Or, would the particles finally become so small that, with any further subdivision, they would cease to be sugar? Investigation has shown us that there *is* such a limit, and this is reached when each particle consists of *one* so-called molecule of sugar; but so fine a subdivision cannot be obtained by mechanical grinding, for the smallest speck of sugar, so obtainable, would still consist of many molecules attached together. The subdivision is, however, far more complete when the sugar is *dissolved* in water; and we may say that, in this condition, the molecules are separated from each other. We could subdivide each molecule still further by chemical means only: thus, we might *burn* the sugar; and the black carbon which is then seen proves that the molecule is complex, and that it contains carbon as one of its constituents.

Porosity.—The manner in which turbid water can be filtered through blotting-paper, and the slow passage of water through an unglazed earthenware bottle, indicate that these materials are full of **pores.** An unglazed brick is porous and permits fresh air to pass through the walls of a dwelling-house built of brick. It may be proved that metals, such as iron and lead, are porous: thus, Francis Bacon (in 1640) observed that when a sphere of lead filled with water is compressed by a great force, the water slowly oozes through the lead and appears on the outer surface.

We may even say that liquids indicate **porosity**. For sugar, or salt, may be dissolved in water without appreciably increasing the bulk of the liquid. Another example is obtained in the following experiment:

EXPT. 46.—**Porosity of liquids.** **Half fill a barometer tube with water; then gently add alcohol until the tube is nearly full. Make**

a mark on the tube at the level of the top of the liquid column, and afterwards shake the tube so as to mix the water and alcohol well together. Observe that the volume of the mixture has diminished, the reason being that some of each liquid has filled up pores between the particles of the other.

Compressibility.—This property follows as a natural consequence of porosity. If pores exist between the indivisible small particles of which matter is built up, it ought to be possible, by the adoption of suitable means, to make these particles go closer together.

This is well known to be the case in gases, and it is also true of solids and liquids. Hence, **compressibility** is not only a consequence of porosity, but actually a proof of its existence.

Elasticity.—Imagine a gas to have been made to assume one-half its size by compressing it. What would happen if the pressure, which is the cause of the diminution, were removed suddenly? The gas would resume its original size or volume, and it would, so far as appearances are concerned, seem to have undergone no change. The gas is said to be perfectly **elastic**, and the property which enabled it to go back to its original state is called **elasticity**. Similar results follow with liquids; they also are perfectly elastic.

Some differences arise when solids come to be examined. Though the property can be developed in solids in at least four ways—by *pressure*, by *pulling*, by *bending*, and by *twisting*—we need consider the first only in this connection, as it is the elasticity which is developed by pressure which is most marked in all forms of matter. Ivory, marble, and glass are examples of elastic solids; while putty, clays, fats, and even lead are instances of solids with scarcely any elasticity. In a scientific sense, glass is more perfectly elastic than india-rubber, because it returns to its original shape after it has been forced out of that shape, whereas india-rubber does not return to its original shape exactly.

A solid will resume its former dimensions only when the pressure is removed, provided that the pressure is within a certain limit. If the pressure be more than this minimum amount, or if it exceeds the **limit of elasticity**, as it is called, the solid will not return to the initial size; it will undergo a permanent change. As has been seen in Expt. 35, this limit of elasticity is exceeded in the case of india-rubber only when the stress applied is very great.

EXPT. 47.—**Compression of a solid.** Procure a slab of polished marble or some similar material and smear it with oil. Drop a billiard ball or a large glass marble from a considerable height on the slab.

Catch it as it rebounds. Notice that a blot of oil is found where the ball came into contact with the slab. Compare the size of the blot with the spot which is formed when the marble is placed in contact with the slab.

Evidently the ball underwent a compression as the result of collision with the slab, and, by virtue of its elasticity, it regained its original shape, and caused the rebound.

Tenacity.—Tenacity is measured by ascertaining what weight is necessary to break solids when in the form of wires.

EXPT. 48.—**Measure of tenacity.** Suspend a balance-pan from the lower end of a thin copper wire attached to a beam. Add weights to the pan until the wire breaks. The force required to break the wire is the joint weight of the balance-pan and the weights in it. Repeat the experiment with wires of the same diameter but of different material.

In making the measurement of tenacity, the area of the cross section of the wire must be estimated first carefully.*

It is found that a wire of twice the cross sectional area of another of the same material is just twice as tenacious. Evidently, then, if we wish to compare the tenacity of two wires of different materials, the experiment is made simpler if wires of the same cross section are selected. Cast steel is the most tenacious of all metals, being about twice as much so as copper and forty times as tenacious as lead. But the tenacity of steel itself is exceeded by that of unspun silk, while single fibres of cotton can support millions of times their own weight without breaking.

The process of wire-drawing increases the tenacity of the wire; hence, a wire cable is stronger than a chain of the same length and weight. The tenacity of wood *along* the fibre is greater than *across* the fibre.

Ductility and malleability.—Several tenacious metals indicate a certain degree of **fluidity**, by which the molecules may alter their relative positions without losing their cohesion. A metal possesses **ductility** if a short thick cylinder can be drawn out into a long thin wire.

Malleability is a property similar to ductility, but the change of form is brought about by the application of pressure; gold, copper, and lead, for instance, can be beaten out into thin plates, and are therefore malleable substances. Lead is an example of a malleable material which is not ductile—it can be beaten out

* Area of cross section $=\frac{22}{7}\times\frac{d^2}{4}$.

but cannot be drawn into wires. Platinum is the most ductile and gold the most malleable metal known. Platinum has been drawn out into wire so fine that a mile of it weighs only one and a quarter grains. Gold has been beaten into plates so thin that it would require three hundred thousand of them placed one above the other to make a layer an inch thick. Antimony is an example of a metal which possesses no malleability.

Hardness.—*Hardness is the property by virtue of which solids offer resistance to being scratched or worn by others.* This property is of great importance in the study of minerals, as it often affords a ready means of distinguishing them. The method of measuring hardness consists in selecting a series of solids, each one of the series being harder than the one above it, and softer than the one below it. At one end of the series, therefore, the hardest solid known is placed; at the other end, the softest which we may wish to measure.

Cohesion.—The term cohesion refers to the force of attraction which exists between molecules of the *same* nature. The phenomenon called into play by the attraction of molecules of different nature, *e.g.* between molecules of glass and molecules of water, is termed **adhesion.** The property of cohesion is well-marked in solids, is less so in liquids, and is absent in gases. The process of welding two pieces of red-hot iron is an application of the phenomenon of cohesion; so also, the process of making the black lead of an ordinary pencil from finely powdered graphite. The force of cohesion is seen in a rain-drop also, or in the formation of a drop of any liquid. The different degrees of tenacity, ductility, and hardness exhibited by different substances depend upon the characteristic differences in cohesion.

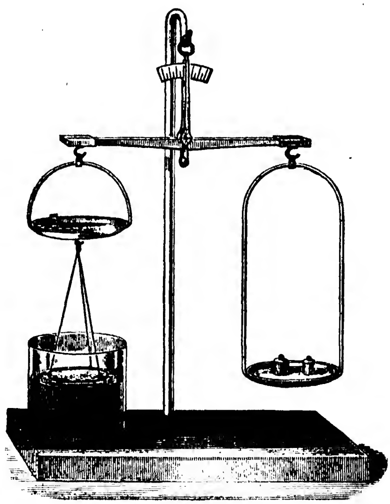


FIG. 44.—Measurement of cohesion of a liquid.

Fig. 44 represents a method of demonstrating the properties of cohesion and adhesion. A glass plate is suspended horizontally

from the beam of a balance, and the plate is counterpoised by means of weights placed on the pan attached to the other end of the beam. A dish containing water is placed just below the plate at such a distance that when the plate is lowered slightly below its normal position it touches the surface of the water. It is found necessary to increase considerably the weight on the pan in order to separate the plate from the water. As the lower surface of the plate is still wetted it is evident that the force of adhesion between glass and water is greater than the force of cohesion of water; and the amount by which the weight was increased is a measure of this cohesion.

Surface tension.—These molecular forces, evident in the property of cohesion, cause peculiar phenomena to appear in the layer of molecules which form the free surface of a liquid. Thus, in the case of any molecule, such as A (Fig. 45), situated well within the mass of a liquid, the forces of attraction due to neighbouring molecules are distributed uniformly in all directions; whereas any molecule, such as B, situated near the surface is acted upon by forces of attraction which are mainly acting downwards and away from the surface. This results in the surface exhibiting a kind of tension, which has the effect of making the area of free surface as small as possible.

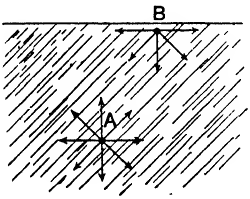


Fig. 45.—Forces acting on a molecule, A, within a liquid and on one, B, near the surface.

Since the sphere is the geometrical figure which has the smallest surface area for a given volume, any mass of liquid would assume this form—providing that internal molecular forces only are acting upon the liquid. Since, except under special conditions, the external force of gravitation is acting upon the liquid always, the form of the free surface is seldom spherical; and the effect due to surface tension alone can be observed only when all external forces are neutralised or eliminated. In the case of very small masses of liquids the force of gravitation is negligibly small compared with the force of cohesion; and in such cases the spherical form of the mass is frequently apparent. Thus, the small globules of condensed moisture which constitute cloud, and

even falling rain-drops, are truly spherical; also, if a small quantity of mercury is dropped upon a horizontal board or sheet of paper, or if water is dropped upon a board sprinkled with powdered resin, the liquid breaks up into drops which have a more or less spherical form.

The same effect can be observed by means of olive oil and a mixture of alcohol and water, the specific gravity of the mixture being adjusted so as to be identical with that of the oil. A tall glass vessel, with flat sides, is nearly filled with the alcohol mixed with water; a pipette filled with the oil is immersed into the mixture, and a small quantity of the oil is allowed to escape from the open end of the pipette. The oil will remain floating within the mixture, and will assume a truly spherical form.

The phenomenon of surface tension and the tendency to spherical form are evident in a soap-bubble. It is easy to observe that when the tube, to which the bubble is attached, is allowed to remain open the bubble slowly diminishes in size.

Capillarity.—If a flat plate of solid material be partly immersed in a liquid vertically, the liquid surface seldom retains its horizontal form in the region quite near where it touches the solid. If the liquid *wets* the solid, as when glass is dipped into water, the surface in this region is concave upwards; but if the liquid does not wet the solid, as when glass is dipped into mercury, the surface in this region is convex upwards.

These effects are due simply to the relative magnitudes of the force of adhesion (between the solid and the liquid) and the force of cohesion in the liquid. When the liquid is water, the former force is greater than the latter; and *vice versâ* when the liquid is mercury.

The phenomena of capillarity are more apparent when glass tubes of narrow bore are used instead of glass plates. When such tubes are supported vertically with their lower ends immersed in water, the liquid rises up the tube to a height which depends upon the diameter of the bore—the narrower the bore the higher the water column (Fig. 46). If the diameter of one tube be one-half that of another, the total upward force exerted by the surface tension is reduced to one-half, since the circumference of the liquid surface is reduced by one-half. But the weight of the

liquid column is only one-fourth of that of a column of the same height in the wider tube, since the volume of liquid varies inversely as the square of the diameter. Hence, in the narrower tube, the height of the liquid column is twice that obtained in the wider tube.

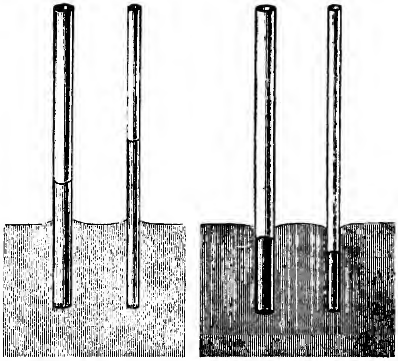


FIG. 46.—Capillary elevation and depression.

When such tubes are dipped into mercury, the liquid surface within the tube assumes a convex form and is depressed below the level of the liquid outside the tube.

Tubes of small internal diameter are termed **capillary tubes**; and the phenomenon observed when such tubes dip into a liquid is termed **capillarity**. To capillarity is due the property by which oil can pass upwards between the threads of a lamp-wick. In the same way, water may be siphoned out of a vessel by allowing a towel to hang over the edge with one end dipping into the water.

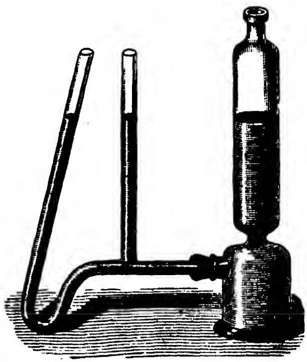


FIG. 47.—Equality of level of liquid in communicating vessels.

Distinctive characters of liquids.—Same level in communicating vessels. If several vessels of varied shapes (Fig. 47) are in communication with one another, and water be poured into any one of them, it is found that as soon as the water has come to rest it will stand at the same level in all the tubes, however different the form of the vessels may be.

The following simple experiments are useful in showing that the surface of a liquid at rest is horizontal and that pressure upon a liquid is communicated equally in all directions :

EXPT. 49.—**Horizontal surface.** Into a shallow glass vessel pour enough mercury to cover the bottom. Attach a ball of lead to the end of a fine string, and so construct a **plumb-line.** Hang it over the surface of the mercury, and notice that the line itself and its reflection are in one and the same line. If this were not the case, that is, if the image slanted away from the plumb-line itself, we should know the surface of the liquid was not horizontal.

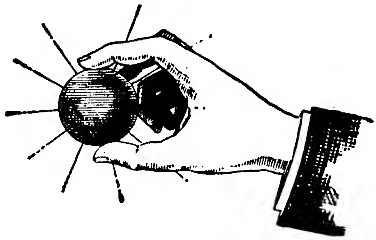


FIG. 48.—Communication of pressure by liquid.

EXPT. 50.—**Communicate pressure equally in all directions.** Bore a hole in a hollow rubber ball and fill it with water. Cover the hole with a finger, and prick several small holes in the ball with a needle. When the ball is squeezed, the water spurts out of each hole straight from the centre of the ball, thus showing that the pressure has been transmitted equally in all directions.

The transmission of pressure by liquids is made use of in the **Hydraulic Press**, which consists essentially of two cylinders in connection, with pistons fitted into them, one much larger than the other. The application of a comparatively small force to the small piston is felt on the larger, and it is as many times greater in an upward direction as the piston of B is larger in area than the piston in A. This great upward force is being used in the instance shown in Fig. 49 to compress bales of wool.

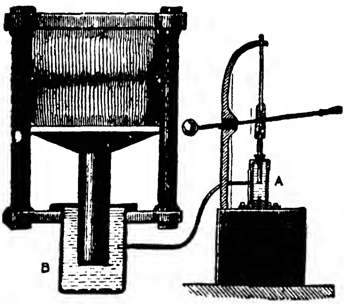


FIG. 49 —Hydraulic Press.

Distinctive characters of gases.—It appears, therefore, that solids tend to retain a definite *size* and *shape*, while liquids tend to retain a definite *size* only. **Gases show no tendency to retain either a definite size or shape.** A small quantity of any gas, when admitted into a large empty vessel, almost instantaneously distributes itself uniformly throughout the interior of a vessel.

Gases are **easily compressible**; and in this they obey a definite law, viz.: that, when the temperature remains constant, the volume increases or decreases in just the same proportion as the pressure decreases or increases (p. 85).

EXERCISES ON CHAPTER V.

1. Describe experiments which prove:

(*a*) That solids are porous.
(*b*) That liquids, too, have pores.

2. How would you show by experiment that the mass of the same portion of matter in different states is constant?

3. What experiment could you perform to show that a solid, say a billiard ball, is elastic? Explain as well as you can what you mean by elasticity.

4. What property in particular is possessed by liquids and not by solids? And what character has a gas which neither liquids nor solids possess?

5. The same portion of matter can, under suitable conditions, assume different states. Describe fully some experiment which illustrates this statement.

6. What evidence can you give that the different states of matter gradually shade into one another?

7. Give reasons for the opinion that all forms of matter are molecular in structure.

8. State some of the properties which are possessed by all kinds of matter, and explain in your own words the meaning of a property of matter.

9. Two narrow glass tubes—one twice the diameter of the other—are dipped into water and mercury in succession. Describe and sketch the effect observed in each case.

CHAPTER VI.

PRESSURE IN LIQUIDS. RELATIVE DENSITY.

Measurement of pressure.—Fig. 50 represents a glass U-tube, open at both ends, and containing mercury. The mercury surfaces are at the same level, since the atmosphere is pressing on both surfaces to an equal extent. If the mouth be applied to the open end of a rubber tube R joined to one limb of the glass tube, and if then air be forced down the tube, the pressure at A increases; the mercury surfaces are no longer at the same level—that at A being depressed, and that at B raised; and the difference in level thus produced is dependent upon the air forced into the tube. On the other hand, if some of the air be removed from the tube by suction,* the pressure at A diminishes: the mercury surface at A rises, and that at B falls. Hence *the position of the mercury in the tube indicates any difference in the pressures acting on the two surfaces.*

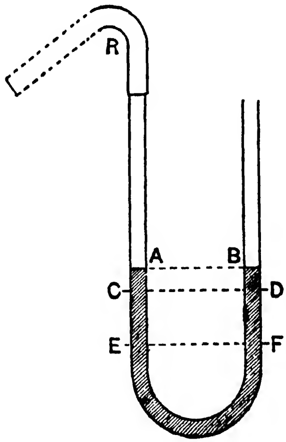


FIG. 50.—Pressure in a liquid.

Let the mercury return to its original position in the tube, and consider the pressures at two points, C and D, each 1 cm. below the surface of the mercury; and, for simplicity, let the cross-section of the tube be 1 sq. cm. Neglecting the pressure of

* The U-tube should be long enough to prevent the mercury from rising up to the top and entering the mouth.

the atmosphere—which, of course, is the same on both sides of the tube—the pressure acting at both C and D is equal to the weight of 1 c.c. of mercury (*i.e.* 13·6 grams); or, *the pressure acting at either point is equal to* **13·6 grams per sq. cm.** Similarly, at any other pair of points at the same level, *e.g.* E and F, the pressures are equal. From such considerations as these we deduce the following rule: **In any U-tube containing a liquid the pressures at the same horizontal levels are the same.**

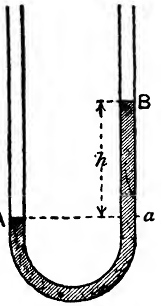


FIG. 51.—Principle of a manometer.

Suppose that the pressure applied to the surface A (Fig. 51) is increased sufficiently to depress it to the position shown. The pressure at A is equal to that at a point *a*—which is at the same horizontal level as A. If the difference of level between the two mercury surfaces be *h* cm., the pressure on each sq. cm. at *a* is equal to *h* × *13·6 gm. per sq. cm.*; and the pressure at A is equal to the same amount. This simple device for measuring the pressure of a gas is termed a **mercury manometer.** If any liquid other than mercury is used, the pressure is equal to *h* × *d gm. per sq. cm.*, where *d* is the density of the liquid.

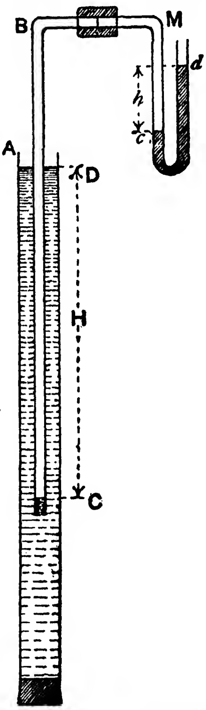


FIG. 52.—Apparatus for measuring pressure in liquids.

A mercury manometer may be used for investigating the pressure beneath the free surface of a liquid. In Fig. 52 A is a long glass tube, about 1 m. long and 4 cm. in diameter, closed at the lower end with a rubber stopper, and filled with water. B is a narrow glass tube bent at the upper end, and connected by thick rubber tubing to a manometer M. When the open end of the tube B is immersed in the water, the pressure exerted by the water at its surface C is transmitted

through the air in the tube to the mercury surface *c*; and this pressure is measured by observing the difference of level in the manometer. A series of observations will prove that **the pressure is proportional to the depth below the free surface.**

EXPT. 51.—**Pressures of liquid columns.** Fit up the apparatus shown in Fig. 52, and measure (i) the depth H of the surface C below the surface D, and (ii) the difference of level *h* in the manometer. Take a series of readings for different positions of the tube B. Tabulate the readings in column, and calculate the ratio H/*h* for each reading.

Plot the readings on squared paper, taking values of H as abscissae, and values of *h* as ordinates.

The magnitude of the liquid pressure at any point below the surface of a liquid may be derived in the following manner: Let A (Fig. 53) be a small horizontal surface, 1 sq. cm. in area, and situated *h* cm. below the liquid's surface. The vertical downward pressure on A is equal to the weight of a vertical column of the liquid *h* cm. long and 1 sq. cm. cross-section; if the density of the liquid be *d* gm. per c.c. this pressure is *hd* gm. per sq. cm.

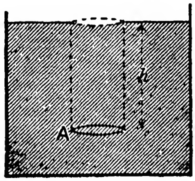


FIG. 53.—Pressure of liquid column.

A further point of great importance is that, at any point, there is an equal pressure *acting upwards*, as well as *downwards*. For if the pressure acting downwards was the only pressure acting on the surface A, the thin layer of liquid represented by A would not remain stationary, but would be thrust downwards in obedience to this force. Common experience tells us that such a layer of water is *not* in this state of constant movement; hence, at any point within a liquid, **there is always an upward pressure equal in magnitude to the downward pressure at the same point.**

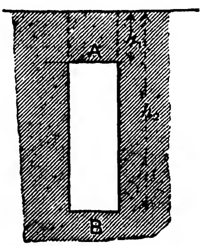


FIG. 54.—Upward pressure upon an object in a liquid.

Apparent loss of weight of a solid when suspended in a liquid.—Let AB (Fig. 54) represent a metal cylinder, of length *l* cm. and cross-section *a* sq. cm., immersed vertically in a liquid of density *d* gm. per c.c. If the upper and

lower surfaces are immersed to depths h_1 cm. and h_2 cm. respectively, the downward pressure on the surface A is equal to the weight of ah_1 c.c. of the liquid, *i.e.* to ah_1d gm.; similarly, the upward pressure on the surface B is equal to ah_2d gm. Hence, the resultant *upward* pressure due to the liquid is equal to $ah_2d - ah_1d$ gm. But $a(h_2 - h_1)$ is the volume of the cylinder; hence, **the upward pressure due to the liquid is equal to the weight of a quantity of the liquid equal in volume to that of the solid immersed.** This is known as the **Principle of Archimedes**: which may be stated as follows: **When a solid is immersed in a liquid, the apparent loss of weight is equal to the weight of an equal volume of the liquid.**

EXPT. 52.—**Upthrust.** Measure the dimensions of a metal cube, and calculate its volume. Suspend the cube, by means of cotton thread, from the hook of a balance, and at such a height that the cube may afterwards hang freely in a beaker of liquid supported on a bridge placed over the pan. Use as little cotton as possible: a single strand with a loop at the top should suffice. Weigh the suspended cube. Now place a beaker of cold water on the bridge so that the cube is immersed totally, taking care that it does not touch the sides of the beaker, and again weigh the cube.

Repeat the above experiment several times, using regular solids of different form and different liquids of known density. Tabulate your results in the following manner, and deduce your own conclusions as to the relationship between the apparent loss of weight of the solid and the weight of an equal volume of the liquid used.

Liquid, and its density (d).	Dimensions of solid.	Volume of solid.	Weight of equal volume of liquid.	Apparent loss of weight.

Relative density (or, **specific gravity**).—It has already been proved by experiment that equal volumes of different materials have different masses, or, in other words, different materials have different densities. Such experiments show that some substances are heavier and others lighter than an equal volume of water. The **relative density** of a material is **the number of times that a fragment of the material is heavier or lighter than an equal volume of cold water.**

It is important to remember that the relative density is the ratio of one mass to another mass; and the ratio is simply a

numerical quantity. On the other hand, the absolute density of a material is not described fully unless the units of mass and volume are given. Thus, it is correct to say that the **relative density of marble is 2·8**; but the **absolute density of marble is 2·8 gm. per c.c.**

Relative Densities of Solids.

To determine the relative density of a solid heavier than water we require to find (i) the weight of the body of which the relative density is required, and (ii) the weight of an equal volume of water. The second quantity is obtained, by the Principle of Archimedes, by observing the apparent loss of weight when the solid is suspended in cold water. Then

$$\textbf{relative density} = \frac{\text{weight of substance}}{\text{weight of equal vol. of water}}$$

$$= \frac{\textbf{weight of substance}}{\textbf{apparent loss of weight in water}}.$$

Expt. 53.—**Solids heavier than water.** Find the relative density of two or three common solids, such as brass, sulphur, copper, lead, and glass.

In applying the Principle of Archimedes to the determination of the relative density of a solid lighter than water it is necessary to attach to the solid an insoluble sinker sufficiently heavy to keep it in the water. Three separate weighings are necessary, viz.: (*a*) the solid, in air, (*b*) the *sinker* alone, in water, and (*c*) the sinker, with the solid attached, in water. It is best perhaps to take the weighings in the order shown in Fig. 55. In Fig. 55 (i) the sinker alone is weighed in water; let this weight be w_1. In Fig. 55 (ii) the solid is supported just below the hook of the balance by looping the thread once round the solid; let this

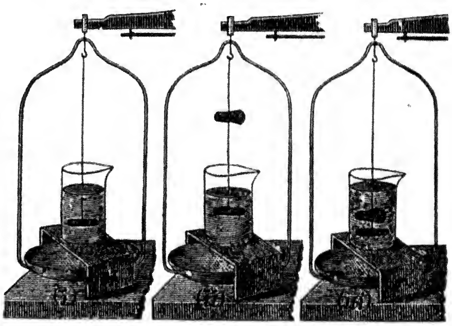


Fig. 55.—Determination of the relative density of a solid lighter than water.

weight be w_2. In Fig. 55 (iii) the solid is rotated downwards until it is close to the sinker and immersed in the water, and another weighing w_3 taken. The only difference between the second and third weighings is *the weight of the water displaced by the light solid.* Hence

$$\text{relative density} = \frac{w_2 - w_1}{w_2 - w_3}.$$

EXPT. 54.—**Solids lighter than water.** Determine the relative density of paraffin wax, of oak, and of cork.

If a solid of less density than water be immersed totally in water, then, since the upward thrust due to the displaced water is greater than the weight of the solid, it must tend to rise to the surface. When it reaches the surface, the downward pressure on the top of the solid becomes zero, but the upward pressure on the under surface continues to act; the body continues to rise until this upward pressure is equal to the weight of the solid, *i.e.* until **the weight of the floating body is equal to the weight of the displaced water.**

Suppose a rectangular block of wood, of area A sq. cm. and height H cm., to be floating in water, and immersed to a depth h cm.;
then, total volume of wood = AH c.c.
and „ „ displaced water = Ah c.c.
Hence, weight of AH c.c. of wood = weight of Ah c.c. of water,
or, „ „ 1 c.c. „ = „ $\frac{h}{\text{H}}$ „ „

$$= \frac{h}{\text{H}} \times \text{weight of 1 c.c. of water.}$$

Therefore, $$\text{relative density} = \frac{\text{weight of 1 c.c. of wood}}{\text{weight of 1 c.c. of water}} = \frac{h}{\text{H}}.$$

EXPT. 55.—**Relative density by flotation.** Fig. 56 represents a method of making the necessary measurements. Let a wooden strip rest across the top of the wide glass beaker in which a block of wood is floating. Fix with wax a needle to one end of a millimetre scale. Take the scale reading of the wooden strip when the needle is just touching the surface of the water; and also the reading when the needle is touching the surface of the block. As the block may not be floating quite horizontally, find the average of four readings taken at opposite corners. From these observations calculate the relative density of the wood.

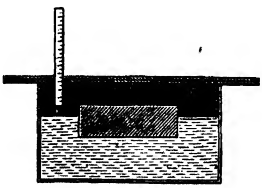


FIG. 56.—Expt. 55.

Relative Densities of Liquids.

To determine the relative density of a liquid it is necessary to find the weight of any volume of the given liquid and the weight of an equal volume of cold water. Several methods are available.

Expt. 56.—**By the principle of Archimedes.** Weigh a glass stopper in air, then immerse it successively in water, turpentine, methylated spirit, and olive oil, and notice the apparent loss of weight in each case. The apparent loss of weight experienced by the glass stopper in each experiment is equal to the weight of a portion of liquid of the same volume as the stopper. The numbers obtained therefore represent the weights of equal volumes of water, turpentine, methylated spirit, and olive oil, and by dividing each by the number obtained in the case of water, the relative densities of the liquids are obtained.

The *relative density bottle* (Fig. 57) consists of a small glass flask, holding about 50 grams of water. It is provided with a nicely-fitting ground stopper, which is in the form of a tube with a very small bore through it, or the stopper may have a groove on its side. Another form of specific gravity bottle has a stopper in the form of a narrow tube or neck with a mark upon it.

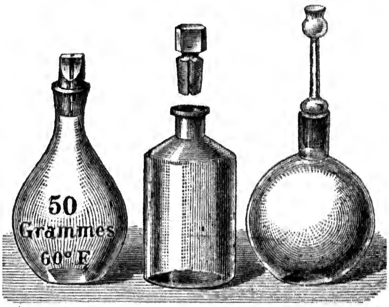


Fig. 57.—Relative density bottles.

Expt. 57.—**By the relative density bottle.** Carefully dry the inside of the bottle, and weigh it. Fill the bottle with the given liquid, insert the stopper, and see that no air bubbles remain in the bottle. Dry the outside of the bottle (holding it in several folds of a duster so as not to warm the bottle by direct contact with the hand), and weigh it. Rinse out the bottle several times with water, and finally weigh it when full of cold water. In this manner you obtain the weights of *equal* volumes of the liquid and of water.

Instead of the bottle, a small flask having a file mark on the neck may be used.

An instructive method of showing the relative densities of liquids is obtained by means of a glass tube bent in the form of a U, and therefore called a U-tube.

EXPT. 58.—**By means of a U-tube.** Cut off two pieces of glass tube, each about 30 cm. long; connect the tubes with india-rubber tubing about 18 cm. long, and fix them upright upon a strip of wood. Or, bend a piece of tubing into the form of a U with arms about 30 cm.

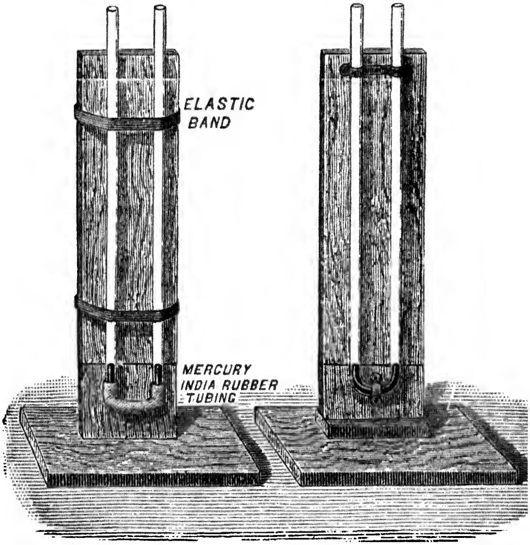


FIG. 58.—U-tubes for determining relative densities of liquids.

long. Pour mercury into one of the tubes until it reaches a horizontal line drawn upon the board (Fig. 58). Now introduce water into one of the tubes by means of a pipette, and notice that the mercury on which the water rests is pushed down; afterwards introduce sufficient water into the other tube to bring the mercury back to its original level. The length of each column of water will be found the same. Repeat the experiment with varying amounts of water.

The mercury in the bend of the U-tube evidently acts as a balance, which enables columns of different liquids in the upright arms to be balanced.

EXPT. 59.—**Spirit and water.** Nearly fill one of the tubes with methylated spirit, and balance it with water introduced into the other tube. Measure the lengths of the two columns.

As these two unequal columns balance one another it will be evident that the liquid in the shorter column, namely, the water, has a greater relative density than the liquid in the longer column.

If h_1 = height of liquid column, and h_2 = height of water column, then

Wt. of h_1 cm. of the liquid = wt. of h_2 cm. of water, or

" 1 cm. " = " h_2/h_1 " or

$= h_2/h_1 \times$ wt. of 1 cm. length of water.

Hence, any volume of the liquid is $\frac{h_2}{h_1}$ times as heavy as an equal volume of water; and

$$\text{Relative density} = \frac{h_2}{h_1}.$$

To determine the relative densities of liquids which mix, an arrangement known as Hare's apparatus may be used. A simple form of this apparatus is represented in Fig. 59. A wide-necked bottle is closed with a rubber stopper bored with three holes. Through two of the holes pass long glass tubes, and the third hole is fitted with a short tube terminating in rubber tubing and a clip. The lower ends of the long tubes dip into beakers containing the liquids the densities of which have to be compared. By applying suction to the rubber tubing, the liquids are drawn up the glass tubes to different heights, the *less* dense being drawn to the *greater* height. The pressure on the surface of the liquids in the beakers is the same, viz. that due to the atmosphere; hence the pressures *inside* the tubes and at these same levels must be equal. Therefore the weights of the liquid columns *measured above the level of the liquids in the beakers* must be equal; just as in the case of liquids in a U-tube, the densities of the liquids are inversely proportional to the heights of the columns.

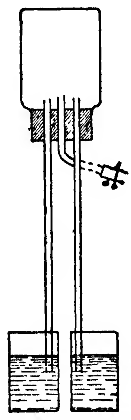


FIG. 59.—Simple form of Hare's apparatus.

EXPT. 60.—**By means of Hare's apparatus.** Using this apparatus, determine the relative densities of brine, solution of copper sulphate, vinegar, and milk.

HYDROMETERS.

A simple form of hydrometer may be made in the following manner: Cut off a piece of thin-walled glass tubing, about 15 cm.

long and 2 cm. diameter. Close one end with a cork; cut off the cork flush with the edge of the tubing, and make the end water-tight by means of sealing-wax. Fix inside the tube a paper millimetre scale, adjusted so as to measure distances from the extreme end of the tube. Put inside the tube sufficient small lead shot to make the tube sink vertically in water to a depth of about 12 cm., and fix the shot in position with wax or a slice of cork. Close the upper end of the tube with a short cork, and make water-tight as before.

EXPT. 61.—**Principle of a hydrometer.** Float a hydrometer, of the form described, in water. Note the depth of immersion. Remove the hydrometer, dry it, and weigh it. Measure the diameter in several different parts, and, taking the average of these readings, calculate the volume of water displaced. Find whether there is any relationship between the weight of the hydrometer and the weight of the displaced water.

When placed in any liquid **the hydrometer will sink until the weight of the displaced liquid is equal to the weight of the hydrometer** Hence, in a less dense liquid, the hydrometer will sink to a greater depth. If a be the cross-section of the tube, and if the tube sinks in water and in another liquid to depths H cm. and h cm. respectively, the volumes of displaced water and liquid are aH c.c. and ah c.c. respectively. Hence

Weight of ah c.c. of liquid = wt. of aH c.c. of water,

or „ 1 „ „ = „ H/h „ „

or „ 1 „ „ $= \frac{H}{h} \times$ wt. of 1 c.c. of water.

Therefore *relative density* of liquid $= \frac{H}{h}$.

EXPT. 62.—**Brine and alcohol.** By means of a simple hydrometer find the relative density of weak brine and of dilute alcohol.

EXPT. 63.—**Verification.** If a commercial hydrometer (*e.g.* a Twaddell*) is available, verify by its means the results of the previous exercise.

Nicholson's hydrometer.—The hydrometers hitherto used are known as **hydrometers of variable immersion.** In another kind of hydrometer, known as **Nicholson's hydrometer,** the instrument is

* The rule for determining a relative density by means of a Twaddell hydrometer is, "Multiply the hydrometer reading by 5, add 1000, and divide by 1000."

always immersed to a fixed mark upon it, and densities are determined by finding the weights necessary to produce this amount of immersion in different cases. The densities of solids as well as of liquids can be determined with this instrument.

EXPT. 64.—**Solids.** Place a Nicholson's hydrometer (Fig. 60) in a tall jar of water.* Load the top pan A until the mark on the stem of the hydrometer is level with the surface of the water. Record the load; then remove it and put in its place a pebble which weighs less, or some other substance of which the density is required. Find the weights which have to be added to depress the hydrometer to the mark on the stem. Then you have

Weight of substance (a) + added weight (b) = weight required to sink hydrometer to mark (c).

Therefore weight of substance $a = c - b$.

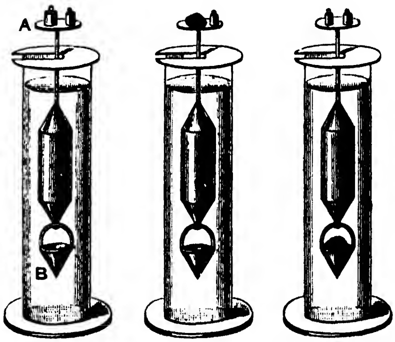


FIG. 60.—Use of a Nicholson's hydrometer to determine the relative density of a solid.

Now put the substance in the lower pan B, and add weights until the mark is again reached. The apparent loss of weight in water is the difference between the weights now added and those required to depress to the mark when the substance was in the top pan. You have thus the weight of the substance and the apparent loss of weight in water, and can therefore determine the relative density of the substance without a balance.

Determine the relative density of sulphur and of lead by means of Nicholson's hydrometer.

EXPT. 65.—**Liquids.** Weigh the Nicholson's hydrometer, and then place it in a jar of water. Add weights until the mark on the stem is level with the water. Then

Weight of hydrometer + weight added = weight of a certain volume of water displaced.

* If the hydrometer does not float in an upright position, place a small piece of lead on the lower pan and keep it in that position throughout the experiment. The lead must not be heavy enough to sink the hydrometer nearly to the fixed mark.

Repeat the experiment with one or two other liquids. The same amount of liquid is displaced in each case, so you obtain the weights of equal volumes of liquids compared with that of an equal volume of water. Determine from your observations the relative densities of the liquids.

EXERCISES ON CHAPTER VI.

1. Find the pressure due (i) to a column of water 1 metre in depth, (ii) to a column of mercury (density 13·6 gm. per c.c.) 1 metre in depth.

Express your answer in grams weight per sq. cm.

2.* Calculate the total pressure (in grams weight) upon the base of a cylindrical vessel one decimetre in diameter, filled with mercury to a height of 40 cm.

3.* The specific gravity of sea-water is 1·025. Calculate the pressure (i) in grams weight per sq. cm. at a depth of 40 metres below the surface of the sea, and (ii) in pounds per sq. ft., at a depth of 50 ft.

4.* To what depth must a surface be sunk in water in order that the pressure upon it may be 60 lb. per sq. inch?

5. In order to determine the pressure of the water supply at a certain tap, it is connected to a mercury manometer. On opening the tap the mercury rises in the open end of the manometer until the difference of level of the mercury surfaces is 110 cm. Express the 'head of water' in feet.

6. Explain why a ship made of iron will float in water, though iron itself is heavier, bulk for bulk, than water.

7. You are given a small rectangular block of brass, and you have at your disposal a measuring rod divided into centimetres and millimetres, a balance and weights, some fine wire, and a vessel of water. How will you determine, by two perfectly independent methods, so that the results may form a check on one another, the volume of the block? Describe exactly what calculations you will have to make, and say upon what scientific principle, if any, each of your methods depends.

8. A piece of lead weighs 150 grams in air, 137 grams in water, and 138·5 grams in paraffin. Find the specific gravity of lead and of paraffin—to two places of decimals.

9. In an experiment to find the specific gravity of copper sulphate by means of Hare's apparatus the following measurements were taken:

Height of water column in cm.:

1st time.	2nd time.	3rd time.	4th time	5th time.
10	12	14	16	20

* In those examples marked with an asterisk the pressure of the air on the liquid surface is disregarded.

Height of copper sulphate column in cm. :

1st time.	2nd time.	3rd time.	4th time.	5th time.
9·5	11	13·5	15	18·8

Find the average specific gravity of copper sulphate correct to two places of decimals, and draw up a neat table of measurements to show how you obtain your result.

10. A block of wood having a volume of 3 cubic feet weighs 93 lb. Find (*a*) the specific gravity of the wood ; (*b*) the weight that must be placed on the block so as just to keep the wood under water. (The weight of 1 cubic foot of water is to be taken as equal to 62 lb.)

11. (i) Why does a solid appear to weigh less in water than in air ?

(ii) Describe any experiment which shows that the apparent loss of weight (expressed in grams) in water is equal numerically to the volume (expressed in c.c.) of water displaced.

(iii) A piece of metal weighs 100 grams in air and 88 grams in water. What is its volume ?

12. Being provided with two pieces of glass tube and a piece of india-rubber tubing, explain how you would proceed to (i) compare the relative densities of olive oil and spirits of wine, (ii) ascertain the relative density of a specimen of milk.

13. If you were provided with a burette, paraffin oil, and pieces of ice, describe how with these things you would determine the relative density of ice.

A piece of ice has a volume of 1000 cubic feet. How many cubic feet would be above the water if the ice floats in (*a*) pure water, (*b*) sea water, assuming that none of the ice melts ? (The relative density of sea water is 1·025 and of ice 0·918.)

14. Describe a practical means of finding (*a*) the weight of one cubic inch of water, (*b*) the number of cubic inches in one pint.

A bar of wrought iron 1 inch wide, 1 inch thick, and 1 yard long, has the same weight as a gallon of water. Find the relative density of the iron. (There are 277·3 cubic inches in a gallon.)

15. If you were asked to determine the specific gravity of a small irregular piece of wood, describe exactly the method you would use.

A beam of oak measures 24 feet long, 18 inches wide, and 1 foot deep. Estimate the weight of the beam in lb., taking as the specific gravity of oak a value which you consider to be roughly correct and stating clearly the value which you take. (The weight of one cubic foot of water is 62·4 lb.)

16. How would you determine by measurement and calculation the volume in cubic centimetres of a rectangular slab of glass, such as that found in a box of weights ? Describe an experiment by which you could check your result.

17. Explain how, using a U-tube, you would determine the relative density of a liquid which does not mix with water.

18. Under what conditions do bodies float or sink in a liquid ?

A piece of iron weighing 275 gm. floats in mercury of density 13·59 with $\frac{5}{9}$ of its volume immersed. Determine the volume and density of the iron.

19. Two blocks of glass, each having a volume of 10 c.c., are hung from the scale-pans of a balance by means of hooks under the pans, and balance one another. Under one is brought a beaker of water, under the other a beaker of alcohol, so that the blocks are immersed in the liquids. The balance is now disturbed, and it is found that 1·82 grams have to be added to one pan to restore equilibrium. To which pan has this weight to be added, what is the explanation of the fact, and how can you determine from the figures now at your disposal the density of the alcohol ?

20. A covered tin canister having a volume of 88 cubic centimetres contains just enough shot to sink it to the top of the cover when placed in cold water. Determine from this information

(i) The weight of the canister and shot.

(ii) The weight of the water displaced by the canister.

21. Explain what is meant by specific gravity. A body of specific gravity 5 weighs 20 grams in air ; what will the body weigh when immersed in water ?

22. A bottle weighs 2 ounces. When holding $3\frac{1}{2}$ ounces of shot it will just float in water, when holding 3 ounces it will just float in oil, and when holding $3\frac{3}{4}$ it will just float in brine. Find the specific gravity of the oil and the brine.

23. A stone, weighing in air one kilogram, is suspended by a piece of cotton so that it is immersed entirely in water. On attempting to lift the stone out of the water the cotton breaks when the stone is partly out of water. Why is this ?

If when the stone is immersed completely the cotton would bear an additional pull equal to the weight of 150 grams, what volume of the stone will be out of the water when the cotton breaks ?

24. A hollow stopper of glass (density, 2·6 gm. per c.c.) is found to weigh 23·4 gm. in air, and 3·9 gm. when suspended in water. What is the volume of the internal cavity ?

25. A solid weighs 8 lb. in air and 5 lb. in water. Find (i) the weight of an equal volume of water, (ii) the relative density of the body, (iii) the volume (in cubic inches) of the body, (iv) the apparent weight of the body when suspended in glycerin (relative density $=1·25$).

26. The relative density of ice is 0·918 and that of sea-water is 1·03. What is the total volume of an iceberg which floats with 700 cubic yards exposed ?

27. A specific-gravity bottle, filled with water, weighed 39·74 gm. Some iron nails weighing 8·5 gm. were introduced, and the bottle filled with water. The bottle and contents now weighed 47·12 gm. Find the relative density of iron.

28. In order to sink a Nicholson hydrometer to the mark in water, it was necessary to add 60·3 gm. to the upper pan. When floating in alcohol only 6·8 gm. were required. If the hydrometer weighs 200 gm., what is the relative density of the alcohol ?

29. How would you determine the volume of an ordinary pen nib ? State all the precautions you would adopt in order to obtain an accurate result.

30. 1 c.c. of lead (sp. gr., 11·4) and 21 c.c. of wood (sp. gr., 0·5) are fixed together. Show whether they will float or sink in water.

CHAPTER VII.

ATMOSPHERIC PRESSURE, AND BOYLE'S LAW.

Weight of the air.—Surrounding the earth in every latitude, over land and sea, is a gaseous envelope which is spoken of as the air or the atmosphere. Its presence when at rest is unperceived, though in motion it becomes apparent by its effects on trees and other bodies free to move. It is easy to prove by direct experiment that the air has weight.

EXPT. 66.—**Determination of weight of air.** Fit a one-holed india-rubber stopper into a fairly large glass flask, and fit into the stopper a short glass tube with rubber tubing and clip (Fig. 61). Put a little water in the flask; open the stop-cock; and boil the water. After boiling for some minutes, close the clip and place the flask on one side to cool. When the flask is cool, weigh it. Then open the clip; air will be heard to rush into the flask, and as it does so the balance will show an increase of weight. Carefully re-weigh the flask. The increase in weight is equal to the weight of air within the flask. Measure the water in the flask by means of a measuring cylinder; *fill* the flask with water, up to the position occupied by the bottom of the stopper, and measure its volume. The difference of these volumes gives the volume of the air which entered the flask. From these results calculate the *weight of one litre of air.*

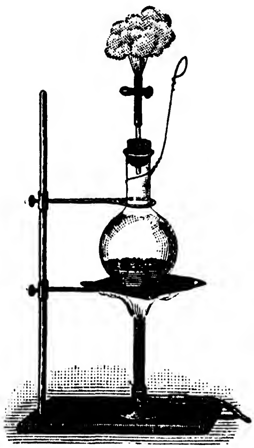


FIG. 61.—Experiment to show that air has weight.

Pressure of the atmosphere.—It is a property of all fluids that they communicate pressure in all directions, and consequently it

is a character of air. In consequence of this fact we are able to move about quite freely in spite of atmospheric pressure.

EXPT. 67.—**Effect of atmospheric pressure.** Procure a thin tin cylindrical can having a central exit-tube, fitted with rubber-tubing and clip. Remove the clip and boil a little water in the can. After the water has been boiling for some time, so that practically the can is filled with steam, remove the can from the flame, and quickly close the clip. Pour a little cold water over the can, and observe how the can collapses.

The explanation of the effect produced in this experiment is that as the can cools the steam inside is condensed into water, and so occupies a much smaller volume. The pressure which the steam exerts on the inside of the can is thus removed, while the pressure of the air on the outside remains practically the same, the result being that the can is crushed. At the sea-level, under ordinary conditions, the pressure of the air is 15 lb. on every square inch.

The following experiments also illustrate effects of atmospheric pressure:

EXPT. 68.—**Action of a syringe.** Dip the open end of a glass syringe or squirt into a bowl of water. Pull up the piston, and notice that the water follows it, owing to the pressure of the atmosphere upon the surface of the water in the bowl. The action of a pump is very similar to this.

EXPT. 69.—**Upward pressure.** Take a tumbler or cylinder with ground edges and fill it completely with water. Slide a piece of stout writing paper across the top and invert the vessel. If the air has been excluded from the cylinder carefully the water does not run out (Fig. 62). Think what keeps the paper in its place.

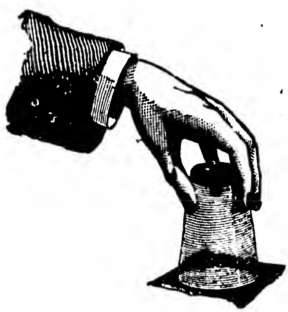


FIG. 62.—Effect of atmospheric pressure.

Principle of the mercurial barometer.—It has been seen that the air has weight, and that it exerts pressure on the earth's surface; we have now to learn how this pressure is measured.

EXPT. 70.—**Construction of a barometer.** Procure a barometer tube about 32 inches long, and bind a short piece of india-rubber tubing upon its open end. Bind the free end of the tubing to a glass tube about six inches long open at both ends. Rest the barometer tube with its closed end downwards and pour mercury into it (being careful to remove all air bubbles) until the liquid reaches the short tube. Then fix the arrangement upright as in Fig. 63.

FIG. 63.—To explain the principle of the barometer.

The mercury in the long tube will be seen to fall so as to leave a space of a few inches between it and the closed end. The distance between the top of the mercury column in the closed tube and the surface of that in the open tube will be found to be about thirty inches.

The instrument used in Expt. 70 is evidently similar to a U tube. Referring to Fig. 63 it is clear that there is a column of mercury supported by some means which is not at first apparent, or else the mercury would sink to the same level in the long and the short tubes, for we know that liquids find their own level. If a hole were made in the closed end of the tube this would happen immediately. There will be no difficulty from what has been said already, in understanding that the column of mercury is kept in its position by the weight of the atmosphere pressing upon the surface of the mercury in the short open tube. The weight of the column of mercury and the weight of a column of the atmosphere with the same sectional area is exactly the same; both being measured from the level of the mercury in the short stem of the apparatus shown in Fig. 63, the mercury column to its upper limit in the long tube, the air to its upper limit, which is a great distance from the surface of the earth. When for any reason the weight of the atmosphere becomes greater, the mercury is pushed higher to preserve the balance; when it becomes less, then similarly the amount of mercury which can be

supported is less, and so the height of the column of mercury is diminished.

The height must in every case be measured above the level of the mercury in the tube or cistern open to the atmosphere. In the arrangement shown in the accompanying illustration, a line is drawn at a fixed point O, and the short tube is shifted up or down until the top of the mercury in it is on a level with the line.

The student will now understand why it is so necessary to remove all the air bubbles in Expt. 70. If this were not done, when the tube was inverted the enclosed air would rise through the mercury and take up a position in the top of the longer tube above the mercury. The reading would not then be thirty inches, for instead of measuring the whole pressure of the atmosphere, what we should be measuring really would be the difference between the pressure of the whole atmosphere and that of the air enclosed in the tube. In a properly constructed barometer, therefore, there is nothing above the mercury in the tube except a little mercury vapour.

An arrangement like that described constitutes a **barometer,** which we can define as **an instrument for measuring the pressure exerted by the atmosphere.**

EXPT. 71.—**Variation of pressure** (Expt. 70) has shown that air pressing upon the surface of the mercury in the short open arm of the U-tube will balance a long column of mercury in the closed arm. Slip a piece of india-rubber tubing upon the open end and notice what happens when you blow sharply into it. Suck air out of the tube, and observe the result.

These experiments illustrate the effect of increasing and decreasing the pressure upon the free surface of the mercury.

Weight of column of atmosphere.—The following is another form of the experiment to show atmospheric pressure by means of a barometer.

EXPT. 72.—**Simple barometer.** Procure a thick glass tube about 36 inches long and closed at one end. The tube must be quite clean and dry. Fill the tube with dry, clean mercury, leaving a small air bubble at the top. Close the tube with the thumb, and slope the tube downwards so that the bubble of air travels along the whole

length of the tube; slope the tube upwards, so that the bubble returns to the open end. Thus all small air bubbles are removed from the sides of the tube. Fill up the tube with mercury, place your thumb over the open end; invert the tube; place the open end in a cup of mercury and take away your thumb.

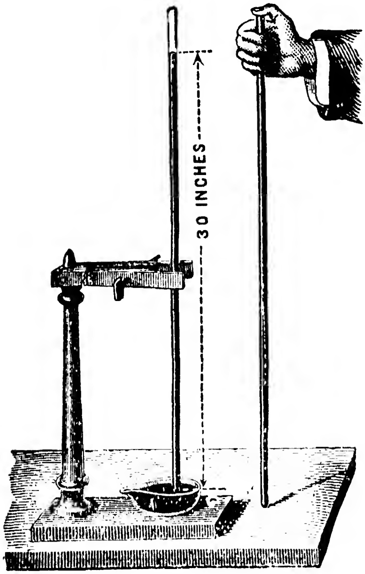


FIG. 64.—A simple barometer.

A column of mercury will be supported in the tube by the pressure of the atmosphere. The distance between the top of the column and the surface of the mercury in the cup will be about 30 inches, or 76 cm., when the tube is vertical. When the tube is inclined so that the closed end of it is less than this height above the mercury in the cup the mercury fills it completely; and when the tube is less than 30 inches long, it is filled by the mercury also. On an average, the atmosphere at sea-level will balance a column of mercury 30 inches in length. No matter if the closed tube be 30 feet long, the top of the mercury column will be about 30 inches only above the level of the mercury in the cistern.

If the tube had a bore with a sectional area of exactly one sq. cm., there would be 76 c.c. of mercury in a column 76 cm. long; and since 1 c.c. of mercury weighs 13·6 gm., the whole column would weigh 1033·6 gm. This column balances a column of air of the same area, so that we find that the weight of a column of air upon an area of one sq. cm. is 1·03 kgm. when the barometer stands at 76 cm. Converting these numbers into British units, it will be seen that the pressure on one square inch of a mercury column 30 in. high is 15 lb.

Mercury a convenient liquid for barometers.—The use of mercury for barometers is a matter of convenience. Since the column of mercury which the atmosphere is able to support is

30 inches high, it is clear that, as water, *e.g.*, is 13·6 times less dense than mercury, the column of water which could be supported would be $30 \times 13{\cdot}6 = 408$ inches $= 34$ feet, which would not be a convenient length for a barometer. The length of the column of glycerin which can be supported similarly is 27 feet. But in the case of lighter liquids like these, any small variation in the weight of the atmosphere is accompanied with a much greater alteration in the level of the column of liquid, and in consequence it is possible to measure such variations with much greater accuracy. For this reason barometers are sometimes made of glycerin.

Pressure of the atmosphere at different altitudes.—The atmosphere being a material substance, the shorter the column of it there is above the barometer, the less is the weight of that column, and the less the pressure it exerts upon the mercury in the barometer. Hence, as we ascend through the atmosphere with a barometer, we reduce the amount of air above it pressing down upon the mercury in it, and in consequence the column of mercury the air is able to support becomes less and less as we ascend. On the contrary, if we can descend from any position, *e.g.* down the shaft of a mine, the mercury column is pushed higher and higher as we gradually increase the length of the column of air above it. Since the height of the column of mercury varies thus with the position of the barometer, it is clear that the variation in its readings supplies a means of ascertaining the height of the place of observation above the sea-level, provided we know the rate at which the height of the barometer varies with an alteration in the altitude of the place. The rule which expresses this relation is not a simple one, but for small elevations it is said that a rise or fall of one inch in the height of the barometer corresponds to an alteration of 900 feet in the altitude of the barometer.

Boyle's Law.

Relation between volume and pressure of a gas.—To understand how and why the density of the atmosphere varies, it is necessary to become acquainted with the rule expressing the relation between the volume and pressure of a gas. This can

be done satisfactorily by one of the forms of apparatus employed in the following experiments, which provide a means of subjecting an enclosed quantity of air to varying pressures, by the addition of smaller or larger quantities of mercury.

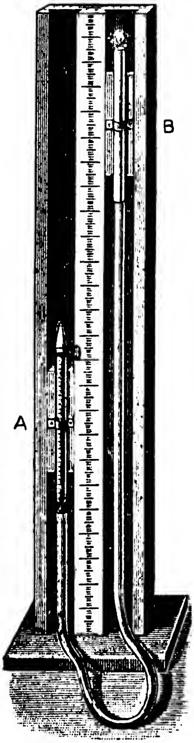


FIG. 65.—Boyle's law apparatus.

In Fig. 65, two glass tubes are supported on blocks which slide in grooves on a vertical stand and are joined at their lower ends by stout rubber tubing; one of the tubes, B, is open at its upper end, and the other, A, is either sealed off with its upper end nearly flat or else it terminates in a glass tap. Before proceeding to take observations, the apparatus is arranged so that, when mercury contained in the apparatus is at the same level in both tubes, the closed tube is about half full of dry air. Under these conditions the enclosed air is acted upon by a pressure equal to that of the atmosphere, which is obtained by reading the height of the barometer at the time of the experiment. If the closed tube be graduated the volume of the enclosed air is obtained at once by taking the scale-reading of the mercury surface in the tube; if the tube be ungraduated but uniform in bore, the *length* of the air column may be taken as numerically equivalent to the volume of the enclosed air. Having taken these readings of pressure and volume, the open tube is raised until the mercury surface in the open tube is several centimetres higher than that in the closed tube. The enclosed air is now under a total pressure equal to the sum of that due to the atmosphere and that due to a column of mercury equal in length to the difference of levels of the mercury in the two tubes. The volume of the enclosed air is now less than before. A series of such readings of pressure and volume are taken, the open tube being raised finally to the greatest extent permitted by the apparatus. Similar readings are taken with the open tube *lower* than the closed tube; in this case the pressure acting on the enclosed air is *less* than that of the atmosphere, and its magnitude is obtained

by *subtracting* the difference of level of the mercury surfaces from the height of the barometer.

EXPT. 73.—**Relation between pressure and volume.** Take a series of readings of the *pressure* acting on the enclosed air and of the *volume* of the air by means of apparatus similar to Fig. 65. The difference of level of the mercury surfaces is obtained readily by measuring the heights of the surfaces above the base-board by means of a metre scale. Record your results thus :

Height of Barometer.	Difference of Level of Mercury.	Total Pressure on the Air, P.	Volume of Air, V.	Total Pressure × Volume, P × V.
Cm.	Cm.			

From these observations it will be found that the volume regularly diminishes as the pressure is increased, and in the same proportion. The converse is also found to be true, viz., that as the volume of a gas increases the pressure upon it diminishes, and exactly in the same proportion. But, in both these cases, it is understood that the *temperature of the gas remains the same*, that is, the temperature of the gas under the different pressures must not alter.

The tabulated results of the experiments reveal another important relation, which is, however, another way of expressing those already noticed. It is found that, when there is no alteration of temperature, the product obtained by multiplying the volume of a given mass of gas by the pressure to which it is subjected is always the same, or remains constant.

These facts were discovered by Robert Boyle in 1662, and are included in what is known as **Boyle's Law.** It can be expressed by saying that **when the temperature remains the same, the volume of a given mass of gas varies inversely as its pressure.** Or, what is the same thing, **the temperature remaining the same, the product of the pressure into the volume of a given mass of gas is constant.**

But it has been learnt that if the volume occupied by a given mass of a substance be increased, its density is decreased, and if the volume be decreased, its density is increased. Therefore, by decreasing the volume of the enclosed air in the above experiment, its density is increased. The increase of density and the increase of pressure are proportional to one another. It is not

difficult to apply these facts to the case of the atmosphere. It has been stated that the pressure of the atmosphere decreases as we ascend, and we are now able to add that its density decreases also and at the same rate. Therefore the densest atmosphere is that at the surface of the earth, leaving out, of course, the air of mines and other cavities below the surface, where the air is denser still. The air gets less dense, or rarer, as we leave the earth's surface, until eventually it becomes so rare that its existence is practically not discernible.

The following experiment provides an alternative method of illustrating Boyle's Law:

EXPT. 74.—**Simple method for Boyle's Law.** Obtain a length of thermometer tubing, (Fig. 66), about 75 cm. long and 1 mm. bore. Seal it at one end and expand the open end somewhat. Clamp the tube in a vertical position, with the closed end below, by the side of a metre scale, and connect a small funnel to the top by means of a short piece of rubber-tubing. Pour a little pure, clean mercury into the funnel and induce it to run down the bore of the tube by inserting a thin, clean, steel wire. In this way any desired volume of air can be enclosed.

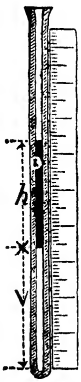


FIG. 66. —Simple experiment on Boyle's law.

The length of the column of enclosed air may be taken to represent its *volume* (V). If H = the height of the barometer, and h = the length of the mercury thread (both expressed in the same units), then the total pressure on the enclosed air = $(H + h)$.

Introduce more mercury in the same manner, and in this way alter the values of V and h. The volume of the air under the pressure of the atmosphere alone can be observed by laying the glass tube flat on the table; and the volume under pressures less than that of the atmosphere can be observed by inverting the tube with its open end downwards.

Perform several experiments and record the results in the following way:

Volume (V).	Pressure $(H \pm h)$.	Volume × Pressure.

SOME INSTRUMENTS DEPENDING UPON ATMOSPHERIC AND FLUID PRESSURE.

The air-pump.—Several forms of air-pumps are in use, but in this place it will be sufficient to describe one of the simplest, that designed by Hawksbee, the essential parts of which are shown in Fig. 67. V is the receiver, from which it is required to remove air. V is connected with a cylinder *cv* by means of a tube, shown on the base in the illustration, bent twice at right angles. At the end of this tube remote from the receiver, and just at the bottom of the cylinder *cv*, is a valve *v* opening upwards. In the cylinder works, in an air-tight manner, a piston provided with a valve *v′* opening upwards; and a handle for pulling the piston up and pushing it down is provided. The action is very simple. Imagine the piston to be at the bottom of the cylinder to begin with, and then that it is pulled up gradually. As this takes place, the air in the receiver and below the valve *v* is subjected to a diminished pressure, and consequently expands, filling the space which is formed as the piston moves upwards. This expansion continues until the piston arrives at the end of its stroke. The piston is now pushed down. This movement compresses the air between *v′* and *v* and increases its pressure, causing the valve *v* to shut. But as the piston descends the pressure on the under surface of the valve *v′* becomes greater than that of the atmosphere upon its upper surface, with the result that the valve *v′* opens upwards and the air in the space *vv′* rushes through the open valve into the outside air. The final result, when the piston reaches the bottom of the cylinder, is that there is less air in the receiver and tube connecting therewith than there was originally. As the piston is worked up and down the same opening and shutting of valves is repeated, with the result, that by and by, nearly all the air is removed from the receiver.

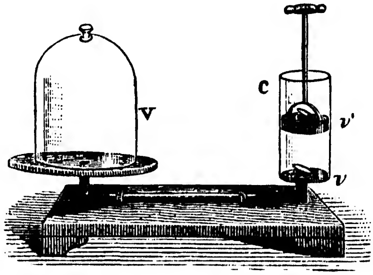


FIG. 67.—Simple form of air-pump.

Sprengel's air-pump.—More perfect vacua can be obtained by a simple form of air-pump, due to Sprengel, in which the piston of Hawksbee's instrument is replaced by drops of mercury and

in which valves are dispensed with. The essential parts of this pump are shown in Fig. 68. The flask, or other vessel, which it is desired to exhaust of air is connected with the tube at A.

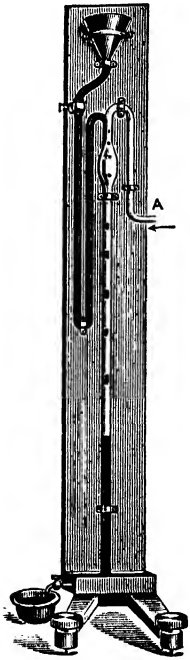


FIG. 68.—The principle of Sprengel's air-pump.

Mercury is poured into the funnel and falls continuously down the long vertical tube. As each drop of mercury passes the opening at the top of the tube connected with the vessel to be exhausted, it carries with it a little air, until eventually, after the stream of mercury has been running for some time, practically the whole of the air in the vessel is removed.

The common pump.—After examining a glass model like that shown in Fig. 69, there is no difficulty in understanding the action of a common pump. To begin with, suppose that the pump is full of air and that the end of the tube below the valve *b* is dipped into a basin of water. The piston *ec* is, to start with, at the bottom of its stroke near the valve *b*. As the piston is raised the air in the cylinder above *b* expands; its pressure consequently decreases; the pressure on the lower surface of the valve *b* is, therefore, soon greater than that on its upper surface, and the valve is pushed upwards by the air below it, the air flowing into the cylinder *acb*. The result is that the air in the cylinder below the piston is at a lower pressure than that of the outside air, and as a consequence water is pushed up the tube *b*. This action continues until the piston reaches the end of its stroke towards the top of the pump.

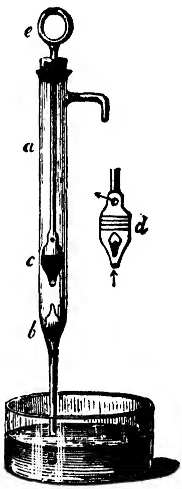


FIG. 69.—Model of a common pump.

As the piston descends, the air in the cylinder below the piston *c* is compressed and its pressure becomes gradually greater. The valve *b* closes and that in *c* opens, through which latter, of course, the air in the cylinder escapes. On raising the piston

again the same effects are repeated until all the air in the pump is removed and the outside air pushes the water up until it reaches the spout and escapes.

As the air is able to support a column of mercury 30 inches in height, and as mercury is about $13\frac{1}{2}$ times heavier than water, the air can support a column of water of a height equal to

$$30 \text{ inches} \times 13\tfrac{1}{2} = 2\tfrac{1}{2} \times 13\tfrac{1}{2} \text{ ft.} = 33\tfrac{3}{4} \text{ ft.}$$

It will be understood at once, therefore, since the efficacy of the common pump depends wholly upon the pressure of the air, that the spout of the pump must never be more than $33\frac{3}{4}$ feet, or, roughly, 33 feet from the level of the water. On account of leakage and friction against the pipe, water cannot be made to rise much more than 30 feet with an ordinary pump.

The siphon.—The siphon is a simple instrument which depends upon atmospheric pressure for its action. It consists usually of a bent tube, one leg of which is longer than the other. It is filled with the liquid to be transferred from one vessel to another, and while both ends of the tube are kept closed, the shorter limb is placed into the vessel of liquid. The result is that the liquid flows until the level of the liquids is the same in both vessels, or the higher liquid has been siphoned to the lower level.

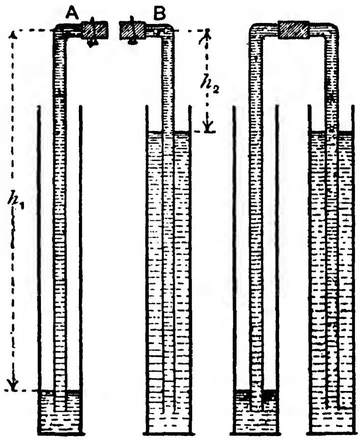


FIG. 70.—Principle of the action of the syphon.

Fig. 70 represents two narrow glass tubes, closed at the upper ends with rubber tubing and clips, filled with mercury, and with their lower ends immersed in mercury contained in tall cylinders. The cylinder on the left is nearly empty, and that on the right is nearly full. If the atmospheric pressure, as observed with a barometer, is H cm. of mercury, and if the vertical distance between the top of tube A and the mercury surface in the cylinder is h_1 cm., then the pressure at the top of tube A is $(H - h_1)$ cm. of mercury. If the corresponding height of the top of tube B is h_2 cm., then the pressure at the top of tube B is $(H - h_2)$ cm. of mercury. Since h_1 is greater than h_2, the pressure at the top of B is greater than the pressure at the top of A, by an amount equal to

$(h_1 - h_2)$ cm. of mercury. Suppose that the two tubes are now joined together, as shown in Fig. 70: the greater pressure at the top of the right-hand tube will push the liquid from right to left. The flow of liquid continues until the level of liquid is the same in each vessel.

EXERCISES ON CHAPTER VII.

1. About what height does the mercury column of the barometer generally stand? How is the reading affected if the tube is not in a vertical position? Calculate the length of the column of a barometer which is inclined at an angle of 30° to the vertical, the true barometric height being 30 inches.

2. What is the height of a water barometer when a mercurial barometer reads 30 inches? If the mercury barometer falls to 29 inches, through what distance would a water barometer fall under the same conditions?

3. If the atmospheric pressure be 15 pounds per sq. inch, what is the pressure in kilograms per sq. centimetre?

4. What is the atmospheric pressure, in pounds per sq. inch, when the height of the barometer is 28·5 inches?

5. A quantity of gas occupies 323 c.c. when the barometer reads 72 cm. What volume will it occupy at 76 cm. if there be no change of temperature?

6. A litre of air weighs 1·293 gm. at normal pressure, and when the temperature is 0° C. What weight of air at the same temperature will a litre flask hold when the barometer stands at 78 cm.?

7. A 20-cm. cube weighs 23·5 gm. in air at 0° C. and normal pressure. What will it weigh (i) in hydrogen, and (ii) in carbonic acid gas, under the same conditions of temperature and pressure? [At normal pressure and 0° C., 1 litre of air weighs 1·293 gm., 1 litre of hydrogen weighs 0·09 gm., and 1 litre of carbonic acid gas weighs 1·98 gm.]

8. When the pressure of the air is normal, what is the greatest height over which you could siphon mercury? At the same atmospheric pressure, what is the greatest height over which you could siphon sulphuric acid (relative density, 1·84)?

9. Describe carefully any experiment you have seen or done to show that the air exerts pressure, and sketch carefully the apparatus used.

One arm of a bent tube containing water is attached by rubber tubing to the gas supply and a difference of level of 6·5 cm. is obtained when the gas is turned on. Find the pressure of the gas in grams per sq. cm., and in lb. per sq. inch. (2·54 cm. = 1 inch, 453 gm. = 1 lb.)

10. Why is mercury generally used in making a barometer? Could a barometer be made using water as the liquid? State and explain what would happen if a hole were bored through the glass of the barometer above the mercury.

11. Describe exactly what you would need to make a rough barometer, and how you would proceed to do it. If you made successively two such barometers, and found on comparing them that the mercury did not stand at the same height in the two, which would more nearly indicate the true pressure of the atmosphere, and what would almost certainly be the cause of the difference?

12. Would the difference in vertical height between the two barometers in the last question be greater when the tubes were vertical, or when they were both sloped away from the vertical to the same extent? Would the difference between them alter if the atmospheric pressure were to change? Why, in each case?

13. Water cannot be raised to a height much greater than 30 feet by means of a common pump. State the reason of this and describe a laboratory experiment by which you could prove your explanation to be correct.

14. State Boyle's Law. Describe an experiment you have performed to verify the law, and mention any precautions you took to ensure accuracy.

15. (*a*) Why does the mercury stand higher in the tube than in the cup of a barometer? (*b*) What is the average height of the mercury in a barometer tube at the sea-level?

16. Describe how to make a simple barometer, and explain its working. What will be seen if (*a*) the tube be sloped away from the vertical, (*b*) the cistern and the lower part of the tube be immersed in a deep vessel of water, (*c*) the whole instrument be taken in a lift to the top of the Eiffel tower? Give explanations in each case.

17. Describe a simple experiment to show that the volume of a quantity of gas changes with the pressure upon it. What is the exact relation between the volume and the pressure? A small india-rubber balloon is partly filled with air, tied at the mouth, and loaded so as to sink in a vessel of water in which it is placed; and the whole is now placed under the receiver of an air pump and the air exhausted from the receiver. What will happen, and why?

18. If I take a barometer tube, the internal sectional area of which is one-fourth of a square inch, calculate, from the known pressure of the atmosphere, the weight of mercury which would be supported in the tube. When the mercury stands at the height of 30 inches in the barometer, what is the weight of air pressing on an acre of ground? (There are 5,280 feet in a mile (linear) and 640 acres to the square mile.)

CHAPTER VIII.

MOTION, FORCE, AND INERTIA.

Definition of motion.—The word *motion* is meant to convey the idea of *change of place.* The simplest forms of motion are changes in the positions of bodies with regard to one another. When a boy runs down the street he is in motion; as regards the houses and lamp-posts he moves. To describe fully the boy's motion it would be necessary to know his **velocity**, that is, the *direction* in which he is moving or the *line* along which he runs, and the *speed* with which he travels. If during every second through which he moves he travels over a distance of five yards, he has a **uniform speed** of five yards a second.

But suppose he does not move regularly over five yards in every second; he sometimes dawdles, sometimes stops to look at a shop, at other times he puts on a spurt to make up for lost time. How should his motion be described now? His rate varies from time to time, or he may be said to have a **variable speed**, and to describe such a variable speed it is usual to speak of the speed *at any instant* as being a certain number of yards per second. Suppose the boy moving with a variable speed had at a given instant a speed of eight yards per second. If he continued to move at the same rate he would travel over eight yards in the succeeding second.

A distinction in meaning between the terms **speed** and **velocity** should be observed always. The former term refers simply to the distance traversed in a given time and irrespective of the direction; whereas the latter term implies that the space is traversed in a known direction. Thus, in the case of a train travelling round a curve with a speed of 60 miles per hour, the

speed may be constant, but the velocity is changing at every instant.

Average speed.—For many problems it is necessary to know the **average speed** of the moving body. Returning to the boy, suppose he travelled 800 yards in 400 seconds; if the first number be divided by the second the boy's average speed is obtained, namely, two yards in a second; this then is the speed with which he would have had to travel, if he moved uniformly, in order to complete his journey in the same time.

The **unit of speed** is generally taken as being a speed of one foot per second. Thus a speed of six means a speed of six feet per second.

Measurement of uniform linear velocity.—It is a very simple matter to calculate the velocity of a body moving uniformly in a straight line when the distance it has travelled, measured in units of length, and the time it has taken to perform the journey, measured in units of time, are known. All that has to be done in order to find its uniform velocity (v) is to divide the number of units of length (s) passed over by the number of units of time (t) taken to complete the distance. Thus:

$$v = \frac{s}{t} \quad \text{or} \quad s = vt.$$

Velocities can be represented completely by straight lines.—To determine a velocity completely its magnitude (or the distance travelled in a given time) and its direction must be known. But, a straight line can be drawn in any direction and of any length; and it can be arranged that the length shall contain as many inches or feet, whichever is more convenient, as there are feet or yards per second of velocity, depending on the way in which we decide to measure our velocities. Velocities can therefore be represented completely by straight lines.

EXPT. 75.—**Same direction.** Taking a line an inch long to represent a velocity of one foot per second, draw lines representing velocities of $3\frac{1}{2}$, $2\frac{3}{4}$, 4, and $1\frac{1}{4}$ feet per second, making the lengths of the lines proportional to the rates of motion.

EXPT. 76.—**Parallel directions.** Draw a line to represent the velocity of a river flowing at the rate of 2 miles an hour. Suppose

a man who can row 6 miles an hour in still water to be rowing in this river. Draw lines to represent his velocity with reference to the bank when rowing (1) with the stream, (2) against the stream.

EXPT. 77.—**Inclined directions.** Draw a large circle upon a sheet of paper. Draw two diameters at right angles to each other. Taking ½ inch to represent a velocity of 1 foot per second, and starting from the centre of the circle, represent by graphic construction the path of a body moving with the following velocities: 2 ft. per sec. N.E.; 2 feet per sec. N.; 3 feet per sec. W.; 4 feet per sec. S.E.

Composition of velocities.—Consider the case of a marble moving along a tube with a uniform velocity, when the tube itself is all the time being moved uniformly across a table (Fig. 71). It is evident that since the marble is in the tube it must have the same velocity *across* the table that the tube has;

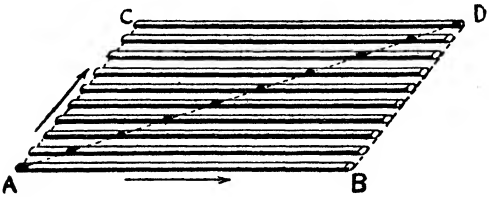


FIG. 71.—To illustrate the combination of two velocities, and the principle of the parallelogram of velocities.

and at the same time it moves along the tube, that is, in a direction at right angles to its former velocity. There are thus two independent velocities to be considered—one the velocity of the tube and the other the velocity of the marble relative to the tube. Similarly, we can think of a ship sailing across the ocean with a man on deck walking from one side of the ship to the other. The man is moving onwards with the ship at a certain velocity, and at the same time he is moving across the ship with another velocity.

Really, a body can only move at any instant in one direction with one definite velocity. The best way to consider the composition of velocities is to think of the velocities as existing successively rather than at the same instant, in order to find the actual change of position in any time in the case of the marble or of the man referred to above. The velocity to be found is called the **resultant** of the two independent velocities, which are themselves spoken of as **components.** If the two

velocities have the same direction, the resultant is their sum; and if they are in opposite directions along the same straight line the resultant is their difference. If they have directions which make an angle with one another, it is clear that the resultant must lie somewhere between the components.

Referring to the case of the marble, let OA (Fig. 72) represent by its length the number of inches the marble moves along the tube in a second, and OB the distance moved by the tube, and consequently by the marble, in the same time across the table. Draw BR parallel to OA and AR parallel to OB, thus completing the parallelogram, then the line OR represents the actual change of position which would be produced if the two velocities in question existed successively for equal intervals of time. In the same way in the second example OA could stand for the ship's velocity and OB for the man's, then OR would represent the resultant of the two velocities. When therefore, the velocity of a body A (such as the marble or the man) relative to a body B (such as the tube or the ship) is represented by one side of a parallelogram and the velocity of B relative to C (such as the table or the ocean) is represented by the other side, the velocity of A relative to C is represented by the diagonal. This principle is called the **Parallelogram of Velocities.**

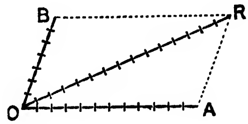


FIG. 72.—Parallelogram of velocities.

Meaning of acceleration.—An express train starting from a terminus begins to move slowly, and, as the journey proceeds, the rate of motion goes on increasing until the train gets its full speed. A stone let fall from a height similarly starts from rest, and as it moves it travels faster and faster until brought to rest again on reaching the ground. Or, we may imagine a cyclist starting for a ride, and regularly increasing his speed until he could not go any faster. In all these examples the velocity of the moving body has increased regularly, and the rate at which the change has taken place is spoken of as **acceleration.**

Acceleration is the rate of change of velocity.—But acceleration may be of an opposite kind to the instances given above. Reverse each of the examples and consider what happens. An

express train going at full speed approaches a station, and its velocity is diminished regularly until it is brought to rest at the platform. A stone is thrown upwards with a certain velocity, it moves more slowly and more slowly until it comes to rest, and then starts falling. A cyclist travelling at full speed slackens his rate regularly until he comes to a standstill. In all these cases we have examples of an acceleration of an exactly opposite kind to the previous instances, but yet an acceleration. In ordinary language this kind of acceleration is given a name of its own, viz. *retardation.*

EXPT. 78.—**Motion down an inclined board.** Obtain a smooth board about six feet long, having a slight groove cut in it from one end to the other. Incline the board slightly at one end. Place a marble or other small sphere near the raised end and let it roll down the board. Notice that as it moves its velocity increases. To show that the space traversed increases every second, fit up a seconds' pendulum and set it in motion. Let the bob of the pendulum strike against a sheet of paper or some other light object at the end of each swing, so that you can hear when the seconds commence. Now start the marble from a mark upon the board exactly when the pendulum taps the paper on one side. Notice how far the marble has rolled by the time the pendulum taps the paper on the other side. Make a mark at the place reached, and do the same for succeeding seconds until the marble rolls off the board. Measure the length of board traversed by the marble in each second. The distances will be found to increase in proceeding down the board from the starting place.

Measurement of uniform acceleration.—In measuring a regular or **uniform acceleration,** it is necessary to know what addition to, or subtraction from, the velocity of the moving body there has been during each second of its journey. Suppose there is an addition of one foot per second to the velocity of a moving body, and that it has taken one second to bring about this change, we should refer to this as an acceleration of one foot per second in a second, or **one foot per second per second.** An acceleration which increases the velocity is referred to as *positive,* while that which diminishes it is *negative.* The first examples given above are instances of positive acceleration, while when we reverse them they afford cases of negative acceleration.

Unit of acceleration.—As in every other measurement, so, when accelerations have to be measured, it is necessary to have a unit in terms of which the quantity under consideration can be expressed. **The unit of acceleration is an increase of unit velocity in a unit of time**; it is generally taken as equal to **an increase of velocity of one foot per second per second.** An acceleration of *two units* would thus be an increase of velocity of *two* feet per second per second; similarly, an acceleration of three units equals an increase of velocity of three feet per second per second, and so on for any number of units.

Acceleration, like velocities, can be represented graphically by straight lines.

Equations of motion.—If we take the unit of acceleration as equal to an increasing velocity of one foot per second in one second, an acceleration of f means an increase of velocity of f feet per second in one second. Suppose a body starts from rest, at the end of the first second it has a velocity of f feet per second, at the end of the next second $2f$, at the end of t seconds ft feet per second. Or if v = change of velocity in t seconds we can write

$$v = ft. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The space travelled over by a body in one second is equal to its average velocity, and that travelled over in t seconds is equal to its average velocity multiplied by t. If it starts from rest and travels for t seconds, finishing with a velocity of v feet per second, its average velocity is $\frac{1}{2}v$ during this time, and

$$\therefore \quad s = \tfrac{1}{2}vt. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Substituting the value of v from equation (1) we get

$$s = \tfrac{1}{2}ft \times t = \tfrac{1}{2}ft^2,$$

or since from (1) $t = \frac{v}{f}$ we can write equation (2) thus:

$$s = \tfrac{1}{2}v \times \frac{v}{f} = \tfrac{1}{2}\frac{v^2}{f},$$

from which

$$v^2 = 2fs. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

If the body be moving freely towards the earth its acceleration is g (see p. 102), and equation (3) becomes

$$v^2 = 2gs. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Acceleration due to gravity.—It has already been shown (Expt. 5) that the time of vibration of a pendulum varies as the square root of the length. The time of oscillation can, by a simple application of dynamical principles, be shown to be given by the expression:

$$\text{Time of oscillation} = 2 \times 3\tfrac{1}{7}\sqrt{\frac{\text{length}}{\text{gravity}}}, \quad \text{or} \quad t = 2\pi\sqrt{\frac{l}{g}},$$

where t stands for the time of oscillation, l for the length of the pendulum, g for the value of the acceleration due to gravitation,* and π for the ratio between the circumference and diameter of a circle.

From this equation it is easy to obtain an expression for the value of g. Thus, squaring both sides, we have

$$t^2 = \frac{4\pi^2 l}{g},$$

so that

$$g = \frac{4\pi^2 l}{t^2}.$$

EXPT. 79.—**Determination of "g."** Using the observations you have made as to the time of a double swing of your pendulum, and taking as the length of the pendulum the distance in feet from the bottom of the support holding the thread to the centre of the ball, determine by means of this equation the value of g.

EXPT. 80.—**Second method.** Calculate the value of g as in the preceding experiment, but express the measurement in centimetres.

Equality of masses.—Motion and mass have hitherto been considered separately, but now their relations to one another will be described. It has been shown that equality of mass can be tested by weighing. The balance thus provides a convenient practical method of comparing masses, but it does not give a fundamental conception of what mass means. To obtain a clear idea of the subject, consider first of all that we are dealing with two variable quantities, namely, mass, or quantity of matter, and motion. Suppose two bodies moving in opposite directions with *equal* velocities to collide with one another and stick together. If the two bodies stopped dead after the impact we could conclude that their masses were equal, and that each exactly destroyed the motion of the other; but if the combined bodies

* The quantities l and g must be expressed in the same units.

moved after the collision, the masses could evidently not have been equal. With this in mind, it will be conceded readily that the following definition of equality of mass holds good: **Two masses are equal, if when they are made to impinge on one another in opposite directions with equal velocities and stick together, they come to rest.***

The effect of the impact of two moving bodies thus depends upon the masses of the bodies and the velocities before the collision. If both the velocities and masses are equal, the bodies come to rest; if the velocities are equal but the masses are unequal, the greater mass predominates after collision, and if the masses are equal while the velocities are unequal the greater velocity will predominate.

Momentum.—The preceding paragraph has introduced the idea of a condition involving both motion and mass. This condition is known as momentum, which is defined as follows:

The momentum of a body is the quantity of motion it has, and is equal to the product of its mass and its velocity.

Expressed as an equation we have

$$\textbf{Momentum} = \textbf{mass} \times \textbf{velocity},$$

or if momentum is represented by $\mathbf{M}$, mass by $\mathbf{m}$, and velocity by $\mathbf{v}$, all expressed in corresponding units, we can write

$$\mathbf{M} = \mathbf{mv}.$$

The unit of momentum is consequently that of a unit of mass moving with a unit of velocity, or if the unit mass be that of the imperial standard pound and the unit velocity a velocity of one foot per second, **the unit of momentum is the quantity of motion in a mass of one pound moving with a velocity of one foot per second.** The meaning of momentum will be better grasped after a concrete example.

When a shot is fired from a cannon, the same momentum is generated in both the cannon and the shot; but since the mass of the cannon is immensely greater than that of the shot, it will be evident that the velocity of the shot must be correspondingly

*This definition, and the general treatment here adopted of matter in relation to motion are based upon Prof. W. M. Hicks's work on *Elementary Dynamics of Particles and Solids.*

greater than that of the cannon in order that the product of the two quantities may be the same. This we know is the case, the velocity of the "kick" or "recoil" of the cannon being very much less than the velocity with which the shot is sent on its journey.

From the point of view of momentum, however, the action and the reaction are equal and opposite. Thus a shot weighing 28 lb. is fired from a cannon weighing 10 tons, that is 22,400 lb., and the shot leaves the gun with a velocity of 200 feet per second. The velocity of the recoil of the cannon is therefore one-quarter of a foot per second (for $28 \times 200 = 22{,}400 \times \frac{1}{4}$).

Third law of motion.—Newton expressed the fundamental principles of the relationship between matter and motion in three laws, known as his Laws of Motion. The third law (which may be considered before the others) states that **action and reaction are equal and opposite.**

Consider the case of a heavy magnet and an iron nail suspended by threads so that they can move toward each other. The nail will move faster than the magnet, and its acceleration will be greater than that of the magnet by exactly the same amount that the mass of the magnet exceeds the mass of the nail. The action and reaction are equal and opposite, so the momentum of the nail is the same as that of the magnet. We can say, therefore, that **the masses of two bodies are inversely proportional to the accelerations which they acquire in virtue of their mutual action and reaction.** This definition of mass involves no assumption, and is true whether we think of the mutual action, or **stress**, as between a nail and a magnet, between the sun and the earth, or between any two bodies or particles.

Force.—Suppose a body to possess a certain momentum; then for the momentum to change or tend to change, something must act upon the body, and that something is termed **force.** In other words: **When a gradual change of momentum is either produced or tends to be produced in a body, that body is acted on by force.**

It must be understood clearly that by thus defining force we do not get to know anything more about it. Nobody can tell what force is. All we can know are the effects produced by a something we call force.

Since a change of momentum is produced by force, the rate at which the momentum changes may be used as a measure of

force, and we can say, therefore, that **equal forces are those which produce equal momentums in equal times.**

Acceleration produced by a force.—The momentum of any particular body is determined by the body's mass and velocity. Since the mass of the body may be regarded as constant, change of momentum can be produced only by changing the velocity. But rate of change of velocity is acceleration, hence when a body is moving with accelerated velocity, the momentum is altered, and an alteration of momentum signifies, as has been explained, that the body is being acted upon by a force. If the acceleration be uniform, the body must be acted upon by a uniform force.

Hence we come to the very important fact that **the number of units of force in any force is equal to the product of the number of units of mass in any body on which it may act and the number of units of acceleration produced in that mass by the force in question.**

The relation between force, mass, and acceleration may be expressed algebraically as follows:—Let F represent the number of units of force in a given force, m the number of units of mass on which it acts producing a units of acceleration, then the definition can be written,

$$F = m \times a,$$

from which equation the third quantity can be obtained whenever we know the other two:

$$\begin{matrix}\text{Number of} \\ \text{units of force}\end{matrix} = \begin{matrix}\text{Number of} \\ \text{units of mass}\end{matrix} \times \begin{matrix}\text{Number of units} \\ \text{of acceleration.}\end{matrix}$$

$$F = ma \text{ ; } \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\therefore \; a = \frac{F}{m}, \ldots\ldots\ldots\ldots (2) \qquad \text{or} \qquad m = \frac{F}{a}. \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The second equation can be expressed in words by saying that the number of units of acceleration produced in the velocity of a moving body is equal to the number of units of force acting upon it, divided by the number of units of mass on which it acts.

Similarly, the third expression means that the number of units of mass in a moving body can be calculated by dividing the number of units of force acting upon it by the number of units of acceleration produced in it.

The second equation tells us. moreover, that if the acceleration produced in a moving body remains the same, or is uniform, that

the value of the force, or the number of units of force it contains, must be the same throughout, or what is the same thing, the force is uniform.

Second law of motion.—This law is stated generally by saying that **change of motion is proportional to the impressed force, and takes place in the direction in which that force acts.** This expression, 'change of motion,' implies something more than the conception of motion as a mere change of place. By change of motion is meant **rate of change of momentum.**

The second law of motion states that the momentum generated in unit time by a force of two units is twice as great as that produced by one unit; and it implies, moreover, that a force of one unit acting for two seconds produces twice the momentum which it would do if it only acted for one second.

Absolute units of force.—It has been shown that the unit of force is the force which, acting on unit mass, produces in it unit acceleration.

In the **British** (or F.P.S.) **system** the absolute unit of force is that which, acting on a mass of one pound, gives to it an acceleration of 1 ft. per sec. per sec. It is called the **poundal.**

In the **metric** (or C.G.S.) **system** the absolute unit of force is that force which, acting on a mass of 1 gram, gives to it an acceleration of 1 cm. per sec. per sec. It is called the **dyne.**

Gravitational units of force.—The **weight** of a body is another name for the **force** exerted by gravity on the mass of a body. It varies slightly at different places on the earth's surface, and it also depends upon the distance above sea-level (p. 35). At any one locality it remains constant, and produces a uniform acceleration of g units. The value of g may be taken as approximately 32·2 ft. (or 981 cm.) per sec. per sec.

For every-day purposes the *weight of* 1 *lb.* and the *weight of* 1 *gm.* are used frequently as the units of force in the British and Metric systems respectively. These are termed the **gravitational units of force.**

Hence, in the formula $\mathsf{F} = \mathbf{ma}$, if $\mathbf{m} = 1$ lb., and $\mathbf{a} = 32{\cdot}2$ ft. per sec. per sec., then

$$\mathsf{F} = (1 \times 32{\cdot}2) \text{ poundals,}$$

or
$$\text{the weight of 1 lb.} = 32{\cdot}2 \text{ poundals,}$$

or
$$1 \text{ poundal} = \frac{\text{weight of 1 lb.}}{32{\cdot}2} = \text{wt. of } 0{\cdot}5 \text{ oz. approximately.}$$

Similarly, if $m = 1$ gram, and $a = 981$ cm. per sec. per sec.,

$$\text{the weight of 1 gram} = (1 \times 981) \text{ dynes,}$$

or $$1 \text{ dyne} = \frac{\text{the weight of 1 gram}}{981} = \text{wt. of } 0{\cdot}001 \text{ gm. approx.}$$

Inertia.—Common experience tells everyone that things do not move of themselves. An object at rest remains at rest until it is forced to move. Moreover, if it is moving it tends to go on moving in the same direction and with the same velocity until either is made to change by the application of force. In a word, dead matter is helpless and conservative. **The inability shown by a material body to change by itself its condition of rest or of uniform motion in a straight line is called its inertia.** It is exemplified when a cyclist is stopped suddenly, for the tendency to continue moving is so great that the cyclist, if he be travelling quickly, falls over the handle-bar of his machine This law of inertia is often referred to as Newton's First Law of Motion.

First law of motion.—**Every object remains at rest or moves with uniform velocity in a straight line until compelled by force to act otherwise.**

This law, which Newton first stated as being obeyed always by bodies in nature, means, first, that if a body is at rest, it will remain still until there is some reason for its moving—until some outside influence or force acts upon it. Consider a mass at rest somewhere in infinite space; evidently it will remain at rest so long as it is not acted upon by an external force. Consider, again, that another body is moving in infinite space in a straight line; it will continue to move in this direction until compelled to deviate from this path by some external force acting upon it.

The force of gravitation.—Experiments and observations made by Newton led him to the conclusion that it was the rule of nature for every material object to attract every other object, and that this force of attraction is proportional to the masses of the bodies; a large mass exerts a greater force of attraction than a small mass. But the farther these bodies are apart the less is the attraction between them, though it is not less in the

proportion of this distance, but in that of the square of the distance. This law may be stated thus:

Every body in nature attracts every other body with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between the bodies; and the direction of the force is in the line joining the centres of the bodies.

Consider the case of a cricket ball on the top of a house. The earth attracts the ball, and, by Newton's law, the ball attracts the earth. The ball, if free to move, falls to the earth; to be correct, however, we must think of the ball and the earth moving to meet one another along the line joining their centres. But the ball moves as much farther than the earth as the earth's mass is greater than that of the ball; and for practical purposes this is the same as saying that only the ball moves and that the earth remains still.

This force of attraction between all material bodies is called the force of gravitation, but we must again point out that this is only a name. Calling this force 'gravitation,' and the rule according to which it acts the 'law of gravitation,' does not teach anything about the nature of the force itself.

Graphic representation of forces.—Every force has a certain magnitude, and acts in a certain direction. It is, therefore, possible to represent a force completely by a line, the length of which is proportional to the magnitude of the force and the direction of which represents the direction in which the force is exerted. If the length of an inch be taken to represent a unit force, then a force of 5 units would be represented by a line 5 inches long, and two forces of 5 and 3 units acting together in the same direction would be represented by a line 8 inches long. If, however, a body were acted upon by a force of 5 units in one direction, and 3 units in the opposite direction, then the effect would be that of a force of 2 units acting in the direction of the force of 5 units; for 3 of the units of this force would be rendered inoperative by the three units acting in the opposite direction.

Parallelogram of forces.—A body can only move in one direction at any given instant, though it may be acted upon by any number of forces. Each force has a certain magnitude and acts in a certain direction, and, in consequence of their joint

action, the body moves with a certain velocity, if it be free to do so. The same velocity could be given to the body by a single force instead of the separate forces, and *the single force which would produce the same effect as the separate forces* is called the **resultant** of the forces. Any system of forces acting upon a particle is equivalent to a single resultant force. When two forces act upon a body at the same time, their resultant usually can be found by means of the parallelogram of forces, which may be expressed thus: **If two forces acting at a point be represented in magnitude and direction by the adjacent sides of a parallelogram, the resultant of these two forces will be represented in magnitude and direction by that diagonal of the parallelogram which passes through this point.**

Let O represent a material body acted upon by two forces, represented both in magnitude and direction by the lines OB, OA (Fig. 73). To find the *resultant* of these two forces, both as regards its amount and direction, complete the parallelogram OBRA and join OR, which will be the resultant required.

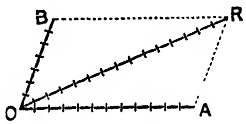


FIG. 73.—Graphic representation of the parallelogram of forces.

The **equilibrant** of two or more forces acting on a body is the single force which, acting with them, maintains the body at rest. It is evident, therefore, that the equilibrant is equal in magnitude, but opposite in direction, to the resultant.

EXPT. 81.—**Demonstration of the principle of the parallelogram of forces.** Lay a large sheet of paper on a flat table (Fig 74). Connect three threads together at a point O. Pass one of the threads over a pulley wheel P, clamped firmly near to the edge of the table, and fasten a known weight to the end of the thread. Join the other threads to spring dynamometers, D_1 and D_2, to which graduated scales are attached. Allow the known weight to hang freely. The point O is now in equilibrium under the action of the *two forces* (exerted by the dynamometers) and their *equilibrant* (which, in this case, is the weight hanging over the pulley). Mark on the paper, by means of a needle point, the direction of the three threads; and write down the magnitudes of the three forces. Remove the paper, and mark off the lengths OE, OF_1, and OF_2 proportional to the three forces. Complete the parallelogram, constructed with OF_1 and OF_2

as adjacent sides; and draw the diagonal Or. According to the principle of the parallelogram of forces, this diagonal should be equal in magnitude and opposite in direction to the equilibrant OE. Produce OE backwards, and mark off a length OR equal to OE. If the experiment is carried out with care the lines OR and Or should be identical in length and in direction.

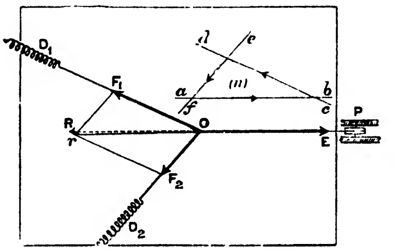


Fig. 74.—Experiment on the parallelogram of forces.

(ii) Alter the directions of the forces OF_1 and OF_2, and again observe whether the true resultant coincides with that deduced by geometrical construction.

Expt. 82.—**The triangle of forces.** On the same sheet of paper used in Expt. 81 draw three lines *ab*, *cd*, and *ef* parallel to the forces OE, OF_1, and OF_2 respectively (Fig. 74). These lines enclose a triangular area. Measure the lengths of the sides of the triangle thus obtained. These three lengths will be found to be proportional to the three forces respectively. Mark, by means of arrow-heads, the directions of the forces. It is evident, therefore, that **if three forces acting at a point are in equilibrium they can be represented in magnitude and direction by the sides of a triangle taken in order.** This is known as the principle of **the Triangle of Forces.**

Geometrical determination of the resultant of two forces.—When the two forces of which the resultant is required act at right angles to one another, the calculation is a simple application of a proposition in the first book of Euclid (I. 47). In these circumstances the triangle ORA (Fig. 73) is right-angled, and it is easily proved that

$$(OA)^2 + (AR)^2 = (OR)^2;$$

consequently

$$(OA^2) + (OB)^2 = (OR)^2,$$

from which when OA and OB are known we can calculate OR.

When the directions of the two forces OB and OA are inclined to each other at an angle which is not a right angle, the calculation involves an elementary knowledge of trigonometry. This can be obviated, however, by the simple expedient of what is called the graphical method. This consists in drawing two lines inclined at the angle at which the directions of the forces

are inclined, and making them of such lengths that they contain as many units of length as the forces do units of force (Fig. 73). The parallelogram is then completed by drawing AR and BR parallel respectively to OB and OA and joining the diagonal OR, the direction of which will be that of the resultant, and its length will contain as many units of length as there are units of force in the resultant force. It is immaterial what lengths are used to represent the units of force so long as the components and the resultant are measured in the same units.

Resolution of forces.—A single force can be replaced by other forces which together will produce the same effect. Such a substitution is called **resolving** the force, or a **resolution of the force.** The parts into which it is resolved are spoken of as **components.** When this has been done it is clear that we have made the original force become the resultant of certain other forces which have replaced it. Referring back to what has been said about the parallelogram of forces, it will be seen that any single force can have any two components in any directions we like; for by trying, the student will be able to make any straight line become the diagonal of any number of different parallelograms. The most convenient components into which a force can be resolved are those the directions of which are at right angles to each other. In this method of resolution, neither component has any part in the other.

A kite at rest in the air affords an example of the principle of the parallelogram of forces (Fig. 75). There are two downward forces—one represented by AB, due to the weight of the kite, and the other represented by AD, due to the pull of the string. The pressure of the air on the face of the kite can be resolved into two forces, one acting along the face and the other at right angles to it. The latter force is an upward one, and if the kite is at rest it is equal

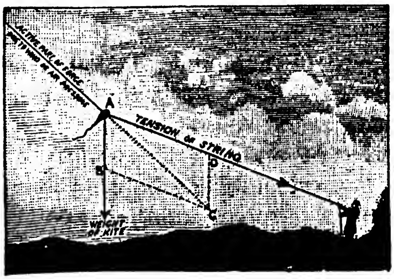


FIG. 75.—Forces acting on a kite at rest in the air.

to the resultant AC of the two downward forces. If it is greater than the resultant AC, the kite rises; if it is less, the kite falls.

EXPT. 82A.—**The polygon of forces.** Place a sheet of paper on a table or bench. Tie four threads together at a point O (Fig. 75A), and attach the free ends to four separate spring dynamometers. Fasten the upper ends of the latter to different points of the table, adjusting the tension of each so that a suitable elongation of the springs is obtained. Let the forces exerted by the springs be denoted by F_1, F_2, F_3, F_4. Mark on the paper the directions of these forces; and mark off lengths OF_1, OF_2, etc., proportional to their magnitudes. The point O is now in equilibrium under the action of the four forces represented.

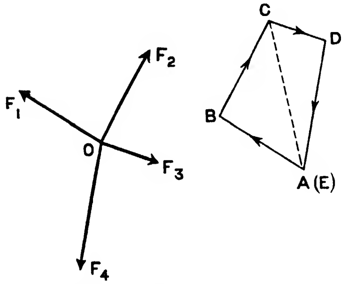


FIG 75A.—The polygon of forces

As in Expt. 82, construct a *force-diagram* by drawing lines AB, BC, CD and DE parallel to, and proportional to the magnitudes of, the forces F_1, F_2, etc. Mark with arrowheads the direction of the forces. Notice whether the point E coincides with the point A. If the measurements are accurate, these points will coincide, and the force-diagram will completely enclose a space.

This experiment is an application of the principle of **the polygon of forces**, which may be stated thus:—**If any number of forces acting at a point are in equilibrium, they may be represented in magnitude and direction by the sides of a polygon taken in order.** It is seen readily that this is simply an extension of the principle of the triangle of forces. For, in Fig. 75A, if the points A and C are joined, CA represents the equilibrant, and AC represents the resultant of AB and BC. Thus the force-diagram is reduced to the triangle ACD; and DA is the equilibrant, and AD is the resultant, of the forces AC and CD.

The principle can be used for finding the resultant of any number of forces acting at a point. For, if a force-diagram be constructed from the given forces, *taken in order*, then the line joining the last point to the first point represents the equilibrant of all the forces. The same line, with the direction reversed, represents the resultant of all the forces.

EXERCISES ON CHAPTER VIII.

1. A line is drawn upon the floor of a railway carriage from door to door. When the carriage is at rest a ball is dropped from the roof and falls upon this line. What difference would be observed :

(*a*) If the train is moving when the ball is dropped?

(*b*) If the train starts when the ball is half way down?

(*c*) If the ball is dropped when the train is in motion, but the train stops suddenly when it is half way down?

2. A man walks backwards and forwards on the deck of a steamer along a line parallel to the direction in which the steamer is moving. If the man walks at the rate of 3·5 miles an hour, and the steamer goes through the water at the rate of 8·4 miles an hour, what is the velocity of the man with reference to the water (1) when he is walking towards the bow of the boat, and (2) when he is walking towards the stern?

3. What is meant by acceleration? Give examples of uniformly accelerated velocities (*a*) where the acceleration is positive, (*b*) where the acceleration is negative.

4. Four forces act at a point. The first of 10 lb. acts due north, the second of 15 lb. due east, the third of 20 lb. due south, and the fourth of 25 lb. due west. Find the magnitude and direction of the resultant.

5. As two ships pass in opposite directions a person in one of them throws a ball to a person on the other. How must he aim? Draw a diagram to explain your answer.

6. Explain what is meant by the inertia of a material body. Give as many of the results of the possession of this property by a material body as you can.

7. The horizontal and vertical components of a certain force are equal to the weights of 60 lb. and 144 lb. respectively. What is the magnitude of the force?

8. Describe an experiment for demonstrating the principle of the parallelogram of forces to a class.

A nail is driven into a wall and two strings are tied to its head. When the two strings are pulled horizontally and at right angles to one another with forces equal to 6 and 8 lb. respectively, the nail is dislodged. What force would be needed if the strings were brought together and the nail pulled straight out? Illustrate your answer with a diagram.

9. Two forces, the magnitudes of which are proportional to the numbers 3 and 4, act on a point at right angles to each other. Draw a parallelogram as nearly to scale as you can to show the direction and magnitude of the resultant, and deduce by measuring your diagram, or in any other way, the magnitude of the resultant.

10. Two forces, P and Q, act upon a body. If P acted alone it would, in two seconds, produce in the body a velocity of 10 feet per second, while if Q acted alone it would in three seconds produce in the body a velocity of 18 feet per second. What velocities will P and Q produce in one second when acting together if the directions in which they tend to move the body are—(1) in the same direction; (2) directly opposed?

11. A weight which is hung by means of a piece of elastic from a nail in the ceiling is pulled some way to one side by a thread which is always kept horizontal. Explain why this operation will increase the stretching of the elastic. Illustrate your answer by a diagram.

12. Forces of 3 gm., 4 gm., and 5 gm. act at a point, and are in equilibrium. What are the angles between their lines of action?

13. Forces of 2 gm., 4 gm., and 5 gm. act at a point, and are in equilibrium. What are the angles between their lines of action?

14. A picture of mass 3 lb. hangs vertically from a nail by a cord attached to rings at the two upper corners of the frame. If the string and the upper edge of the frame form an equilateral triangle, find the tension in the string.

15. A mass of 30 gm. is suspended by a string. What horizontal force is required to displace it until the string makes an angle of 30° with the vertical?

16. A heavy weight is suspended from the end of a piece of string. A piece of the same string is attached to the lower surface of the weight. If the latter string is pulled with a sudden jerk it will snap; but if it is pulled gradually the upper string will be the first to break. Explain this.

17. A projectile weighing 560 lb. is fired from a gun weighing 40 tons with a velocity of 1600 ft. per second. Find the velocity of recoil.

18. A hammer head of $2\frac{1}{2}$ lb. moving with a velocity of 50 ft. per second is stopped in 0·001 second. What is the average force of the blow?

19. What do you understand by

(*a*) velocity, (*b*) acceleration, (*c*) average velocity?

A body starting from rest moves with an acceleration of 3 centimetre second units; in what time will it acquire a velocity of 30 cm. per second, and what distance does it traverse in that time?

20. A body falling freely under gravity drops s ft. in t sec. from the time of starting. If corresponding values of s and t at intervals of half a second are as follows:

t	0·5,	1,	1·5,	2,	2·5,	3,	3·5,	4,
s	4,	16,	36,	64,	100,	114,	196,	256,

draw a curve connecting s and t, and find from it (i) the distance through which the body has fallen after 1 min. 48 sec.; (ii) the distance through which it drops in the 4th second.

CHAPTER IX.

WORK. FRICTION. ENERGY.

Work.—When a man endeavours to raise a heavy mass from the floor, he applies a force, acting vertically upwards, by means of the muscles. At first the mass does not move, because the force of gravitation pulling the mass downwards is greater than the force exerted by the muscles upwards; so far no work has been done by the muscles. When the force applied is exactly equal to the force of gravitation, the mass no longer exerts any pressure on the floor, but it does not move; in such a case it is only necessary to increase the upward pull to the slightest degree in order to make the mass commence to move upwards; then the muscles begin to do work.

We thus obtain the following definition of work: **Work is done when the point of application of a force moves.**

The work done is proportional to the force overcome and to the distance through which the force has been overcome; or,

Work = force overcome × distance.

Unit work is done when unit force is overcome through unit distance.

For practical purposes the **unit of work** which is adopted is **the work done in raising the mass of one pound through a vertical distance of one foot**, and it is called the **foot-pound.** This is not a strictly constant unit, for it will be evident, in the light of what has been said about the weight of a body, that where the weight is greater the amount of work done will be greater. The unit of work will vary slightly in different latitudes in a precisely similar manner to that in which the weight of a mass varies.

Another unit adopted frequently in practical work is the **Kilogram-metre,** which is equivalent to **the work done in raising a mass of one kilogram through a vertical distance of one metre.**

In the metric system, the **absolute unit of work,** called the **erg,** is **the quantity of work done when a force of one dyne is overcome through a distance of one centimetre.** Thus, nearly 1 erg of work is done when 1 mgm. is raised vertically through 1 cm. In practice the erg is often found to be inconveniently small, and a unit called the **Joule,** which is equal to 10^7 ergs, is then used.

EXAMPLE.—How much work is done when an engine weighing 12 tons moves a mile on a horizontal road, when the total resistance is equal to a retarding force of 10 lb. weight per ton?

The total resistance equals $12 \times 10 = 120$ lb. weight, the distance traversed is 5280 feet.

$$\therefore \text{ work done} = (120 \times 5280) \text{ foot-pounds.}$$

It is important to bear in mind that the *distance* through which a force is overcome must be measured *in the direction in which the force is acting*: thus, in conveying a weight to the top of a building, the work done on the weight is the same when it is lifted vertically by means of a pulley as when it is carried to the same height by means of a sloping ladder or a spiral stair-case.

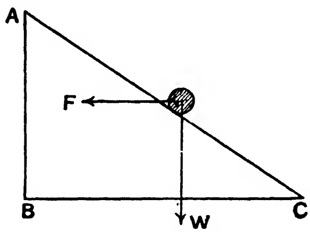


FIG. 76.—Work done by a force.

Similarly, if a weight W be raised up an inclined plane (Fig. 76) from B to A by means of a force F acting horizontally, the work done by the force is measured by the product $F \times BC$, since BC is the distance through which the point of application of the force moves *in the direction of* the force. On the other hand, the work done against gravity is equal to the product $W \times AB$. Hence, if the whole of the work done is spent in raising the weight, and none of it is absorbed in overcoming resistance due to friction,

$$F \times BC = W \times AB,$$

or

$$F/W = AB/BC.$$

This result introduces one of the fundamental principles of the *inclined plane*, which is discussed more fully in Chap. XI.

EXAMPLE.—How much work is done against the force of gravity

by a horse in drawing a load of $\frac{1}{2}$ ton along a road 1 mile long which rises 1 in 30?

The distance through which the vertical force of gravity is overcome is equal to $(\frac{1}{30} \times 5280)$ ft.; and the force overcome is equal to 1120 lb. Hence,

$$\text{Work done} = 1120 \times (\tfrac{1}{30} \times 5280) = 197{,}120 \text{ ft. lb.}$$

Power.—It will have been noticed that the question of time does not enter into an estimation of the amount of work done. It is manifest that the same quantity of work is accomplished whether a day is spent in raising a weight to a given height from the ground or only a minute. If we introduce the time taken to perform the work we begin to consider what is called the **power** of the agent. We should measure this power by the quantity of work the agent can perform in a given time; or **power is the rate of doing work** and is measured by the work done in a second. Thus, engineers use the expression **horse-power,** by which they mean the rate at which a good horse works. James Watt estimated this at 33,000 foot-pounds per minute, or 550 foot-pounds a second.

EXAMPLE.—A man, weighing 10 stone, runs up a flight of stairs to a height of 30 ft. in 5 seconds. Express, in horse-power, the rate at which he does work against the force of gravity.

$$\text{Work done in 5 seconds is } (140 \times 30) \text{ ft. lb.}$$

$$\text{,, \quad ,, \quad 1 second is } \frac{140 \times 30}{5} \quad \text{,,}$$

$$\therefore \text{ rate of doing work} = \frac{1}{550} \times \frac{140 \times 30}{5} = 1 \cdot 70 \text{ horse power.}$$

FRICTION.

Friction.—When a rectangular wooden block is resting on a horizontal table, a small force may be applied horizontally to the block without causing it to move. The reason for the block remaining at rest is that the force applied to it is neutralised by an equal and opposite force which tends to keep the block at rest and is located between the two surfaces in contact: this latter force may be expressed more accurately as a *stress*, and it is called into play by **friction** between the two surfaces.

When the force is increased gradually, the opposing stress due to friction increases at the same rate until a certain maximum is

reached, which the stress cannot exceed; if the applied force slightly exceeds this maximum the block begins to move. The magnitude of this maximum force measures what is termed the **limiting friction.** When motion has commenced it will be found that a smaller force is sufficient to maintain the body in motion. Hence, the stress due to friction between two surfaces in relative motion—termed the **sliding friction**—is less than the limiting friction.

EXPT. 83.—**Friction between wood and wood.** Fix a small staple into a middle point *a* (Fig. 77) of one of the smaller faces of a rectangular block of wood. Weigh the block, and lay it on a clean horizontal wooden surface. Attach a spring dynamometer to the staple, and gradually apply an increasing horizontal force P to the block. Note the reading of the dynamometer (i) at the instant when motion is just beginning, and (ii) when motion is just maintained. Repeat the observation several times, being careful to use in each case the same portion of the table top.

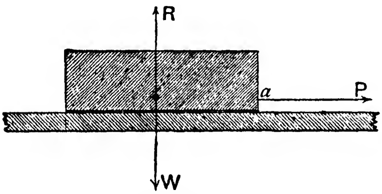


FIG. 77.—Experiment on friction.

Increase the load W by placing weights of 100, 200, 300, ... grams on the top of the block; and measure for each load the limiting and sliding friction. Tabulate the observations, and plot them on squared paper, taking loads as abscissae and the limiting friction as ordinates. What conclusion may be derived from the curve obtained?

Turn the block so that it rests on one of its smaller faces. Repeat the previous observations, and determine whether the limiting friction, when the load is constant, depends upon the area of the surfaces in contact.

EXPT. 84.—**Friction between glass and glass.** Repeat Expt. 83 using a glass slab resting on a clean horizontal glass surface. Plot on squared paper, to the same scale as that adopted in the previous experiment, the readings obtained for the load and the limiting friction.

Smear a few spots of oil over the glass surfaces and determine whether the limiting friction and the sliding friction are influenced thereby.

Laws of friction.—By means of these experiments it is possible to verify, though perhaps roughly, the main laws of friction. These may be stated as follows:

(i) The stress due to friction is greater between two surfaces at rest than when in relative motion.

(ii) The friction is proportional to the load.

(iii) The friction is independent of the area of the surfaces in contact.

The coefficient of friction.—The experiments will have shown also that the limiting friction depends upon the nature of the surfaces in contact: thus, it is evidently greater between wood and wood than between glass and glass. Also, it will be evident from the curves obtained that the ratio

$$\frac{\text{limiting friction (P)}}{\text{pressure between the surfaces (W)}}$$

is a constant quantity for any given pair of surfaces. This ratio is termed the **coefficient of friction** (μ).

ENERGY.

Energy.—All moving bodies possess energy. Moving air or wind drives round the sails of a windmill and so works the machinery to which the sails are attached; it drives along a ship, thus overcoming the resistance of the water. The running stream works the mill-wheel and the energy it possessed is expended in grinding corn. The bullet fired from a rifle can pierce a sheet of metal by overcoming the cohesion between its particles. It may therefore be said that **the energy of a body is the power of overcoming resistance or doing work.**

EXPT. 85.—Stretch a piece of tissue-paper over the top of an empty jam-pot. Carefully place a bullet on the paper and notice the paper will support it. Now lift the bullet and allow it to drop on to the paper. It is seen that the bullet pierces the paper.

EXPT. 86.—Support a weight by a thin thread. Show that though the thread will support the weight at rest it will be broken if the weight is allowed to fall.

EXPT. 87.—Show that a falling weight attached by a string to a spring balance extends the balance beyond the point which it indicates when the weight is at rest.

All these examples are cases of the energy of moving bodies, or the energy of motion, or *Kinetic Energy*. **Kinetic Energy is the energy of matter in motion. All energy which is not kinetic is known as Potential Energy.** Potential energy is capable of becoming kinetic or active when the conditions become suitable. Imagine a mass raised from the ground and placed upon a high shelf. We know that to place it in this position we must expend a certain amount of work, which is measured by multiplying its weight by the height through which it is raised. Further, we know that just so soon as we release it from its position of rest, making it free to move, it will travel with an ever-increasing velocity until it reaches the ground. On the shelf the mass, *by virtue of its position*, possessed a certain amount of *potential energy* exactly equal to the work expended in placing it there.

Similarly, an ordinary dining-room clock, which is worked by a spring, affords us an example of potential energy. The wound-up spring possesses potential energy exactly equal to the amount of work done in winding it up. This potential energy is being converted into kinetic energy continually as the spring becomes unwound in working the clock.

The kinetic energy of a mass $\mathbf{m}$ moving with a velocity $\mathbf{v}$ is expressed numerically by half the product of the mass and the square of the velocity. For, suppose the mass to be initially at rest, and to be acted upon by a constant force; then, after any interval of time, **the kinetic energy of the mass is equal to the work done by the force.** If, at any moment, the body has acquired a velocity $\mathbf{v}$, then

$$\begin{aligned}\text{Kinetic energy} &= \text{force} \times \text{distance through which it has acted}\\ &= (\text{mass} \times \text{acceleration}) \times (\text{average velocity} \times \text{time})\\ &= \left(\text{mass} \times \frac{\mathbf{v}}{\text{time}}\right) \times \left(\frac{0+\mathbf{v}}{2} \times \text{time}\right)\\ &= \frac{\mathbf{mv}^2}{2}.\end{aligned}$$

If $\mathbf{m}$ be given in lb. and $\mathbf{v}$ in ft. per second, then the kinetic energy is in *foot-poundals*. This can be reduced to foot-pounds by dividing by 32.

The motion of a pendulum affords an interesting example of the two forms of energy. At the end of its swing, in the position

A (Fig. 78), the bob of the pendulum possesses potential energy enough to carry it through half a single vibration, that is, until it reaches its lowest position N, when the whole of the energy of position which it possessed at A is expended, as it can reach no lower position. But though it lacks potential energy, since it is a mass moving with the velocity it has gained in its passage from A to N, it possesses energy of motion or kinetic energy enough to carry it up to its next position of rest at A′—where the only energy it will have will be again potential. Through the next vibration from A′ to A it will pass through just the same transformations again.

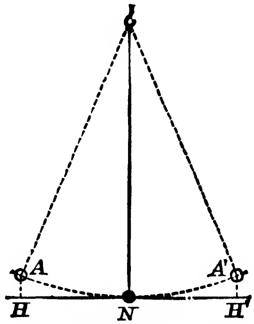


FIG. 78.—Pendulum in oscillation.

At any point in the swing the pendulum will possess a certain amount of energy due to position and a certain amount due to motion, but the total amount of energy—the sum of the potential and the kinetic energy—is always the same, the loss of one form of energy being equalised exactly by the gain of the other.

Forms of energy.—A body may possess energy due to other causes than that of the actual motion of the body as a whole. When it is in rapid vibration, or when it is heated, or when it is electrified, it is endowed with energy in consequence of these conditions. But when a body is in rapid vibration it gives out sound or becomes a sounding body, hence we may regard sound as a form of energy. We shall see that work may be done by the passage of heat from a hot body to a cold one, and, in consequence, heat is regarded properly as another form of energy. An intensely hot body emits light, hence it would seem that light and heat have a common cause and that we must also regard light, like heat, as a manifestation of energy. When a body is electrified it has the power of attracting unelectrified and certain electrified bodies also, and when such bodies are attracted as a result of this electrification we see that electrification must be looked upon similarly as still another kind of energy. But it must be borne in mind that electrification is not electricity. Then, too, there is the attraction of magnetism, which is capable

of accomplishing work, and hence must be looked upon as a form of energy. Chemical combinations, again, are accompanied always by the development of heat, and resulting as they do from the chemical attraction of two more or less unlike forms of matter, we shall be right in saying chemical combinations are accompanied always by energy changes, and so in regarding chemical attraction as another kind of energy. Therefore, **in addition to the energy of moving bodies we have energy manifested as sound, heat, light, electrification, magnetism, and chemical action.**

Transformation of energy.—Energy can cease to exist in one particular form and can assume another form. Thus, the energy of moving bodies can give rise to sound and heat; heat can be changed into the energy of moving bodies, electric currents, chemical action, and so on. The general tendency of all forms of energy is gradually to get converted into heat. In the course of ages the change may become complete; the universe would then exist at one temperature and no further transformation would be possible. When this imaginary condition is realised by the degradation of all energy to one heat-level the universe will be dead and no movements of any kind will take place.

Conservation of energy.—**Energy, like matter, is indestructible.** The total amount of energy in the universe remains the same. One form may be changed into another, but we can create no new energy. We may be unable to trace and account for some of it in the numerous transformations which it undergoes, but we are sure, from many considerations, that if the methods of experiment were only refined enough, we should be able to account for the whole amount.

Work done in overcoming friction.—When two surfaces are rubbed together—as, for example, when a wooden block is pulled slowly along a table—work is done; yet neither of the bodies rubbed together acquires either kinetic energy or an increase of potential energy. The work is spent in overcoming friction (p. 113), and the energy equivalent in amount to this work done appears in the form of heat generated between the two surfaces.

Since the total amount of available energy is constant, a certain quantity of mechanical work must be capable of conversion into a certain quantity of heat—or, in other words, they

must be equivalent to each other. This relationship has been subjected to most rigorous experimental proof; and the quantity of mechanical work which corresponds to unit quantity of heat is termed the **mechanical equivalent of heat** (p. 225).

EXERCISES ON CHAPTER IX.

1. What is the meaning of the terms velocity, mass, force, work, energy, inertia?

2. Explain the terms 'foot-pound' and 'horse-power.' How much work is done in raising 25 weights of 56 lb. each from the ground to a height of 4 feet?

3. A man can pump 25 gallons of water per minute to a height of 16 feet. How many foot-pounds of work does he do in an hour?

4. A ladder 20 ft. long rests against a vertical wall and is inclined at 30° to it. How much work is done by a man weighing 10 stone in ascending it?

5. What should be the indicated horse-power of an engine which is intended to pump 250 gallons of water per minute to a height of 40 yards?

6. The mass of a train is 250 tons, and the resistances to its motion on a level line amount to 15 lb. per ton. Find the horse-power of the locomotive which can maintain a speed of 40 miles per hour on the level.

7. What is the difference between kinetic energy and potential energy?

8. Define work, and describe an experiment to prove that a falling ball is capable of doing work.

9. What is meant by a foot-pound of work? What is the value of a horse-power in terms of this unit?

10. How is kinetic energy measured? If we wish to express the result in foot-pounds, how do we proceed?

11. A man weighing 140 lb. puts a load of 100 lb. on his back and carries it up a ladder to a height of 50 feet. How many foot-pounds of work does he do altogether and what part of his work is done usefully?

12. A body weighing 10 lb. is placed on a horizontal plane and is made to slide over a distance of 50 feet by a force of 4 lb. What number of units of work is done by the force?

13. If a man can work at the rate of 210,000 foot-pounds an hour, how long would it take him to raise a weight of 10 tons through 150 feet, supposing him to be provided with a suitable machine?

14. A horse pulling a horizontal trace with a force equal to the weight of 72 lb., draws a cart along a level road at the rate of $3\frac{3}{4}$ miles per hour. What amount of work is done by the horse in 5 minutes?

15. A cannon-ball the mass of which is 60 lb. falls through a vertical height of 400 feet. What is its energy at the end of its fall?

16. What is the kinetic energy of a mass of 5 lb. moving with a velocity of 10 feet per second? State clearly what the unit is in terms of which your answer is expressed.

17. A body having a mass of 10 lb. is carried up to the top of a house 30 feet high. By how many foot-pounds has the change of position increased its potential energy? If it be allowed to fall, what number of foot-pounds of kinetic energy will it have when it reaches the ground?

18. Describe an experiment to prove that energy due to visible motion can be transferred from one body to another.

19. What proof can you adduce that the energy of visible motion can be transformed into heat?

CHAPTER X.

THE LEVER. PARALLEL FORCES. CENTRE OF GRAVITY.

The lever.—A lever is a rigid bar which can be turned freely about a fixed point. The **fulcrum** of a lever is the fixed point about which the lever can be turned. The force exerted when using a lever is often described as the **Power** and the body lifted or resistance overcome as the **Weight.** These words are convenient, but they are not used correctly in connection with levers, as their true meanings are confused by so doing. It is better to substitute the word **effort** for power, and **resistance** or **load** for weight. It should be borne in mind, that, so far as mechanical principles are concerned, there is no difference between the power and the weight; both represent forces, and as such they must be considered in the action of levers.

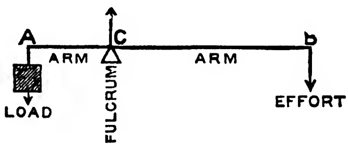


Fig. 79.—Terms used in connection with levers.

The perpendicular distances from the fulcrum to the lines of actions of forces acting upon a lever, are known as the **arms** of the lever. In Fig. 79 the distance AC is the arm of the end at which the load acts, and BC is the arm of the end at which the effort acts.

The principle of the simple lever has been learnt previously, in Expt. 27. It was proved that the

$$\begin{array}{c}\text{Weight on one}\\\text{side of fulcrum}\end{array}\times\begin{array}{c}\text{Perp. distance}\\\text{from fulcrum}\end{array}=\begin{array}{c}\text{Weight on}\\\text{other side}\end{array}\times\begin{array}{c}\text{Perp. distance}\\\text{from fulcrum.}\end{array}$$

Each of these products is termed the **moment of the force** about the fulcrum : the following definition is important:

The moment of a force about any point is the product obtained by multiplying the force by the perpendicular distance between the point and the line of action of the force.

The law of moments.—Refer to the diagram (Fig. 80), where a lever AB is in equilibrium under the action of two forces, M_1 and M_2. These forces *tend* to rotate the lever in an anti-clockwise and a clockwise direction respectively. It is agreed generally that these directions of rotation shall be denoted by the algebraic symbols *plus* and *minus*. Hence,

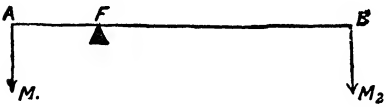


FIG. 80.

The moment of M_1 about F is written $+(M_1 \times AF)$, and
" " M_2 " " $-(M_2 \times BF)$.

Since these moments are numerically equal, it follows that the algebraic sum of the moments is zero. Hence, **when a lever is in equilibrium under the action of two (or more) forces the algebraic sum of the moments of the forces about the fulcrum is zero.** This deduction is termed the **Law of Moments.**

EXPT. 88.—**Sum of moments.** A slight modification in Expt. 27 will afford a proof that the Law of Moments is true when three or more forces are acting upon the lever. Thus, three weights may be suspended from the lever, and their position adjusted so that the lever is in equilibrium. The *algebraic* sum of the moments will be found to be equal to zero.

Classes of levers.—For convenience, levers are sometimes divided into three orders or classes, according to the relative positions of the fulcrum, resistance, and effort. This classification is, however, of no real consequence; for the principle underlying the action of all levers is the same. The following experiments illustrate the relation between the fulcrum, resistance, and effort in each of the three cases:

EXPT. 89.—**Fulcrum between resistance and effort.** Using the lever described on page 36, show that a weight placed at a short distance

from the fulcrum on one side can be moved by a much smaller weight placed at a proportionally greater distance on the other side of the fulcrum. A lever of this kind is represented by a see-saw, a pump-handle, a balance, and a spade used in digging.

Find the weight of a piece of metal by means of a simple lever and a box of weights.

EXPT. 90.—**Resistance between effort and fulcrum.** Pivot a metre scale at its centre (Fig. 81). Suspend a weight R from any point A. Tie one end of a long thread round the lever at any point B, pass the thread over a pulley, and fasten a scale pan (of known weight) to the other end. Adjust the weights in the scale pan until the lever is in equilibrium. The effort P is then equal to the sum of the weights of the scale pan and of the weights supported on the pan. Show that the algebraic sum of the moments, $+(P \times FB) - (R \times FA)$, is equal to zero.

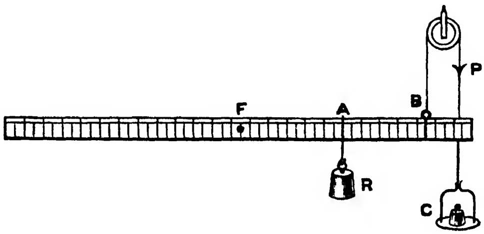


FIG. 81.—Experiment with a lever.

Modify the magnitudes of the weights and of the arms several times; and in each case find whether the same deduction is obtained. A common kind of nutcrackers and a wheelbarrow are examples of levers of this type.

EXPT. 91.—**Effort between fulcrum and resistance.** Use the same apparatus as in the previous experiment, but interchange the points of application of the effort and of the resistance. Take a series of observations, and determine whether the Law of Moments still holds good.

Examples of this class of lever are furnished by sugar tongs, ordinary fire-tongs, and the pedal of a grindstone.

Parallel forces.—It has been seen that the earth exerts a downward pull upon all objects on its surface, and that in consequence of this all things fall to the ground if unsupported. It follows, therefore, that everything which is supported above the earth's surface is being pulled downwards constantly, even though it does not fall. When a beam, for instance, is supported

horizontally by resting the ends upon two posts, each particle of it may be regarded as being pulled earthwards by an attractive force. The direction of the pull is everywhere towards the centre of the earth, so, in view of the distance of the earth's centre from its surface, for any one spot on the earth's surface we may consider the attractive forces due to gravity to be parallel to one another.

When a stiff lath or rod of uniform thickness rests upon two letter balances, or is supported by hanging each end from a spring balance, the experiment represents on a small scale the case of the beam referred to before ; and by using spring balances it can be proved that the weight supported at its ends is equally divided between the two supports. In other words, the two upward forces exerted by the balances are together equal to the downward force represented by the weight of the beam.

If a load be placed anywhere upon the lath, the balances still show that when the lath is in equilibrium the sum of the upward forces is equal to the sum of the downward forces.

In the experiments with a lever having the fulcrum between the resistance and effort, the lever remains at rest although acted upon by two forces tending to pull it downwards : evidently the fulcrum must be exerting a force exactly equal, and opposite in direction to the combined forces acting downwards ; or, in other words, the force exerted by the fulcrum must be equal and opposite to the resultant of the downward parallel forces. The following experiment will serve to demonstrate, (i) the magnitude, and (ii) the point of application, of the resultant of two or more parallel forces.

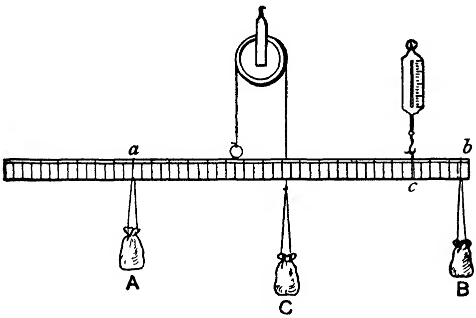


FIG. 82 —Parallel forces in equilibrium.

EXPT. 92.—**Parallel forces.** Suspend the lath by a string which passes over a pulley and has attached to the other end a weight

equal to the weight of the lath (Fig. 82). The rod can then move as if it had no weight.

An upward force can be applied to it by a spring balance, or by a weight attached to a string passing over a pulley, and downward forces by hanging weights from it.

Attach a spring balance to any convenient point on the lath by passing the hook of the balance through a hole in the lever or by means of thread. Suspend weights A and B from the lath so that they counterpoise one another (Fig. 82). Notice the reading of the spring balance, and the weights used.

Since the lath is in equilibrium, the experiment would not be affected if the lath were pivoted *at any point along its length.* Suppose that the lath were pivoted at the point *c*; then the algebraic sum of the moments of the weights of A and B about the point *c* should be equal to zero. Calculate these moments, and verify this deduction. Record your observations thus:

Sum of Weights (A+B).	Reading of Spring Balance.	Moment of A about *c*.	Moment of B about *c*.

The force exerted by the spring balance must, necessarily, be equal and opposite to the resultant of the downward parallel forces; and the point of application of the resultant is determined by the fact that the algebraic sum of the moments of the component parallel forces round that point must be equal to zero.

Centre of gravity.—Consider a large number of weights, some heavier than others, suspended from a horizontal rod. A certain position can be found at which a spring balance has to be attached in order to keep the rod in equilibrium. When the rod is hung from this point the tendency to turn in one direction is counteracted by the tendency to turn in the other, so the rod remains horizontal. The weights may be regarded as parallel forces, and the pull of the spring balance as equal to their resultant. Now consider a stone, or any other object, suspended by a string. Every particle of the stone is being pulled downwards by the force of gravity, as indicated in Fig. 83. The resultant of these parallel forces is represented by the line GF, and the centre of the forces is the point G. The point G, through

which the resultant (GF) of the parallel forces due to the weights of the individual particles of the stone passes, is known as the **centre of gravity.** For the stone to be in equilibrium, the string must be attached to a point in the line GF, produced upwards.

FIG. 83.—Parallel forces due to gravity.

Every material object has a centre of gravity, and the position of this point for a particular object is the same so long as the object retains the same form. The centre of gravity need not be, however, a point on the actual object.

Geometrical determination of centres of gravity.—It has been explained sufficiently that the centres of gravity of straight lines, circles, squares, and other regular figures are at their geometrical centres. Hence, the geometrical constructions for determining these central points also locate the position of their centres of gravity.

The centre of gravity of a parallelogram is at the intersection of its diagonals.

The centre of gravity of a triangle is determined by bisecting any two sides and joining the middle points so obtained to the opposite angles. The intersection of the lines so drawn gives the centre of gravity. The centre of gravity is found, by measuring, to be one third the whole length of the line drawn from the middle point of the side to the opposite angle, away from the side bisected.

To find the centre of gravity of a quadrilateral by construction, divide the figure into two triangles by drawing a diagonal. By the method just described the centre of gravity of each triangle is found, and the points so obtained are joined. The centre of gravity of the quadrilateral lies on this line. Repeat the process by drawing the other diagonal. Join the centres of gravity of the second pair of triangles, the centre of gravity of the quadrilateral lies on this line. Hence, it is situated at the point of intersection of this line and the first one obtained in the same way.

Experimental methods of determining centres of gravity.—In the case of unsymmetrical figures the centre of gravity cannot be found easily by geometry, and is best determined by experiment.

EXPT. 93.—**Thin uniform sheet.** Procure a disc or an irregular piece of sheet cardboard and find by trial the point on which it may be balanced, that is, the centre of gravity of the card. Make a hole in the card near the edge, and make a plumb-line consisting of a thread with a piece of lead tied at one end and a hook of thin wire at the other. Hang the card from the hook, and then suspend both as shown in Fig. 84, so that the card and lead are both suspended and the thread passes over the point of suspension. The thread also passes through the centre of gravity. Do this for various holes in the edge of the card, and see that in all cases the *vertical line through the point of suspension passes through the centre of gravity.*

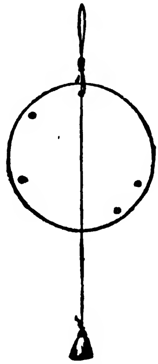


FIG. 84.—A method of determining centre of gravity.

After the centre of gravity of a sheet of metal, or other stiff material, has been determined by hanging it from a support in the manner described in Expt. 93, it will be found that if this sheet be so arranged that a pointed upright is immediately under the centre of gravity, the plate will be supported in a horizontal position. This affords a convenient means of checking the correctness of the experiment performed.

Relation of centre of gravity to base of support.—A circular disc, in which the centre of gravity coincides with the geometrical centre, will not rest upon a table if the centre is beyond the edge of the table, but will topple over. In a similar way, if any plane figure lies flat upon a table the centre of gravity of the figure must be within the edge of the table. The same conditions apply to any object resting upon a support. For an object resting upon a base to be in equilibrium, a vertical line drawn from the centre of gravity downward must fall within the base. When this vertical line falls outside the base, the body topples over.

Consider the case of an omnibus on level ground. The centre of gravity of an omnibus ought to be kept as low and central as possible, so that a vertical line drawn from it downwards will fall well within a line traced around the omnibus upon the ground. But when the outside of the omnibus is filled with people and

the inside is empty, the centre of gravity is much higher, and if the vehicle happens to be running across a sloping road, it may cause so great a change of position of the centre of gravity as to make the vertical line from the centre fall outside the base of support; in such a case the omnibus must topple over (Fig. 85).

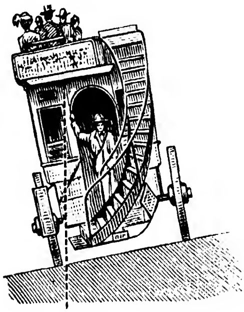


FIG. 85.—Conditions of stability of an omnibus.

Equilibrium.—When a body is at rest, all the forces acting upon it balance one another (or, what is the same thing, any force is equal and opposite to the resultant of the remaining forces), and it is said to be in equilibrium. It is in **stable equilibrium** when any turning motion to which it is subjected raises the centre of gravity; in **unstable equilibrium** when a similar movement lowers the centre of gravity, and in **neutral equilibrium** when the height of the centre of gravity is unaffected by such movement. Consequently, if a body in stable equilibrium be disturbed, it returns to its original position; if in unstable equilibrium, it will, if disturbed, fall away from its original position; while if the condition of equilibrium be neutral it will, in similar circumstances, stay where it is placed.

EXPT. 94.—**Base of support.** Place upon a square-edged table or board one of the cardboard figures of which you have found the centre of gravity. Gradually slide the figure near the edge until it would just topple over; keeping it in this position, draw a line along the under side of the cardboard where the edge of the table touches it. Then place the cardboard in another position and again mark where the edge of the table touches it when it would just topple over. The intersection of these lines is the centre of gravity, and it will be noticed that the cardboard will just topple over when the centre of gravity falls outside the edge of the table.

EXPT. 95.—**Suspension and equilibrium.** Procure an oblong strip of wood or cardboard (Fig. 86). Support the strip as at A by a long pin pushed through it; it is then in stable equilibrium, for the slightest turn either to right or left raises the centre of gravity. When supported as at B, the strip is in neutral equilibrium; and

when supported as at C, it is in unstable equilibrium, for the slightest movement lowers the centre of gravity.

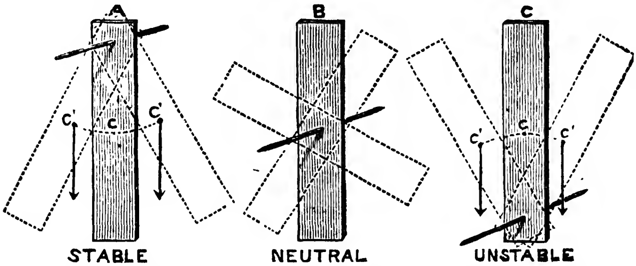


FIG. 86.—Relative positions of centre of gravity and point of support for stable, neutral, and unstable equilibrium.

Conditions of stability.—The centre of gravity must in every case be below the point of support for a suspended object to be in equilibrium. The greater the distance between the point of support and the centre of gravity the greater is the tendency to return to the position of equilibrium.

When the centre of gravity and the point of support of a suspended object are close together the equilibrium of the object is disturbed easily. A good balance partly owes its sensitiveness to this condition, the centre of gravity and point of support being brought close together designedly.

It has been shown that in the case of a freely suspended object the centre of gravity is at its lowest point when the object is in equilibrium. Let us see how this applies to a body supported upon a surface below the centre of gravity. A body is least liable to be upset when the centre of gravity is at a considerable distance from all parts of the edge of the base; for when this is the case the body has to be tilted through a large arc before the centre of gravity falls outside the base.

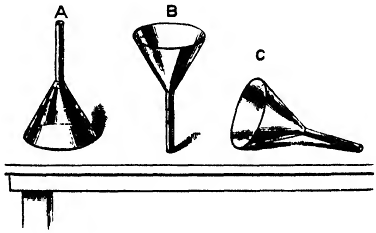


FIG. 87.—A funnel in (A) stable equilibrium, (B) unstable equilibrium, (C) neutral equilibrium.

A funnel standing upon its mouth is an example of a body which cannot be overturned easily on account of the low centre

of gravity and its distance from the edge of the base (Fig. 87). It is then in stable equilibrium. If the funnel be stood upon the end of the neck it can be overturned easily, because very little movement is required to bring the centre of gravity outside the base. It is then in unstable equilibrium. When the funnel lies upon the table it is in neutral equilibrium, for its centre of gravity cannot then get outside the points of support.

The steelyard.—The steelyard is a modification of the simple lever. In weighing an object by means of it, the object is suspended from one arm of the lever, at a fixed distance from the fulcrum, and its weight is determined by varying the distance from the fulcrum of a constant mass which hangs from the other arm of the lever. A scale marked on the variable arm enables the weight of the object to be read directly.

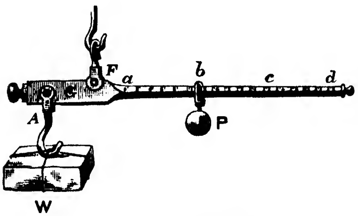


FIG. 87A.—A steelyard.

The instrument consists of a steel bar block supported by knife-edges at F (Fig. 87A). The object (W) to be weighed is attached to a hook suspended from the point A; and the constant mass P is moved along the other arm until the lever remains in a horizontal position, when the scale-reading (*b*) of its point of support gives the weight of the object.

If the centre of gravity (G) of the lever coincided with F the scale would commence at F. In actual practice, however, the centre of gravity is usually between A and F. The weight P, therefore, has to be suspended from some point *a* in order to balance the weight of the lever; and the scale must start from this point.

By making the distances *ab*, *bc*, *cd*, ... each equal to AF, the weight W is equal to that of P, 2P, or 3P, ..., according as P is suspended from the 1st, 2nd, or 3rd, ... division of the scale. These divisions on the scale are sub-divided so as to give intermediate fractions of the weight P.

EXERCISES ON CHAPTER X.

1. Describe the principle of the action of a simple lever.

A stiff wooden rod, six feet long, and so light that its weight may be neglected, lies upon a table with one end projecting four feet over

the edge. Upon the end of the rod lying on the table a weight of 8 lb. is placed. What weight must be placed upon the other end so as just to tip the rod?

2. What is a lever? What is the "fulcrum" of a lever?

Name four or five levers in common use, and say where the fulcrum of each may be?

3. What is meant by the resultant of two forces?

Describe an experiment to prove that the resultant of two parallel forces is equal to the algebraic sum of the forces.

4. A uniform rod is pivoted at its middle point, and a weight of 20 grams is attached at a point 25 centimetres from the fulcrum. To what point on the rod must a weight of 15 grams be attached in order that the rod may balance in a horizontal position?

5. A lever, two feet long, has a force equal to the weight of 10 lb. acting at one end, 18 inches from the fulcrum. What is the greatest weight it will support at the other end?

6. How do you define the moment of a force about a point, and how can you apply the principle of moments to find the resultant of parallel forces?

Two men A and B support the ends of a wooden beam six feet long and weighing 1 cwt. A weight of $2\frac{1}{4}$ cwt. hangs from the beam at a distance of 2 ft. from A. What are the total weights supported by A and B respectively?

7. A piece of cardboard, nine inches long and six inches broad, is divided into six equal squares by means of a ruler and pencil. One of the two squares that are not corner squares is cut away with a penknife. Find the centre of gravity of the remaining piece of cardboard.

8. A man with a bucket in one hand, stands with his feet close together. Why is it that in order to preserve his balance the man has to stand with his body leaning to one side? Illustrate your answer by a sketch.

9. A square sheet of cardboard weighing 8 oz. is suspended by a thread fastened to one corner, and a weight of 4 oz. is fastened to one of the corners adjacent to the corner of suspension. Draw a diagram to show the position in which the sheet will hang, and say what is the total weight that the thread supports.

10. How would you determine the centre of gravity of an iron hoop made by joining together two semicircles, one thicker than the other? Explain how the observations could be used to find out which was the thicker half of the hoop.

11. A solid hemisphere made of uniform material is placed with any part of its curved surface upon a horizontal plane. Show that, however thus placed, it will tend always to a position of stable equilibrium with its flat surface horizontal and uppermost. What other positions of equilibrium are there? Which of them are stable and which unstable?

CHAPTER XI.

THE PULLEY, INCLINED PLANE, AND SCREW.

Machines.—The term **machine** is applied to **any contrivance by which a force acting at a given point and in a given direction may be rendered available at some other point and in some other direction.** In some machines a *small* applied force may give rise to a *greater* force acting at another point and in another direction; in such cases, the ratio of the resulting force to the applied force is termed the **mechanical advantage** of the machine. Also, in any machine, the total work given out is never greater than the work put into it: as a general rule, owing to work absorbed in overcoming friction of the working parts, the available work is *less* than the work put in, and the ratio of the former to the latter is termed the **efficiency** of the machine.

Use of a single fixed pulley.—With a single *fixed* pulley, no mechanical advantage is obtained. All that the pulley does is *to change the direction of the pull*; if one of the loads, for instance, be pulled down, the other rises. The pulley thus acts in the same way as a lever balanced at its centre; the distance from the centre to the circumference, in other words, the radius of the pulley, being regarded as one arm of the lever. A pulley having a radius of three inches has therefore an equivalent lever-arm three times as great as one with a radius of one inch.

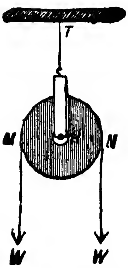


FIG. 88.—Parallel forces acting on a pulley.

A pulley supported as in Fig. 88 may be considered as an example of the action of parallel forces. The forces W + W + the weight of the pulley, acting downwards, are

kept in equilibrium by the single force T acting upwards. If a single mass, the weight of which is equal to W + W + **pulley**, were hung from the cord, it would produce the same tension as the three forces, that is, it would be their resultant.

Use of a single movable pulley.—The fixed pulley is of no advantage in reducing the force required to raise a mass; the advantage gained is derived from the use of a movable pulley. Thus, one half of the mass W (Fig. 89) is supported by the part of the string hooked to the beam, and the other half is supported by the part of the string which goes to the spring balance. There are several different combinations of pulleys, but the principle exemplified by the following experiments, namely, that every movable pulley reduces by one-half the effort required to support or raise the mass below it, is utilised in them all.

EXPT. 96.—**One movable pulley.** Place a weight in one of the pans previously used (p. 132), and weigh the pan and a pulley together by suspending them from a spring balance. Record the reading of the balance. Now arrange the pulley and balance as shown in Fig. 89, and again record the reading. Increase the total load by adding other weights to the pan, and repeat the observations.

Total load, W.	Reading of Spring Balance, P.	Ratio $\frac{P}{W}$.

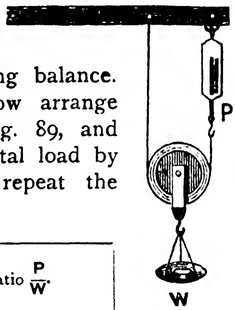


FIG. 89.
Expt. 96.

It will be noticed that P is only $\frac{1}{2}$W (roughly) in every case; in other words, the tension in a cord supporting a movable pulley having a mass hung from it is equal to one-half the total mass supported.

EXPT. 97.—**Two movable pulleys.** Arrange two pulleys in connection with the spring balance as shown in Fig. 90, and observe the reading. Add a weight W to the pan, and observe the *increase* (P) of the tension in the spring balance. Take a series of readings, using heavier weights. Tabulate your observations as in Expt. 96.

In this case, that is, with two movable pulleys, it will be noticed that the force required to support the pulleys and load is less than before. Consider this experiment from a theoretical point of view. The upper pulley is supported by two cords, in each of which the tension is P. If the pulley had no weight the single force required to counteract this tension would be equal to 2P, but taking the pulley into account, the force is $2P - W_1$, where W_1 represents the weight of the pulley. The tension in the string around the lower pulley is thus equal to $2P - W_1$, and if the pulley had no weight the load W would be equal to $2(2P - W_1)$. But taking the weight (W_2) of the second pulley into account it is found that W is equal to $2(2P - W_1) - W_2$. In the same way the relation between the load and sustaining force for any number of pulleys can be determined.

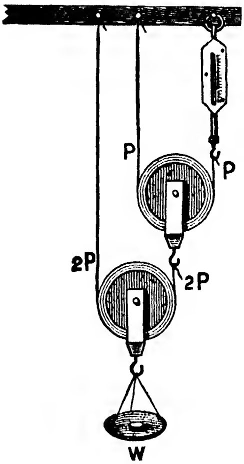


FIG. 90.—Expt. 97.

The principle of work applied to pulleys.—With pulleys, as with levers, there is neither loss nor gain of work. If, in any combination of pulleys, a force of 10 lb. wt. balances a force of 120 lb. wt.—the mechanical advantage or resistance ÷ effort, thus being 12—the effort will have to be exerted through twelve feet in order to move the resistance through one foot. For it is an invariable rule that

$$\text{Effort} \times \frac{\text{the distance through}}{\text{which it acts}} = \text{Resistance} \times \text{Distance moved.}$$

EXPT. 98.—**Principle of work.** Remove the spring balance used in Expt. 97, pass the string over a fixed pulley supported from the beam, and attach a scale pan to the free-end of the string. Add weights to this pan until the system is in equilibrium. Place a weight (W) of 100 gm. in the lower pan and add weights (P) to the upper pan until the system is again in equilibrium. Measure the height of the bottom of each tray from the table. Then move the weight P through a certain distance, and observe how much the weight W is moved. It will be found that:

$$P \times \text{distance moved} = W \times \text{distance moved.}$$

The inclined plane.—A plane in mechanics is a rigid flat surface, and an inclined plane is one that makes an angle with the horizon.

The reason for the decrease of tension in an elastic cord attached to a mass resting on an inclined board, compared with the tension when the mass hangs freely, will be best understood by applying the principle of the triangle of forces to the inclined plane. Suppose an object O (Fig. 91) is kept in position upon a smooth inclined plane by a force acting up the plane. The object is acted upon by three forces, namely, W due to its weight, acting vertically downwards, P the force exerted up the plane, R the reaction of the plane.

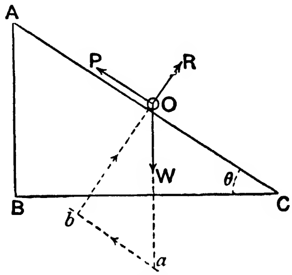


Fig. 91.—Equilibrium upon an inclined plane.

By the principle of the **triangle of forces** (p. 106) these three forces can be represented in magnitude and direction by the sides of a triangle taken in order. Then, supposing the weight (W) of the object to be known, if a vertical line O*a* be drawn to scale so that its length is proportional to W, and the triangle O*ba* is completed so that the sides *ab* and *b*O are parallel to the forces P and R respectively, this triangle may be regarded as a **force-diagram**, and the lengths of its sides are proportional to the forces to which the sides are parallel. Hence,

$$\frac{\mathrm{P}}{\mathrm{W}}=\frac{ab}{a\mathrm{O}}.$$

But the triangles *ab*O and ABC are similar, since their sides are mutually perpendicular; therefore,

$$\frac{\mathrm{P}}{\mathrm{W}}=\frac{ab}{a\mathrm{O}}=\frac{\mathrm{AB}^*}{\mathrm{AC}}, \qquad \text{or,} \qquad \mathrm{P}=\mathrm{W}\times\frac{\mathrm{AB}}{\mathrm{AC}}.$$

* It is more convenient to express this ratio in terms of the inclination (θ) of the plane. The ratio $\frac{\text{perpendicular}}{\text{hypotenuse}}$ is termed the *sine* of the angle (θ); hence, the above relationship may be written $\mathrm{P}/\mathrm{W}=\sin\theta$.

The ratio $\frac{\text{perpendicular}}{\text{base}}$ and $\frac{\text{base}}{\text{hypotenuse}}$ are termed respectively the *tangent* and the *cosine* of the angle; hence, in Fig. 91, $\mathrm{AB}/\mathrm{BC}=\tan\theta$, and $\mathrm{BC}/\mathrm{AC}=\cos\theta$.

The same result may be deduced from the **principle of work.** Thus, if the object O starts from C, and is moved up the plane to A, it is lifted through a vertical height AB. For this to take place, the effort has to be exerted through a distance equal to AC, the length of the plane. Therefore,

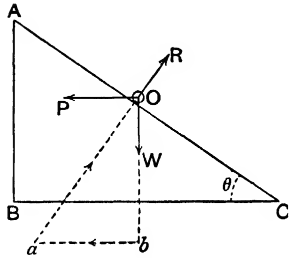


FIG. 92.—Equilibrium on an inclined plane.

$$\mathrm{P} \times \mathrm{AC} = \mathrm{W} \times \mathrm{AB},$$

or

$$\mathrm{P} = \mathrm{W} \times \frac{\mathrm{AB}}{\mathrm{AC}}.$$

The magnitude of the reaction R which the plane exerts on the object may be determined from the equation

$$\frac{\mathrm{R}}{\mathrm{W}} = \frac{\mathrm{O}b}{\mathrm{O}a} = \frac{\mathrm{BC}}{\mathrm{AC}},$$

or,

$$\mathrm{R} = \mathrm{W} \times \frac{\mathrm{BC}}{\mathrm{AC}}.$$

Fig. 92 represents the relationship of the forces P, W and R when the effort P acts horizontally. The triangle abO is the force-diagram, and it is similar to the triangle ABC. Hence,

$$\frac{\mathrm{P}}{\mathrm{W}} = \frac{ab}{b\mathrm{O}} = \frac{\mathrm{AB}}{\mathrm{BC}},$$

or,

$$\mathrm{P} = \mathrm{W} \times \frac{\mathrm{AB}}{\mathrm{BC}}.$$

EXPT. 99.—**Inclined plane with effort acting up the plane.** Fig. 93 represents a suitable form of apparatus for demonstrating the principle of the inclined plane when the effort is parallel to the inclined surface. Clamp the inclined plane firmly in position, weigh the roller, and attach it to the thread by which the effort is applied. Determine the weights which must be hung from the end of the thread and are just sufficient (i) to prevent the object from rolling down the plane, and (ii) to cause the roller to start rolling up the plane. The average of these weights represents the effort P. Read by means of the plumb-line the angle of inclination of the plane.

Draw a horizontal line of any convenient length to represent the base of the plane and construct upon it an angle equal to the inclination of the plane. At any convenient point drop a perpendicular to the horizontal line from the inclined line, so as to construct a right-angled triangle ABC, the angle at C representing the inclination of the plane.

Measure AB and AC and calculate the ratio AB/AC. Calculate also the ratio P/W. The two results will be found approximately the same.* Test further this relationship by repeating the experiment with the plane incline at different angles.

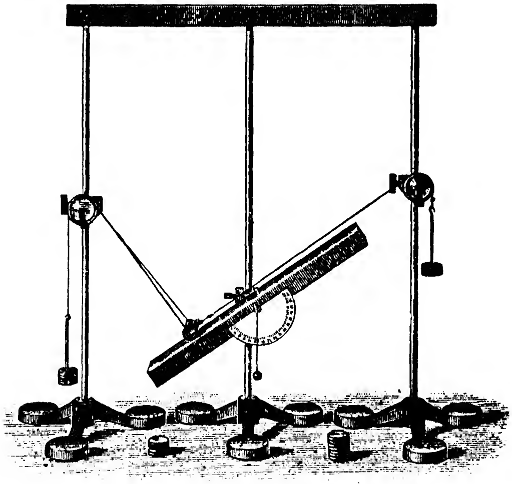


FIG. 93.—"Capstan" apparatus for inclined plane experiment (G. Cussons, Manchester).

EXPT. 100.—**Measurement of the reaction of an inclined plane.** Attach to the roller the thread which passes over the pulley on the left of the apparatus; and adjust the position of the roller so that this thread is perpendicular to the surface of the plane. Adjust the weight attached to this thread so that the roller is just on the point of being raised from the plane. This weight represents the magnitude of the reaction R. Calculate the ratio R/W. Construct a force-diagram as in the preceding experiment, and calculate the ratio BC/AC. Notice that the result is the same as that of R/W. Repeat the observations with the plane inclined at other angles.

* The values of the ratio AB/AC, that is, of sines of angles, are given in trigonometrical tables, and can be used instead of determining them by constructing a triangle as described and measuring the lengths of the perpendicular and hypotenuse. In the same way, the values of cosines and tangents required for Experiments 100 and 101 can be found by referring to trigonometrical tables instead of by construction.

EXPT. 101.—**Inclined plane with effort horizontal.** Fig 94 represents a similar form of apparatus, but the inclined plane has a slot cut along

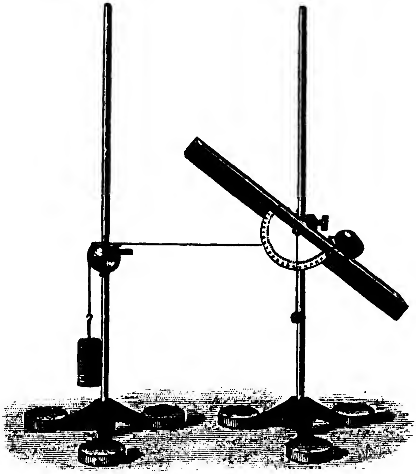


FIG. 94.—"Capstan" apparatus for inclined plane experiment (G. Cussons, Manchester).

its axis so that the effort may be applied horizontally. Take a series of observations in order to demonstrate that the ratio P/W is equal to the ratio AB/AC in a force-diagram constructed as in the preceding experiments.

Principle of the screw.—The principle of the action of a screw is similar to that of an inclined plane. Thus comparing a screw with an inclined plane, it will be seen that

Height of inclined plane represents distance between threads,
Base of " " " circumference of screw.

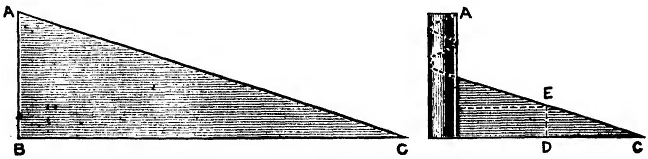


FIG. 95.—Formation ot a screw thread by the slope side of an inclined plane.

The angle of inclination of the inclined plane is represented by the angle ECD (Fig. 95), and this determines the *pitch* of the screw.

EXPT. 102.—Cut out of paper a right-angled triangle such as ABC (Fig. 95) and wind it round a pencil. The slant side of the triangle forms a spiral upon the pencil, similar in appearance to the thread of a screw. If the inclination of the triangle be small, the threads appear close together, and if it be large they occur farther apart. Mark where the end C of the paper touches the base of the triangle and draw a line DE, perpendicular to the base, from this point to the slant side. The small triangle CDE thus formed is similar to the large one, and it represents one turn of the screw thread.

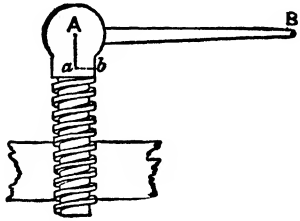


FIG. 96.—A screw turned by a lever.

In considering the use of a screw, the resistance to be overcome can be regarded as a load upon an inclined plane. With a screw such as is shown in Fig. 96 the effort is exerted in a direction parallel to the base of the plane. Under this condition

$$\frac{\text{Load}}{\text{Effort}} = \frac{\text{Base of plane}}{\text{Height of plane}}.$$

Or, expressing the equality in terms which apply to screws:

$$\frac{\text{Resistance}}{\text{Effort}} = \frac{\text{Circumference of screw}}{\text{Distance between successive threads}}.$$

When force is applied at B (Fig. 96), leverage is gained in the ratio of AB to *ab*, and so further mechanical advantage is obtained on this account. But, in order to advance the screw by a distance equal to that between two successive threads, the end of the handle B has to be turned through a complete circumference. This fact can be used to deduce the mechanical advantage of a screw from the principle of work. For

$$\text{Effort} \times \begin{array}{c}\text{Circumference of}\\ \text{circle described}\\ \text{by it}\end{array} = \text{Resistance} \times \begin{array}{c}\text{Distance between}\\ \text{two successive}\\ \text{threads.}\end{array}$$

Hence the mechanical advantage, or resistance ÷ effort, can be found from the relation:

$$\text{Mech. adv.} = \frac{\text{Circumference of circle described by effort}}{\text{Distance between two successive threads}}.$$

EXERCISES ON CHAPTER XI.

1. State the principle of the action of a simple pulley.
How would you show that the strain or tension in a cord supporting a pulley is equal to half the weight hanging from the pulley?

2. Explain by reference to an inclined plane what you understand by the "mechanical advantage" of a machine.

3. Give in a few words the principle of the screw. On what does the ratio of the resistance overcome to the effort exerted depend?

4. Explain how a weight, a pulley, and a chain can be arranged so as to make a door shut when it is let go. Draw a diagram. What forces has the weight to overcome as it shuts the door?

5. A penny lies at rest on a sloping desk. What forces are acting on it? Draw a diagram showing clearly the direction of each.

6. What must be the inclination of an inclined plane so that a given force, whether it acts horizontally or parallel to the length of the plane, will support the same mass? (This question should be answered by means of diagrams.)

7. Part of a chain rests on an inclined plane, and the remainder hangs over the back. Draw a diagram to show in what position the chain will rest if the plane is so smooth that friction need not be taken into account. Give any explanation you think necessary.

8. What is the mechanical advantage of a lever, the load arm of which is 30 cm. long and the effort arm 135 cm. long?

9. By means of a lever, an effort equal to a weight of 100 gm. moving through 16 cm. lifts a weight of 290 gm. through a vertical distance of 5 cm. What is the efficiency of the lever?

10. What is the inclination of a plane when a force equal to the weight of 120 gm. can move a weight of 170 gm. with uniform velocity up a smooth plane, the force being parallel to the plane? If the length of the plane is 70 cm., how much work, expressed in kilogram-metres, is done in moving the weight up the entire length of the plane?

11. What work is done by a horse against the action of gravity in drawing a carriage with its load, all weighing 1000 lb., 100 yd. up a slope of 1 in 25?

12. A cyclist and his machine weigh 180 lb. Assuming the absence of frictional resistance, at what horse-power must he work in order to ascend a slope of 1 in 30 at 5 miles an hour?

PART III.

HEAT.

CHAPTER XII.

EXPANSION. THERMOMETERS.

Change of size.—As a rule all bodies, whether solid, liquid, or gaseous get larger when heated, and smaller when cooled. The change of size which a body undergoes is referred to as the amount it expands or contracts; or, heat is said to cause expansion in the body. This expansion is regarded in three ways. When dealing with solids, expansion may take place in length (linear expansion), in area (superficial expansion), and in volume (cubical expansion). In the case of liquids and gases we have only cubical expansion. Similar terms can be used with reference to contraction.

The expansion which substances undergo when heated has to be allowed for in many cases. Railway rails, for instance, are usually not placed with their ends in actual contact, but a little space is allowed between the separate rails, so that they can expand in summer without meeting. Steam pipes used for heating rooms are also not firmly fixed to the walls at both ends, but are left slightly loose or are loose-jointed, so that they can expand or contract without doing any damage. For the same reason the ends of iron bridges are not fixed to the supports upon which they rest. Iron tyres are put on carriage wheels by first heating the tyre and, while it is hot, slipping it over the wheel. As the tyre cools it contracts and clasps the wheel very tightly.

The common occurrence in domestic life of the cracking of thick glasses when boiling water is poured on them, is explained by this expansion of solids by heating. The part of the glass with which the hot water comes in contact is heated and expands; but the effect is quite local; the heating is confined to one spot, because glass does not allow heat to pass through it readily. It is this local expansion of the glass which results in the cracking of the vessel. On the contrary, as silica expands only very slightly when heated, a flask constructed of this substance may be made red-hot in a flame and then plunged into cold water without cracking.

EXPT. 103.—**Expansion of metal.** Select a rod of copper, iron, or brass, about 30 cm. long. See that the ends are planed truly at right angles to the length of the rod. Lay the rod on a steel millimetre scale, and note its length. Place the rod on a tripod stand, and heat it with a Bunsen flame. Transfer it, by means of tongs, to the scale again, and note its length while hot.

EXPT. 104.—**Expansion apparatus.** Fit up the apparatus shown in Fig. 97. W is a wooden box with a sheet of paper pinned upon one of its sides. AB is a long knitting-needle fixed vertically with its upper end A held firmly in a clamp. CD is a strip of wood, about 12-15 cm. long and with square cross-section (like a very long household match). A shallow notch is made in the strip near its end D, in which the lower end of the needle can rest. The strip is pivoted at B, by burning a small hole through the strip with a red-hot needle; and a loosely fitting pin is passed through the hole and driven into the box. Mark the position of the end C of the strip by a pencil mark on the paper. Raise the temperature of the needle by passing a Bunsen flame up and down it. Note the change in the position of the pointer, and mark the final position of the end C. Allow

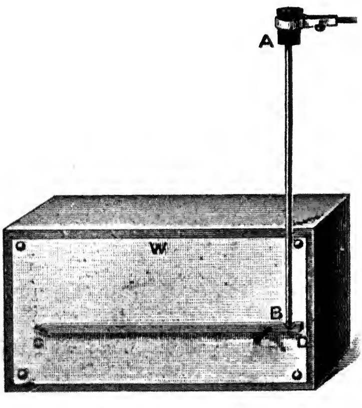


FIG. 97.—Simple apparatus to show expansion by heat.

the needle to cool, and notice how the wooden strip returns to its original position.

Expt. 105.—**Compound bar.** Make a compound strip (Fig. 98) of brass and of soft iron, by soldering or riveting the strips together. If necessary, straighten the strip by hammering, then heat it. Notice that the strip bends, because the brass expands more than the iron.

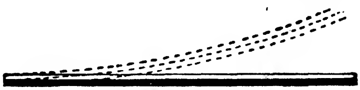


Fig. 98.—Effect of heat on a compound bar.

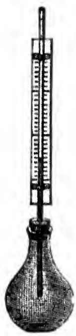


Fig. 99.—To show the expansion of a liquid when heated.

Expt. 106.—**Water.** Procure a 4-oz. flask and fit it with a cork. Bore a hole through the cork and pass through it a long glass tube which fits tightly. *Fill* the flask with water to which red ink has been added. Push the cork into the neck of the flask and so cause the coloured water to rise up the tube. There should be no air between the cork and the water. Now dip the flask in warm water, and notice that the liquid soon expands and rises up the tube (Fig. 99). Take the flask out of the warm water, and see that the coloured water contracts as it cools, and that it sinks in the tube.

Expt. 107.—**Other liquids.** Arrange two other flasks as in the last experiment, but filled respectively with alcohol and turpentine. Push in the corks till the liquid stands in each tube at the same height. Put all three flasks to the same depth into a vessel of warm water. Notice that the expansion of the glass causes a momentary sinking of the liquids ; and that ultimately the expansions of the three liquids are very different.

Fig. 100.—An air thermometer.

Expt. 108.—**Air thermometer.** Tightly fit a cork, through which a straight tube passes, into the neck of a 2-oz. flask. Support the flask so that the tube dips into a beaker containing coloured water (Fig. 100). Warm the flask with the hand or a flame so as to expel some of the air, and let the liquid rise in the stem. This instrument constitutes an *air thermometer*.

EXPT. 109.—**Differential thermometer.** Fasten two bulbs or flasks together (air-tight) by a tube bent six times at right angles, and containing some coloured liquid in the middle bends (Fig. 101). Show that the liquid moves if one flask is warmed more than the other. (This instrument is known as a *differential thermometer.*)

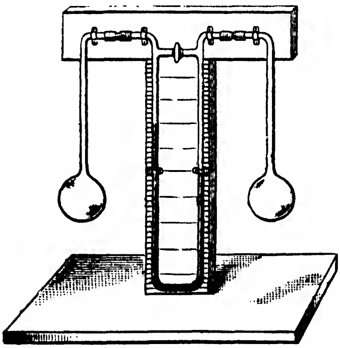


FIG. 101.—A differential thermometer.

Measurement of change of temperature.—Change of temperature means change in the state of hotness of a body. The change of size which takes place when a thing is heated provides a means of measuring the change of temperature which it undergoes. Consider the experiment with the coloured water in the flask with a long tube attached to it. Suppose the coloured water in the tube rises through a certain number of inches after the water has been heated somewhat, and that when the flask is placed into some other liquid, or some more water, the coloured water is found to rise up the tube to just the same place, we should have every right to say that the second liquid was exactly as hot as the first. This is measuring its temperature. The flask and tube with the water have become a "temperature measurer," that is, a **thermometer.**

A flask filled with water, and having a stopper through which a glass tube passes, could thus be used to show the expansion produced by warming and the contraction by cooling. But this flask and tube make only a very rough temperature measurer. The water does not get larger by the same amount for every equal addition of heat. Neither is it very sensitive, that is to say, it does not show very small increases in the degree of hotness; in other words, it does not record very small differences of temperature, and for a thermometer to be any good it must do this. Then, too, as every one knows, when water is made very cold it becomes ice, which, being larger than the water from which it is made, might crack the flask. For many

reasons, therefore, water is not a good substance to use in a thermometer.

Mercury and spirit thermometers.—There are many reasons for selecting mercury as the liquid for an ordinary thermometer. It is a liquid the level of which can be seen easily; it does not wet the vessel in which it is contained; it expands a considerable amount for a small increment of temperature; it is a good conductor of heat, and consequently it assumes very quickly the temperature of the body with which it is placed in contact. Very little heat is required to raise its temperature, and there is therefore little loss of heat due to warming the thermometer. Another liquid frequently used is spirits of wine, which is particularly valuable for measuring temperatures below that at which mercury would be frozen.

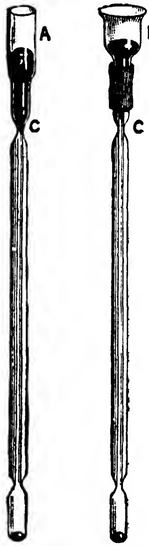


FIG. 102.—Thermometers in course of construction: (i) tube enlarged at top; (ii) tube with small funnel attached.

EXPT. 110.—**Construction of a thermometer.** Procure or make a thermometer tube, with a bulb at one end. With a little practice it is easy to blow a bulb upon a piece of thermometer tubing. One end of the tubing is held in a blow-pipe flame and twirled round until the glass melts and runs together so as to seal up the tube. A small blob of glass is then allowed to form, and while the glass is molten the tube is taken out of the flame and blown into steadily. Fit a small funnel to the open end, as shown in Fig. 102; pour clean dry mercury into the funnel, and fill the bulb by heating and cooling it several times in succession. Arrange the quantity of mercury so that, when cool, the bulb and *part* of the stem are filled with mercury.

EXPT. 111.—**Use of a thermometer.** Place in hot water the bulb of the instrument just constructed, and make a mark at the level of the mercury in the tube. Now place the instrument in cold water, and notice that the mercury sinks in the tube. The mercury is thus seen to expand when heated and contract when cooled.

EXPT. 112.—**Degrees of temperature.** Examine a thermometer. Notice that it is similar to the simple instrument already described, but the top is sealed up, and divisions or graduations are marked upon it, so that the height of the mercury in the tube can be recorded easily. These divisions are called *degrees* (Fig. 103).

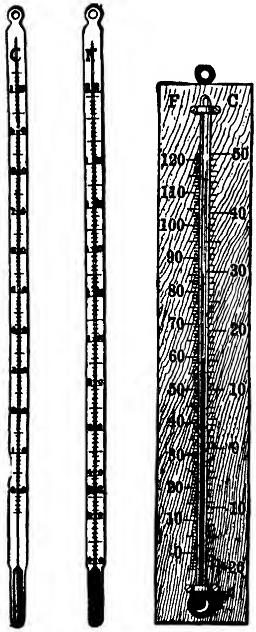


FIG. 103.—Forms of mounted and unmounted thermometers.

The fixed points on a thermometer.—In the graduation of a thermometer the plan always adopted is to choose two 'fixed points' from which to number the degrees of temperature. A convenient lower fixed point is the temperature at which ice melts, or water freezes, for this is always the same if the ice be pure, and remains the same so long as there is any ice left unmelted. Whenever the thermometer is put into melting ice the mercury in it always stands at the same level, or melting ice is always at the same temperature, and thus may be used to give one fixed point. The higher 'fixed point' chosen is that at which pure water boils at normal atmospheric pressure (viz. 760 mm. of mercury). This condition is necessary because the boiling point of a liquid is altered when the pressure upon it is changed, being raised when the pressure is greater, and lowered when the pressure is less. When the water boils, the temperature of the steam is the same as that of the water, only providing that the water is perfectly pure, but the temperature of the steam is dependent upon the pressure alone; hence, in determining the higher fixed point, it is usual to suspend the thermometer so that its bulb is just *above* the surface of the boiling water.

Thermometer scales.—It is necessary to give definite values to the fixed points, and to divide the interval between the two points according to some accepted plan, in order to be able

to compare observations made with different thermometers. The thermometers used in this country are graduated in two ways—(1) the Centigrade scale, (2) the Fahrenheit scale. A third scale—the Réaumur scale—is used extensively in Germany.

The Centigrade scale.—Here the freezing point is called *zero* or *no degrees Centigrade*, written 0° C. The boiling point is called *one hundred degrees Centigrade*, and is written 100° C. The space between these two limits is divided into 100 parts, and each division is called a *degree Centigrade*. This scale was devised by Celsius, who, however, proposed to call the boiling point of water 0° and the freezing point 100°.

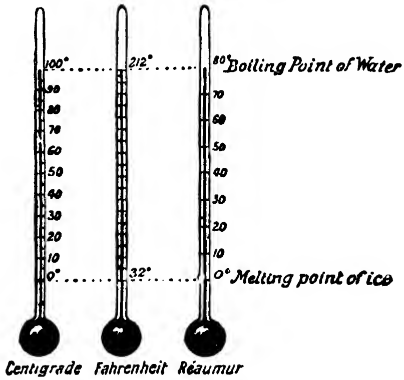


FIG. 104.—Thermometric scales.

The Fahrenheit scale.—On thermometers marked in this way the freezing point is called *thirty-two degrees Fahrenheit*, written 32° F., and the boiling point *two hundred and twelve degrees Fahrenheit*, written 212° F. The space between the two limits is divided into 180 parts, and each division is called a *degree Fahrenheit*. The reason of this difference is interesting.

Newton made a thermometer before Fahrenheit and divided into twelve equal parts or degrees the space through which the liquid in it expanded when the instrument was placed (1) in ice-water, (2) in the mouth of a healthy human being. Fahrenheit obtained a lower temperature by means of a mixture of ice-water and sal-ammoniac or sea-salt, and he used this temperature to mark the starting point or zero of his thermometer. He first divided the interval between this temperature and the temperature of the body into twice the number of parts used by Newton, namely 24; and he found that when the thermometer was placed in ice and water the liquid stood at the mark 8. The spaces between the degrees were however found to be too large, so later he divided each into four, thus making the freezing point of

water 32° and the body-temperature 96°. When the scale was extended by dividing the tube of the thermometer into equal spaces the mark 212° was found to coincide with the point at which the top of the liquid stood when the thermometer was placed in boiling water. There is no evidence that Fahrenheit used the boiling point of water as one of his fixed points, so that the existence of 180° between freezing point and boiling point may be accidental.

The Réaumur scale.—Upon thermometers graduated according to this scale the freezing point is marked 0° and the boiling point 80°. Réaumur adopted the number 80° as the temperature of steam, because he found that alcohol, diluted with one-fifth water, expanded in volume from 1000 to 1080 when raised from the freezing to the boiling point. The relation between the three scales is shown in Fig. 104.

Conversion of scales.—It should be clear from what has been said that the interval between the boiling and freezing points, that is, the same temperature difference, is divided into 100 parts on the Centigrade scale and 180 parts on the Fahrenheit, and consequently 100 Centigrade degrees are equal to 180 Fahrenheit degrees, which is the same as saying one degree Centigrade is equal to nine-fifths of a Fahrenheit degree, or one degree Fahrenheit is equal to five-ninths of a degree Centigrade.

$$100 \text{ C. degs.} = 180 \text{ F. degs.}; \quad \therefore \; 5 \text{ C.} = 9 \text{ F.}$$
$$\therefore \; 1^\circ \text{ C.} = \tfrac{9}{5}^\circ \text{ F. or } 1^\circ \text{ F.} = \tfrac{5}{9}^\circ \text{ C.}$$

In converting Fahrenheit readings into Centigrade degrees, we must subtract 32 (because of what has been said of the freezing point on the former scale) and multiply the number thus obtained by 5 and divide by 9. To change from Centigrade to Fahrenheit, multiply the former reading by 9 and divide by 5 and add 32 to the result.

EXAMPLE.—What temperature on the Fahrenheit scale corresponds to 20° C.?

Answer.—20° C. is 20 C. degs. above the temperature of melting ice, *i.e.* $20 \times \frac{9}{5}$ Fahr. degs. above 32° F. $= (36 + 32)^\circ$ F. $= 68^\circ$ F.

When it is necessary to refer to temperatures lower than the freezing point of water, a minus sign is placed before the temperature, thus, three degrees below the freezing point of water on the Centigrade scale is written -3° C.

Marking the lower 'fixed point.'—For this purpose an arrangement like that shown in Fig. 105 is suitable. The funnel is filled with shavings or fragments of ice, which before using has been washed carefully; or snow, if more convenient, may be used. The glass dish catches the water which is formed from the melting of the ice or snow. A hole is made in the ice-shavings by thrusting in a pencil or glass rod about the size of the thermometer, and into this hole the thermometer is put and is so supported that the whole of the mercury is surrounded

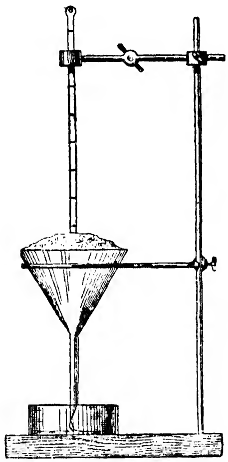


Fig. 105.—Thermometer in ice for the observation of freezing point.

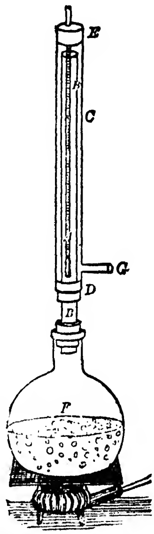


Fig. 106.—Apparatus for determining the boiling point on a thermometer.

by the ice or snow. The arrangement is left for about ten or fifteen minutes, until it is quite certain that the tube and mercury are at the same temperature as the melting ice. When this is so, the tube is raised until the mercury is just above the ice, and the level of the mercury is marked by a scratch on a thin layer of varnish or wax with which the stem has been covered previously.

Marking the higher 'fixed point.'—Satisfactory determinations can be made by means of the apparatus shown in Fig. 106. A can or flask F is fitted with a cork, through which a glass or brass tube B passes. Surrounding this tube is a wider tube C, fitted upon the inner tube by means of a piece of thick india-rubber

tubing D. At the top of the outer tube is a cork E having a hole in which a thermometer can be fitted. When the water in the flask is boiled, steam passes up the inner tube B, and down the wide tube C, and escapes at the outlet G into the open air. To use the apparatus, the top of the stem of the thermometer is pushed gently into the cork which fits in the outer tube, and adjusted so that the 100° point is just below the cork. The cork is then fitted into its place, the water boiled, and when steam has been coming off for about a quarter of an hour, the thermometer is raised so that the mercury surface is just above the cork; and, while the steam continues to pass through the apparatus, the position of the surface is indicated by means of a file mark on the outside of the stem.

EXPT. 113.—**Graduation of a thermometer.** Mark on the stem of the thermometer which you are making, the lower and the upper fixed points by the methods described in the previous paragraphs. Read and note the height of the mercury barometer at the time of the experiment. Fix the thermometer to a suitable strip of thin well-planed wood, the face of which has been previously covered with plain paper. Mark off on the paper the positions of the fixed points. If a *Centigrade* scale be desired, divide the interval into 100* equal divisions, and mark the fixed points 0° and 100° respectively

EXPT. 114.—**Thermometer corrections.** Determine the errors, if any, in the positions of the fixed points of a Centigrade and of a Fahrenheit thermometer. Enter your observations thus:

Height of mercury barometer,......
Calculated temperature of steam,......

	Centigrade thermometer.		Fahrenheit thermometer.	
	Error.	Correction.	Error.	Correction.
Lower fixed point -				
Upper " "				

* This is only approximately correct when the reading of the barometer is between 758 and 762 mm. When the pressure is outside these limits, the temperature of the steam must be calculated. For pressures between 750 and 770 mm., it may be assumed that the temperature of dry steam is increased or diminished by 0°·037 Centigrade for each millimetre change in the barometric height. Thus, when the atmospheric pressure is 752 mm., the temperature of the steam is

$$100° - \{0°{\cdot}037\,(760 - 752)\} = 100° - 0°{\cdot}296 = 99°{\cdot}70\ C.$$

If the atmospheric pressure is 770 mm., the temperature of the steam is

$$100° + \{0°{\cdot}037\,(770 - 760)\} = 100°{\cdot}37\ C.$$

It may be necessary, therefore, to modify the above number 100.

Carefully note that when the reading is too high, the error is *positive*, and should be denoted by the sign +; when the reading is too low, the error is denoted by the sign −. The *corrections* to the readings of a thermometer are *the quantities which must be added to or subtracted from the readings in order to give the correct temperature.* Hence the signs attached to the corrections are the reverse of those attached to the errors.

Maximum and minimum thermometers.—There are several forms of thermometer which show the highest or lowest temperatures reached since they were last set. One of these is Six's self-registering thermometer represented in Fig. 107. The bulb A is filled with alcohol, and is separated from the top bulb D by a mercury thread BC. Above C there is more alcohol; but there is sufficient space left in D to allow for expansion. Expansion or contraction of the alcohol in A causes movement of the mercury thread, and the extreme positions of the thread are indicated by two steel indexes provided with springs just strong enough to prevent them from slipping.

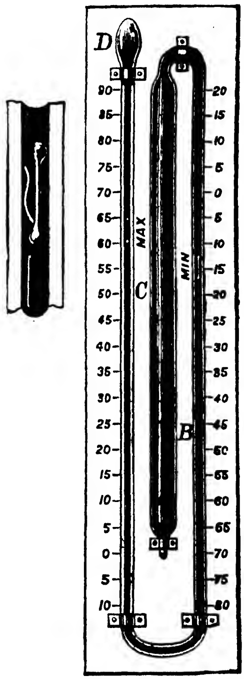


FIG. 107.—Maximum and minimum thermometer.

The doctor's thermometer.—For the measurement of the temperature of the body, what is termed a **clinical thermometer** is best (Fig. 108). As the temperature of the living human body in a state of health is never many degrees above or below a temperature of 98° F., a clinical thermometer is only graduated from about 95° F. to 110° F. When the bulb of such a thermometer is put into the mouth, or under the armpit, of a person in health, and left there for two or three minutes, it will be found, on taking it out, to indicate a temperature from 97°·8 F. to 98°·6 F. The thread of mercury in the stem of the thermometer remains in one position, though the air is cooling the mercury while the

thermometer is being read. This is because of the constriction at the top of the bulb, which causes the thread of mercury in the stem to be left behind while the mercury in the bulb contracts.

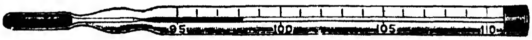


FIG. 108.—A clinical thermometer.

To "set" the thermometer for a fresh observation, it is only necessary to jerk it slightly, when the thread of mercury will again join up to the liquid in the bulb.

EXERCISES ON CHAPTER XII.

1. What is a thermometer, and what information concerning heat does it supply?

2. How would you test whether the two fixed points on a mercurial thermometer were marked accurately?

3. Describe how you would graduate a thermometer. Would any correction be necessary if you did it on the top of a mountain, or at the bottom of a coal mine?

4. I take two equal flasks, the mouths of which are fitted with bored corks carrying long glass tubes, and fill one with water coloured blue, and the other with methylated spirits coloured red; I then plunge them both into boiling water. Explain what will take place, giving reasons.

5. Take a glass tube open at one end and having a bulb at the other. Hold the tube so that the open end dips into water. Heat the bulb gently with a spirit lamp for a minute or two, and then take the lamp away. What will be observed? How can you account for the facts observed?

6. One-fifth of a bottle is filled with cold water; the bottle is tightly corked; the cork is pierced by a bent tube, one end of which dips into the water of the bottle and the other into water standing in an open vessel. Describe the results that may be observed if the bottle and its contents are heated up to a temperature of 99° Centigrade, and then allowed to cool.

7. A nurse cleanses a doctor's thermometer which reads up to 105° F. in boiling-hot water. The doctor now finds that the thermometer is useless. Why is this?

8. What do we mean by the 'boiling point' of a liquid? Explain why good tea cannot be made at the top of a very high mountain.

9. Convert the following into temperatures on the Centigrade scale: $71°$ F., $0°$ F., $-40°$ F.

10. Find the temperatures on the Fahrenheit scale corresponding to: $83°$ C., $15°$ C., $-5°$ C.

11. Convert the following Centigrade temperatures into Fahrenheit.

	Melting Point.	Boiling Point.
(*a*) Hydrogen	$-259{\cdot}5°$C.	$-252°$ C.
(*b*) Mercury	$-39{\cdot}2$	357
(*c*) Sodium	$97{\cdot}6$	825

12. How does the distance between the two fixed points of a thermometer vary with the size of the bore, when the size of the bulb remains the same?

13. When the bulb of a thermometer is plunged into hot water, the mercury at first falls a little, and then rises. Why is this?

14. A Fahrenheit thermometer registers $110°$ while a faulty Centigrade thermometer registers $44°$. What is the error in, and what is the correction for, the latter?

15. Two thermometers are hung up in a room. One registers a temperature of $15°$ and the other $59°$. Explain fully the meaning of this difference.

16. In some liquid-in-glass thermometers mercury is used, in others alcohol. Discuss the advantages and disadvantages of the use of each of these two substances as the thermometric liquid.

17. Explain what you understand by the "temperature of a substance." State what conditions an instrument should fulfil which is intended to measure accurately the temperature of a liquid.

18. Why are thermometer tubes usually of very fine bore, and why are they provided with bulbs?

19. How would you test the accuracy of the 'fixed points' of a thermometer? Explain how Six's thermometer shows both maximum and minimum temperatures.

CHAPTER XIII.

COEFFICIENTS OF EXPANSION.

Coefficient of expansion.—While a definite rise of their temperatures causes most bodies to expand, the amount of such expansion varies within wide limits. In the case of certain special alloys the expansion is almost negligible; while gases expand more than double their volume on being heated from 0° C. to 300° C. In the case of solids we are concerned usually with the **linear** coefficient of expansion, while for liquids and gases the coefficient of **cubical** expansion is of importance.

EXPANSION OF SOLIDS.

Coefficient of linear expansion.—**The increase in length experienced by a rod of unit length, at 0° C., when its temperature is raised through 1° C. is called its coefficient of linear expansion.** The actual expansion being small in the case of solids, it is not necessary to know the length of the solid at 0° C.; hence a simpler definition is sufficiently accurate. It may be said that the fraction of its length which a solid body expands on being heated through 1° C. is called its *coefficient of linear expansion.*

To obtain a measurement of the linear expansion of a rod when heated the form of apparatus shown in Fig. 109 may be used. A rod of glass or metal about 18 inches long is surrounded by a glass tube having an inlet for steam at C and an outlet at D. The end A of the rod rests in a V-shaped groove and against a weight W. The other end rests on a needle which is free to roll on a glass base. To the needle is attached a cork having a split straw pointer. Any movement of the needle will be observed against the scale E.

When steam is passed through the tube it heats the bar, and the expansion shows itself at B—the end A being fixed—by causing the needle to roll. To secure good contact the rod may be roughened with emery where it rests on the needle, and be pressed to it by means of an elastic band attached to the support.

After the steam has circulated for 5 or 10 minutes, the fraction of a complete circle the index has moved through is observed.

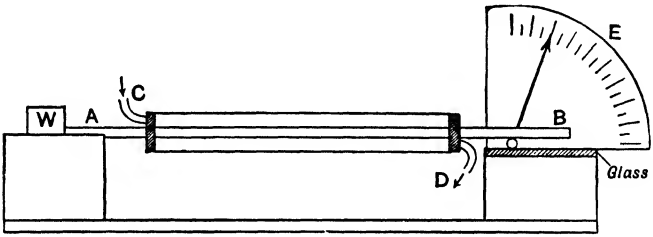


FIG. 209.—Determination of the coefficient of linear expansion of a rod.

This shows what part of a complete rotation the needle has made. To find the horizontal expansion of the rod corresponding to this rotation it is necessary to determine the diameter of the needle. To do this several similar needles are placed in a row, the total breadth is measured and divided by the number of needles.

If the diameter of the needle be d cm., the distance through which the needle rolls in one complete rotation is $(\pi \times d)$ cm. Let θ be the angle through which the index moves; then the advance of the rod, being equal to *twice* the distance through which the needle travels, is

$$2\pi d \times \frac{\theta}{360}.$$

Suppose the measured expansion to be E, the length of the rod up to the needle L, and its temperature at the commencement of the experiment 15° C. The fraction of its original length which the rod expands will be E/L.

The rise of temperature having been from 15° C. to 100°C., that is 85° C., the fraction of its original length which the rod expands for *each degree rise of temperature* will be $\frac{E}{L \times 85}$.

This value is the coefficient of linear expansion.

EXPT. 115.—**Measurement of linear expansion.** Procure and examine the apparatus shown in Fig. 109, if necessary fitting the parts together. Notice that AB is a glass, or metal, rod resting in a groove at A, against a weight W, and rolling on a needle on a glass bed at B. Fix a split straw pointer to the needle to move against the graduated quadrant E. Observe that CD is a tube of wide bore fixed on the rod by means of corks and having an inlet for steam at C and an escape at D. When the apparatus is in adjustment, take the temperature of the room in the neighbourhood of the apparatus, and then pass steam through CD for 5 or 10 minutes. Note what part of a whole turn the pointer moves through. Obtain the diameter of the needle by placing several similar needles in a row, measuring the total breadth and dividing by the number of needles. Obtain the actual movement of the end B of the rod by multiplying together the fraction of a rotation shown by the index and the circumference of the needle. Assuming the steam to be at 100° C. obtain the rise of temperature of the rod.

Then calculate thus :

A length of......cm. of the metal or glass rod when raised......degrees expanded......cm.

Therefore 1 cm. of the rod when raised 1 degree expands......cm.

The value so obtained is the coefficient of expansion of the material of the rod used.

The *coefficient of linear expansion* is usually denoted by the symbol α. From the above definition of the coefficient, it may be expressed thus :

$$\alpha = \frac{\text{change in length}}{\text{original length} \times \text{change in temperature}},$$

or, if L_t = length at $t°$ C., and L_T = length at a higher temperature T° C., then

$$\alpha = \frac{L_T - L_t}{L_t(T - t)},$$

$$\text{or} \quad L_T - L_t = \alpha L_t(T - t),$$

$$\text{or} \quad L_T = L_t\{1 + \alpha(T - t)\}.$$

EXAMPLE.—A platinum wire is 3 metres long at 0° C. What will be its length at 100° C.? (Coefficient of linear expansion of platinum, 0·000009.)

1 cm. of the wire at 0° C. becomes (1 + 0·000009) cm. at 1° C., and

" " " 1 + (0·000009 × 100) cm. at 100° C.

Hence, 300 cm. at 0° C. becomes 300(1·0009) cm. at 100° C., or

Length, at 100° C., is 300·27 cm.

Coefficient of superficial expansion.—The coefficient of superficial expansion of a solid is the increase in area of a sheet of the substance, initially having unit area, when warmed through 1° C.

Suppose the length of each edge of a small square sheet of a metal to be 1 cm. at 0° C. If the coefficient of linear expansion of the metal be α, then, when the sheet is warmed to 1° C., the length of each edge will be $(1+\alpha)$ cm., and the area will be $(1+\alpha)^2$ sq. cm. or $(1+2\alpha+\alpha^2)$ sq. cm. But, since α is extremely small, the last term may be neglected; hence the total increase in area may be regarded as equal to 2α; or, the coefficient of *superficial* expansion is numerically equal to *twice* the *linear* coefficient.

EXAMPLE.—A sheet of brass is 40 cm. long and 20 cm. wide at 0° C. If it be heated to 50° C., what will be its increase in area? (Coefficient of linear expansion of brass, 0·000019.)

$$\begin{aligned}
\text{Original area, at } 0^\circ\text{ C.,} &= 40\times 20 = 800 \text{ sq. cm.}\\
\text{Length of sheet, at } 50^\circ\text{ C.,} &= 40\{1+(0{\cdot}000019\times 50)\}\\
&= 40\times 1{\cdot}00095\\
&= 40{\cdot}038 \text{ cm.}\\
\text{Width of sheet, at } 50^\circ\text{ C.,} &= 20\{1+(0{\cdot}000019\times 50)\}\\
&= 20{\cdot}019 \text{ cm.}\\
\text{Area of sheet, at } 50^\circ\text{ C.,} &= 40{\cdot}038\times 20{\cdot}019\\
&= 801{\cdot}52 \text{ sq cm.}\\
\text{Hence, increase of area} &= 1{\cdot}52 \text{ sq. cm.}
\end{aligned}$$

Alternate method. Since the coefficient of superficial expansion is $(2\times 0{\cdot}000019)$,

$$\begin{aligned}
\text{the area of sheet, at } 50^\circ\text{ C.,} &= 800\{1+(0{\cdot}000038\times 50)\}\\
&= 800\{1+0{\cdot}019\}\\
&= 801{\cdot}52 \text{ sq. cm.}
\end{aligned}$$

Cubical expansion of solids.—When a solid body is heated each of its dimensions is increased in the same proportion. Suppose each edge of a cube of metal, of which the coefficient of linear expansion is α, to have a length of 1 cm. at 0° C., and that it is warmed to 1° C. The length of each edge will now be $(1+\alpha)$ cm., and the volume of the cube will be $(1+\alpha)^3$ c.c. Hence,

$$\begin{aligned}
\text{Increase of volume} &= (1+\alpha)^3 - 1\\
&= (1+3\alpha+3\alpha^2+\alpha^3) - 1\\
&= 3\alpha+3\alpha^2+\alpha^3.
\end{aligned}$$

But, since a is extremely small, all terms beyond the first may be neglected; hence, increase in volume $= 3a =$ three times the coefficient of linear expansion.

By definition, **the coefficient of cubical expansion of a substance is the increase in volume of unit volume when it is heated through 1° C.** Hence, the coefficient of cubical expansion of a substance is equal numerically to three times the coefficient of linear expansion of the substance.

Expansion of Liquids.

Different rates of expansion.—The following experiment will serve to illustrate the great difference in the rate of expansion of different liquids, and to show that the rate is regular over a wide range of temperature.

Expt. 116. **Mercury, water, and spirit.** Fit three 4 oz. flasks with rubber stoppers and narrow glass tubes of the same bore. Fill the flasks respectively with the liquids; insert the stoppers, and see that

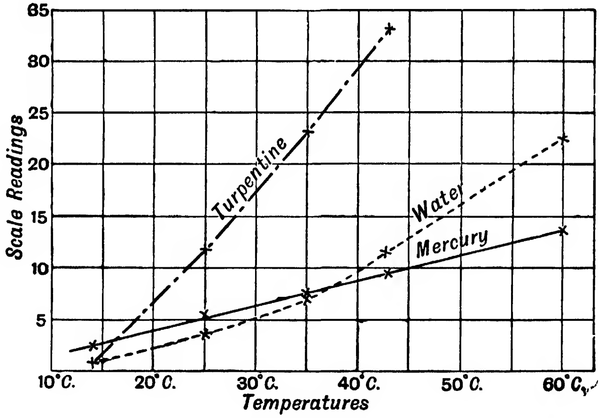


Fig. 109 a.—Relative expansions of mercury, water and turpentine.

no air-bubbles remain beneath the stoppers. Place the flasks in a deep dish of cold water. Stir the water, and note its temperature. Measure the height of each liquid column above the top of the stopper. Raise the temperature of the water about 10° C., keep it constant for several minutes, and repeat the measurements. Repeat at higher temperatures, up to about 60° C.

Plot your readings on a piece of squared paper and state your conclusions. In Fig. 109A are shown the results obtained with mercury, water and turpentine. It will be noticed that the rate of expansion of the two latter liquids appears to increase as the temperature rises.

Real and apparent coefficients of expansion.—As in the case of solids, **the coefficient of cubical expansion of a liquid is the increase in volume of unit volume when it is heated through 1° C.** Hence, if β is the coefficient, V_T and V_t the volumes occupied by the same mass of liquid at temperatures T° and $t°$, then

$$\beta = \frac{V_T - V_t}{V_t(T - t)},$$

or
$$V_T - V_t = V_t\beta(T - t),$$

hence
$$V_T = V_t\{1 + \beta(T - t)\}. \quad \text{(i)}$$

Since the density of a liquid is *inversely* proportional to the volume occupied by the same mass of liquid, equation (i) may be written
$$D_t = D_T\{1 + \beta(T - t)\}. \quad \text{(ii)}$$
where D_t and D_T are the densities at the temperatures $t°$ and T°.

As, in most methods of finding the coefficient, the liquid is contained in a glass vessel, the capacity of which necessarily changes with change of temperature, the amount by which the liquid expands appears to be less than it really is. The above coefficient β therefore is less than it ought to be, and it is termed the **apparent coefficient.** It will be seen that in using a thermometer, and in Expts. 106, 107 and 116, the apparent coefficient only is observed. It can be proved, by simple mathematics, that

$$\beta_r = \beta_a + g,$$

where β_r and β_a are the real and apparent coefficients respectively, and g is the cubical coefficient of expansion of glass.

FIG. 110.—Determination of the coefficient of expansion of a liquid.

EXPT. 117.—**Apparent coefficient of expansion.**—Fig. 110 represents a thin glass tube (about 30 cms. × 0·3 cm. bore) partly filled with the liquid. The tube is firmly attached to a thermometer. Place the arrangement in iced water,

contained in a deep cylinder, and observe the degree on the thermometer scale which is level with the top of the liquid column. Transfer the tube to a cylinder of warm water, keep the temperature constant for several minutes and again take the reading of the top of the liquid. Remove the tube and, with a scale, measure the distance from the bottom of the tube to the points on the thermometer scale previously observed. If the bore is uniform the volume of liquid may be taken as proportional to the *length* of the column. Calculate the apparent coefficient.

EXPT. 118.—**Alternative Method.** Find the weight (w_1) of a clean, dry 'specific gravity' bottle. Fill the bottle with the given liquid, and suspend it for 5 minutes immersed to the neck in a water bath at 18°-20° C. Keep the water stirred, and its temperature constant. Note this temperature (t). Remove the bottle, dry it in a folded cold duster, and weigh it (w_2). Re-suspend it in a water-bath at 50°-60° C., and proceed as before. Note the temperature (T), allow the bottle to cool, and find its weight (w_3).

Assuming that the capacity of the bottle is constant, the density of the liquid is proportional to the weight of the liquid contained in the bottle; hence, from equation (ii),

$$(w_2 - w_1) = (w_3 - w_1)\{1 + \beta(\mathrm{T} - t)\}$$

or

$$\beta = \frac{w_2 - w_3}{(w_3 - w_1)(\mathrm{T} - t)}.$$

EXAMPLE.—A litre glass flask is calibrated at 15° C. If the diameter of the neck is 1·6 cms., how far above the calibration mark will the surface of the water be when the flask contains 1 kilogram of water at 35° C.? [Linear coefficient of glass, $8{\cdot}3 \times 10^{-6}$. Volume of 1 gm. of water, at 15° C., 1·0009 c.c.; at 35° C., 1·0059 c.c.]

(i) Capacity of flask, at 15° C., = 1000·9 c.c.

" " at 35° C., $= 1000{\cdot}9\{1 + (3 \times 8{\cdot}3 \times 10^{-6}) \times 20\}$

$= 1001{\cdot}4$ c.c.

Increase in capacity of flask, 0·50 c.c.

(ii) Vol. of 1 kgm. of water at 35° C. = 1005·9 c.c.

Increase in vol. of water = 1005·9 − 1000·9 = 5 c.c.

(iii) Vol. of water above calibration mark = 5 − 0·5 = 4·50 c.c.

Hence, height $\times \pi \times \overline{0{\cdot}8}|^2 = 4{\cdot}50$; or, height = 2·24 cms.

The anomalous expansion of water.—The first effect of cooling water is to make it get smaller in volume, or contract. This

goes on steadily until the temperature of four degrees Centigrade (4° C.) is reached. From this point, though the cooling is continued the water no longer contracts, but begins to expand. This expansion continues until the temperature 0° C. is reached, when the water begins to change into solid ice, which is much larger than the water from which it is formed.

Conversely, if some water be taken at a temperature of, say, 1° C., and allowed to get warmer, it will steadily get smaller in volume up to a temperature of 4° C., but after this temperature is reached the volume will increase as the temperature rises.

It has been shown already that if the volume of a body gets greater while its mass remains the same, the density of the body must get less and less. Since the same mass of water gradually gets smaller and smaller in volume as it is cooled down or warmed up to 4° C., we may say that its density becomes greater and greater as the temperature approaches 4° C. This temperature (4° C.), at which a given volume of water weighs more than the same volume at any other temperature, is known as the temperature of the **maximum density of water.**

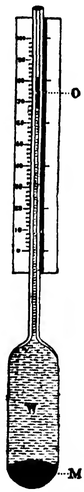


FIG. 111.—Apparatus for expansion of water.

Determination of temperature of maximum density of water.—In order to observe the changes in volume occupied by water near the temperature of maximum density, it is necessary to use, as a containing vessel, some device which has a constant capacity. This can be obtained by using a glass vessel partly filled with mercury, as shown in Fig. 111. When such a vessel is cooled, the contraction of the glass diminishes the capacity of the vessel; but the contraction of the contained mercury *increases* the capacity of the space within the tube and above the mercury. Since the coefficient of expansion of mercury is about seven times as great as the coefficient of cubical expansion of glass, a vessel of constant capacity is obtained if about one-seventh part of the total capacity is occupied by mercury. In order to fit up the apparatus, the empty tube is weighed, filled with mercury, and

again weighed. The tube is inverted, and alternately heated and cooled, until six-sevenths of this mercury has been expelled. The tube is then fixed in an upright position and filled with *boiled* distilled water.

EXPT. 119.—**Expansion of water near freezing point.** Support the apparatus shown in Fig. 111 in a wide test tube containing mercury, so as to secure uniformity of temperature. Place a thermometer in the mercury, and support the wide tube containing it and the apparatus in a beaker of cold water. Notice the position of the top of the liquid in the tube, and read the temperature shown by the thermometer. Add ice to the water, and as the temperature falls notice the level of the liquid in the tube for every degree down to 1° or 2° C.

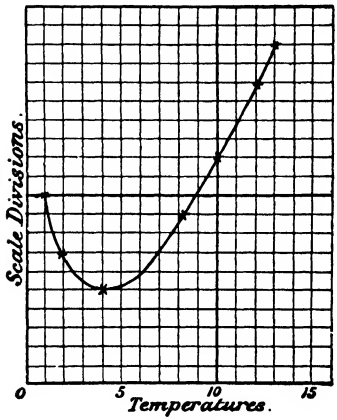


FIG. 112.—Graphic representation of changes in volume of water near the temperature of maximum density.

Then let the temperature of the water in the beaker gradually rise, adding a little warm water, if necessary, and again observe the positions at the same temperatures as before. The mean of the two positions observed for each temperature should be taken as the true reading for that particular temperature. Construct a curve like that shown in Fig. 112 to represent the observations of the changes of volume of water at temperatures near the freezing point.

Hope's apparatus.—An experiment with what is known as Hope's apparatus shows very well that water is at its maximum density at 4° C. A metal cylinder provided with two side necks in the way shown in Fig. 113 is filled with water at 10° C. Into the side necks, corks with thermometers passing through them are fitted. The sides of the vessel should be wrapped round with cotton wool, and a piece of cardboard should be placed on the top of the cylinder. A freezing mixture, which can be made by mixing salt with pounded ice, is applied to the middle of the cylinder. This is done by filling a vessel, fixed round the middle of the outside of the cylinder, with the mixture in a way which the illustration makes quite clear. The freezing mixture, of

course, at once cools the water in the middle of the cylinder. On watching the thermometers it is found that the first effect of the cooling is to cause the temperature of the lower thermometer to fall. The temperature of the upper thermometer, however, remains unaltered. The only way in which this can be explained is by supposing that as the water in the middle of the cylinder is cooled it gets denser and sinks to the bottom. As the cooling proceeds it is found that the water at the bottom of the cylinder *never gets below* 4° C. But soon after the water at the bottom of the cylinder has reached 4° C., the temperature of the upper thermometer begins to fall and goes on getting lower till it actually reaches 0° C. But all this time the water at the bottom remains at 4° C.

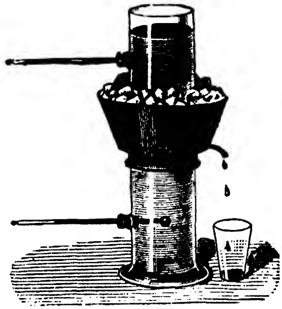


FIG. 113.—Hope's apparatus for the observation of the temperature of maximum density of water.

Now it is quite clear that the densest water will sink to the bottom, and as the temperature of the water there remains at 4° C. it may be concluded that water at this temperature is denser than any other.

These considerations are summed up in the statement that **water at a temperature of 4° C. expands whether it is heated or cooled.**

Expansion of Gases.

Coefficient of expansion of gases.—In the case of the expansion of solids and liquids, the coefficient is so small that, although the strict definition refers to the increase in length when heated from 0° C. to 1° C., yet, in conducting an experiment, it suffices to determine the increase in length due to an increase of temperature of 1° C. at any part of the thermometer scale. But, **in the case of the expansion of gases, the coefficient is so great that it has to be defined in terms of the volume occupied at 0° C.**

Owing to the compressibility of gases (*see* Boyle's Law, p. 85), the pressure to which the gas is subjected has to be taken into consideration. An increase of temperature may give rise, according to the arrangement of the containing vessel, either to an increase in *volume* of the gas or to an increase in its *pressure*. As a general rule, it is usual either (i) to keep the pressure constant, and to measure the increase in volume when

the temperature is raised, and so to determine the coefficient of expansion *at constant pressure*, or (ii) to keep *the volume constant* and measure the increase in pressure when the temperature is raised.

Expansion of gases, at constant pressure.—John Dalton, at Manchester in 1801, made the first important experiments on the expansion of gases. He claimed that the increase in volume for each equal rise in temperature is a constant fraction of the volume *at the temperature immediately preceding*. He also proved that *all gases expand equally when heated*. Shortly afterwards, J. A. C. Charles (a professor of Physics in Paris) found that the expansion is a constant fraction of the volume *at some arbitrary fixed temperature* (and *not* at the temperature immediately preceding, as stated by Dalton). The law, generally known as **Charles's Law**, according to which gases expand when heated, may be expressed thus: **Equal volumes of all gases under the same constant pressure, expand by the same fraction of the volume at 0° C. for every degree rise in temperature.**

Charles's Law is practically true for the so-called permanent gases, such as hydrogen, air, nitrogen and oxygen, the increase in volume for an increase of 1° C. in temperature being in each case $\frac{1}{273}$ or 0·00366 of the volume at 0° C.; and this fraction may be termed **the coefficient of expansion of a gas at constant pressure.**

Consider a volume of 273 c.c. at 0° C.; this would become 274 c.c. at 1° C., 275 c.c. at 2° C. and $(273+t)$ c.c. at $t°$ C. Or, expressed in another way, if the volume at 0° C. be taken as unity, the volume at other temperatures may be stated as follows:

0° C.	1° C.	2° C.	30° C.	$t°$ C.
1	$1+\frac{1}{273}$	$1+\frac{2}{273}$	$1+\frac{30}{273}$	$1+\frac{t}{273}$

Let the volume of gas at 0° C. and $t°$ C. be V_0 and V_t respectively, and the coefficient of expansion be represented by α, then

$$V_t = V_0(1+\alpha t). \quad \text{.........................(1)}$$

This may also be written,

$$V_0 = \frac{V_t}{1+\alpha t}. \quad \text{.............................(2)}$$

The student must carefully bear in mind that the fraction $\frac{1}{273}$ refers to the volume of the gas *at* 0° *C.*; if the volume at some higher temperature, say 50° C., is given, it is not correct to say that it will increase by $\frac{1}{273}$ of this volume when warmed to 51° C., but it *is* correct to say that it would expand by $\frac{1}{273}$ of the volume *which it would occupy at* 0° *C.*

When the volume at some temperature other than 0° C. is given, it is necessary to calculate, by equation (2), the volume which it would occupy at 0° C., and then, by equation (1), the volume it would occupy at 51° C. An alternative method is as follows:

Suppose V_t to be the volume of gas at $t°$ C., and V_T the volume at T° C., then

$$V_T = V_0(1 + \alpha T) \quad \text{and} \quad V_t = V_0(1 + \alpha t).$$

Hence
$$\frac{V_T}{V_t} = \frac{1 + \alpha T}{1 + \alpha t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$= 1 + \frac{T}{273} \Big/ 1 + \frac{t}{273} = \frac{273 + T}{273 + t} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

or
$$V_T = V_t \times \frac{273 + T}{273 + t}.$$

It is explained in a later paragraph (p. 168) that the expressions $273 + T$ and $273 + t$ represent the temperatures T° C. and $t°$ C. on the **absolute scale**, the zero of which is 273 degrees Centigrade below 0° C.

Determination of coefficient of expansion of a gas.—In carrying out an experiment to determine the coefficient of expansion of a gas it is not always convenient to observe the volume occupied by the gas at 0° C., and it would be necessary to *calculate* the volume which the gas would have occupied at 0° C. An alternative method of calculating the result is to find the *apparent* coefficient of expansion of the gas between the two temperatures used in the experiment. It can be shown, as indicated below, that when the lower temperature used in the experiment is $t°$ C., the apparent coefficient of expansion is

$$\frac{1}{273 + t}.$$

Suppose a to be the apparent coefficient of expansion between the temperatures $t°$ and T° C., and V_t, V_T, to be the observed volumes at $t°$ C. and T° C., then

$$V_T = V_t\{1 + a\,(T - t)\}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

But (p. 165)
$$\frac{V_T}{V_t}=\frac{273+T}{273+t}.$$

Hence, from equation (5),
$$1+a(T-t)=\frac{273+T}{273+t},$$

$$\text{or}\quad a(T-t)=\frac{T-t}{237+t},\qquad \text{or}\quad a=\frac{1}{273+t}.$$

EXPT. 120.—**Expansion of gas at constant pressure.** Dry thoroughly a 500 c.c. flask, and fit it with a one-holed *rubber* stopper through which passes a short glass tube with rubber tubing and clip attached. Fix the flask in an open metal vessel (as shown in Fig. 114) sufficiently large for the flask to be immersed completely in boiling water. The flask can be held immersed by means of an inverted stout glass jar held in a clamp at its upper end. In order to prevent steam from entering the flask it is well to attach a long glass tube to the open end of the rubber tubing. Keep the clip open, and gradually heat the water to boiling. Let the water boil for about 5 minutes, and close the clip.

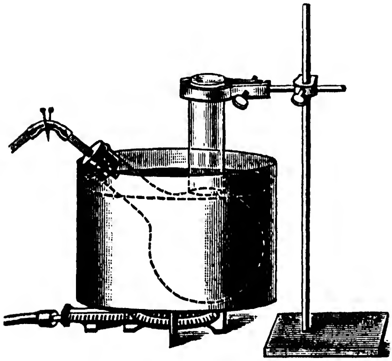


FIG. 114.—Determination of the coefficient of expansion of a gas at constant pressure.

Note the temperature of the water. Remove the flask, immerse it in a bath of cold water with its neck downwards. Open the clip, and keep the flask moving in the water so that the remaining air acquires the same temperature as the water. Lower the flask till the level of the water is the same inside and outside; then close the clip. Note the temperature of the cold water. Remove the flask, and measure the volume of water which has entered the flask. Find also the volume of the whole flask up to the lower surface of the stopper.

A method of calculation is indicated by the following example of an experiment:

Volume of whole flask - - - - -	340 c.c.
„ water drawn into flask - - -	77 c.c.
Temperature of boiling water - - - -	98° C.
„ cold „ - - - -	15° C.

340 c.c. of air, in cooling through 83° (*i.e.* from 98° C. to 15° C.), contract 77 c.c.

340 c.c. of air, in cooling through 1°, contract $\frac{77}{83}$ c.c.

" " " 98°, " $\frac{77}{83}\times 98 = 90{\cdot}9$ c.c.

Hence,

(340 − 90·9) c.c., at 0° C., expand 90·9 c.c. when warmed to 98° C.

or 1 c.c., at 0° C., expands $\frac{90{\cdot}9}{249{\cdot}1}$ c.c. " "

or 1 c.c., at 0° C., " $\frac{90{\cdot}9}{249{\cdot}1\times 98}$ c.c. " through 1° C.

or " " " 0·00372 c.c. " "

An alternative method of calculating the result is to determine the *apparent* coefficient of expansion between the temperatures 15° C. and 98° C. The result should be approximately $\frac{1}{273+15}=\frac{1}{288}$. Thus, by equation (5) (p. 165),

$$\alpha=\frac{V_T - V_t}{V_t(T-t)}$$

$$=\frac{77}{(340-77)(98-15)}=\frac{77}{263\times 83}=\frac{1}{283{\cdot}5}.$$

Fig. 115 represents another method of determining the coefficient of expansion of a gas at constant pressure.

EXPT. 121.—**Direct method.** Obtain a piece of thermometer tubing of about 1 mm. bore and 20 cm. long. Suck into it a length of about 1 cm. of mercury. Seal one end of the tube and arrange that the index of mercury comes near the middle of the tube when the end has been closed and the tube is cool. Fasten the tube to a thermometer, closed end downwards (Fig. 115). You have in it a certain volume of air, and can find the volume at different temperatures as you did with liquids. Place the combined thermometer and tube in melting ice and notice the position of the air column with reference to the thermometer scale. Repeat the operation for every 10° up to 100° C., taking care that the air column is immersed completely

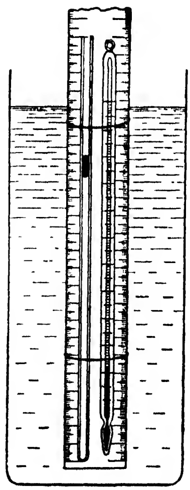


FIG. 115.—Determination of the coefficient of expansion of a gas at constant pressure.

in each case, and giving the tube two or three taps before making an observation, in order to make sure that the mercury is not sticking to the tube. Record your observations thus :

Temperature.	Length of air column.	Expansion for 10° C.	Average expansion for 1° C.

As the tube is cylindrical and uniform in bore, the volume of the air in it is proportional to the lengths of the air column. The average increase of volume for 1° C., expressed as a fraction of the volume at 0° C., is the *coefficient of expansion*. Calculate from your results, by means of equation (3) (p. 165), the coefficient of expansion of air.

When a gas is heated in circumstances where, as in these experiments, free expansion is possible, it is said to expand *under a constant pressure*. Both at the beginning of the experiment and after the gas has been heated, the pressure to which it is subjected is simply that of the atmosphere.

It should be remembered carefully that the same results would be obtained if any other gas, instead of air, were experimented with.

The absolute scale of temperature.—Charles's Law holds good whether a gas is expanded by heating or contracted by cooling. We may therefore construct a table, such as the following, which gives the volume at different temperatures of a mass of gas which occupies 273 c.c. at 0° C. :

283 c.c.	at	10° C.
278 c.c.	„	5° C.
273 c.c.	„	0° C.
268 c.c.	„	− 5° C.
263 c.c.	„	− 10° C.

We may thus imagine that, if such a degree of cooling were possible, a temperature would be reached at which the volume of the gas entirely disappeared. From the above Table this temperature will be at − 273° C. This is termed the **absolute zero of temperature**; and temperatures reckoned in Centigrade degrees from this zero are called **absolute temperatures.** The constant coefficient of gases thus provides an absolute scale of temper-

ature. The following are corresponding temperatures on the absolute and Centigrade scales:

ABSOLUTE SCALE.	CENTIGRADE SCALE.
$0°$	$-273°$
$200°$	$-73°$
$273°$	$0°$
$373°$	$100°$
$273+t°$	$t°$

Referring back to equation (4), on p. 165, where it is shown that the ratio of the volumes, V_t and V_T, occupied by a gas at temperatures $t°$ C. and T° C. is given by the equation

$$V_T/V_t=(273+T)/(273+t),$$

it is evident that **the volume of a given mass of gas is proportional to its absolute temperature.**

The coefficient of increase of pressure of a gas, when its volume is kept constant.—If a gas be heated while its volume is kept constant, its pressure increases according to the same law as that which holds good for the increase of volume when the pressure is kept constant. If P_0 be the pressure exerted by any volume of a gas at 0° C., its pressure P_t at $t°$ C. is given by the equation

$$P_t=P_0(1+\alpha t),$$

providing that the volume remains constant. The symbol α denotes the coefficient of increase of pressure at constant volume; and it has the same value ($\frac{1}{273}$) as the coefficient of increase of volume at constant pressure.

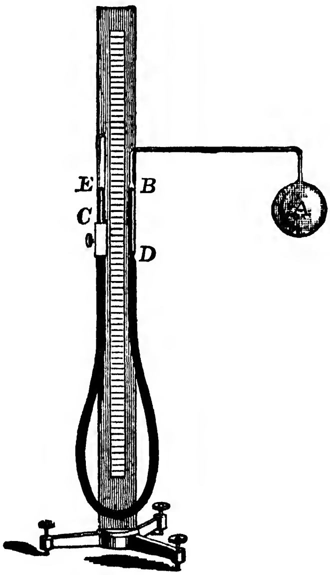


FIG. 116.—Determination of the coefficient of expansion of a gas at constant volume.

Fig. 116 represents an arrangement of apparatus which may be used for determining the coefficient of increase of pressure. It consists of a glass bulb A connected by a short horizontal tube to the Boyle's tubes

EC and BD. The tube EC can be moved up and down the wooden stand. The bulb A is surrounded with water contained in a metal vessel, and the temperature of the water is observed by means of a thermometer. The tube EC is adjusted so that the mercury surface at B coincides with a mark made near to the top of the tube. If H be the height of the barometer, and if h be the difference of level of the mercury surfaces at E and B, then the total pressure on the air in A is equal to $(H \pm h)$, the sign used depending upon whether the surface at E is above or below the surface at B. The temperature of the water bath is now raised; the temperature is noted, and the tube EC is adjusted so that the mercury surface at B occupies its previous position.

If P_1 and P_2 are the total pressures at temperatures $t_1°$ C. and $t_2°$ C.,

then
$$P_1 = P_0(1 + \alpha t_1) \text{ and}$$
$$P_2 = P_0(1 + \alpha t_2).$$

Hence
$$\frac{P_2}{P_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1}.$$

By means of this formula the value of α can be derived from the above readings.

Coefficients of Expansion.

Linear (Mean, between 0° C. and 100° C.).

Copper, - - -	0·0000168	Silver, - - -	0·0000192
Iron, - - -	0·0000109	Zinc, - - -	0·0000292
Lead, - - -	0·0000292	Brass, - - -	0·0000193
Platinum, - -	0·0000089	Glass, - - -	0·0000086

Cubical.

Benzene, - - -	0·00138	Alcohol (between 0° C. and 80° C.), -	0·00104
Carbon bisulphide, -	0·00147	Olive oil, - - -	0·00074
Ether (between −15° C. and 38° C.), -	0·00215	Turpentine, - -	0·00105
Glycerin, - - -	0·00053	Water (between 10° and 100°), - -	0·00043
Mercury, - - -	0·00018		

EXERCISES ON CHAPTER XIII.

1. A vessel of water at the freezing point contains two small glass bulbs. One is at the bottom, the other floats, but is almost wholly below the surface. The water is heated gradually; soon the bulb that was at the bottom rises, but after a while sinks again, and remains sunk. What is the meaning of this behaviour? How will the other bulb behave during the heating of the water?

2. Why may a glass stopper sometimes be loosened by warming the neck of the bottle?

3. A copper rod, the length of which at 0° C. is 2 metres, is heated to 200° C. What length will it be now? At what temperature will its length be 200·51 cm.?

4. Assuming the highest summer temperature to be 40° C., and the lowest winter temperature to be −20° C., what allowance should be made for expansion in one of the 1700 feet iron spans of the Forth Bridge?

5. A sheet of brass is 20 cm. long and 15 cm. broad at 0° C. What is its superficial area at 80° C.?

6. The brass pendulum of a clock beats seconds exactly at 25° C. How many seconds a day will the clock gain if the temperature falls to 0° C.?

7. The density of a liquid at 0° C. is D_0, and its coefficient of cubical expansion is k. Show that its density at t° C. is

$$D_t = D_0/(1+kt).$$

If the density of mercury at 0° C. be 13·596 grams per c.c., find its density at 125° C. if the coefficient of cubical expansion of mercury between these two temperatures be 0·00018.

8. The volume of a gram of water at 10° C. being 1·000269 c.c. and 1·002935 c.c. at 25° C., find the mean coefficient of expansion between these two temperatures.

9. A graduated glass flask has a capacity of 100 c.c. at 10° C. If the mean coefficient of expansion of water between 4° C. and 25° C. is 0·00014, what weight of water will the flask hold at 25° C.?

10. An empty specific gravity bottle weighs 38·5 grams. When filled with mercury at 25° C. it weighed 360·25 grams. It was then heated to 100° C. When cool it weighed 356·67 grams. Calculate the apparent coefficient of expansion between the above temperatures.

11. The bulb of a Centigrade thermometer has a volume of 1 c.c. at 0° C., and the sectional area of the bore is 0·1 sq. mm. If the lower fixed point be marked 1 cm. above the top of the bulb, and the coefficient of cubical expansion of mercury in glass is 0·00015, find the distance between the points marked 20° C. and 70° C.

12. A glass rod which weighs 90 grams in air is found to weigh 49·6 grams in a certain liquid at 12° C. At 97° C. its apparent weight in the same liquid is 51·9 gm. Find the coefficient of expansion of the liquid, taking the coefficient of cubical expansion of glass as 0·000024.

13. A hollow lead vessel, internal capacity 20 c.c. at 10° C., terminating in a narrow capillary tube of 1 mm. diameter, is filled with water so that the water surface is visible and below the top of the tube. If the vessel be cooled from 10° C. to 4° C., will the water fall or rise ? Calculate the distance through which it moves. (Mean coefficient of expansion of water between 4° C. and 10° C. is 0·000045.

14. 100 c.c. of air are measured at 20° C. If the temperature be raised to 50° C., what will the volume be, the pressure remaining constant ?

15. 15 litres of air, measured at 27° C., are cooled to 7° C. By how much will the volume diminish ?

16. On heating a certain quantity of mercuric oxide it is found to give off 380 c.c. of oxygen gas, the temperature being 24° C. and the barometric height 74 cm. What would be the volume of the gas at normal temperature and pressure (0° C. and 76 cm.) ?

17. A quantity of air is contained in a straight vertical tube closed at the lower end, the air being shut off by a pellet of mercury, the weight of which may be neglected. When the temperature is 13° C., the mercury is 66 cm. from the bottom of the tube ; and when the temperature is raised to 52° C. this distance is increased to 75 cm. Calculate the coefficient of expansion of air.

18. If the density of air at normal temperature and pressure be 0·001293 gm. per c.c., prove that the density at 15° C. and 76·8 cm. pressure is 0·001239.

19. Calculate the weight of air in a room 20 × 10 × 2 metres when the temperature is 15° C. and the pressure 77 cm.

CHAPTER XIV.

QUANTITY OF HEAT AND ITS MEASUREMENT; SPECIFIC HEAT.

Difference between heat and temperature.—Temperature is not heat; it is only a state of a body, for the body may be cold one minute and hot the next. A hot body is one at a high temperature, a cold body one at a low temperature. When a hot body and a cold body are brought into contact there is an exchange of heat until they are both of the same degree of hotness or coldness, that is, at the same temperature. Hence, **temperature** may be defined as **that which determines the transference of heat from one part of a body to another part, or from one body to other bodies in its neighbourhood.**

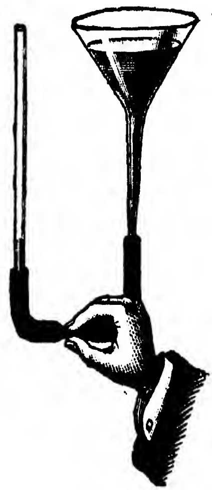


FIG. 117.—Tendency to equality of level of a liquid in communicating vessels.

Hydrostatic analogy.—When two vessels containing water and arranged at different levels are connected by means of a piece of india-rubber tubing, there is a flow of water from the vessel of water at the higher level towards the vessel at a lower level. This is a consequence of a property possessed by all liquids which makes them, as we say, *seek their own level.* This flow of water continues until the water in the two vessels is at the same level. Evidently this is a similar state of things to that which we have in the case of a hot and cold body in contact. In one case there is a flow of water until the level is the same in the two vessels. In the other there is a passage of heat until the temperature

of the two bodies is the same. *Temperature corresponds to water-level.* It may also be said that just as different vessels may have different capacities for holding water, so different substances may have different capacities for heat. The same amount of water produces different changes of level in vessels of different sizes (Fig. 118); and, in a similar way, the same quantity of heat produces different changes of temperature (or heat-level) in substances having different capacities for heat.

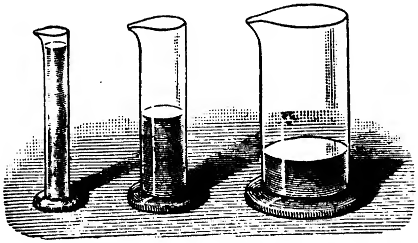


FIG. 118.—The same amount of liquid produces different rises of level when put into vessels of different capacities.

EXPT. 122.—**Mixture of equal weights of cold and warm water.** Put a certain weight of warm water in a beaker, and an equal weight of cold water in another beaker. Observe the temperature of each by means of a thermometer. Pour the cold water into the hot. It will be found on stirring them together with the thermometer (taking care not to break the thermometer), that the temperature of the mixture is about midway between the two original temperatures.

From the observations construct a table like that below, to show that the temperature, produced by mixing equal weights of the same liquid at different temperatures, is approximately equal to half the sum of the temperatures :

Temperature of water A.	Temperature of water B.	$\frac{A+B}{2}$.	Temperature of mixture.

Quantity of heat in water at different temperatures.—Quantity of heat may be measured by its heating effect, so that we can say that the quantity of heat in a certain quantity of water depends upon the *weight* of the water and its *temperature.* For our purpose the amount of heat in 100 grams of water at a temperature of 60° C. may be regarded as double that in 50 grams of water at 60° C., if for the sake of simplicity water at 0° C. is considered to contain

no heat. When equal or unequal weights of water at different temperatures are mixed, the quantity of heat lost by the hot water is the same as the quantity gained by the cold water, neglecting the loss of heat that occurs during the experiment and the effect upon the vessels containing the water. When these influences are taken into consideration, it is found that the fall of temperature multiplied by the weight of hot water is equal to the rise of temperature multiplied by the weight of cold water.

EXPT. 123.—**Loss and gain of heat.** Weigh about 200 gm. of cold water into a beaker, and observe its temperature. Put an equal weight of water into another beaker; heat it to about 45° C. Now place the beaker of warm water upon a table, with a thermometer in it, and observe its temperature. When the temperature has fallen, to say 40° C., take hold of the beaker with a duster, and quickly pour the warm water into the cold. Stir up the mixture with the thermometer, and observe the temperature after mixing. Record your observations as below:

Weight of cold water, - - - -	......gm.	
Temperature „ - - - - -	......° C.	
„ of mixture, - - -	......° C.	
Number of degrees through which the temperature of the cold water was raised, - - - - - -		......° C.
Weight of warm water, - - -	......gm.	
Temperature of warm water, - -	......° C.	
Number of degrees through which the temperature of the warm water fell, -		......° C.

Tabulate the gain and loss of heat that occur, as shown below:

GAIN.	LOSS.
Weight of cold water × its rise of temperature	Weight of warm water × its fall of temperature
.........×...............	...............×.........
....	....

The gain will be found to be slightly less than the loss. This is not really the case, and it only appears so because the amount of heat required to raise the temperature of the glass of the beaker containing the cold water has not been taken into consideration.

EXPT. 124.—**Equality of gain and loss of heat.** Repeat the experiment, using unequal weights of hot and cold water. Notice that in each case the weight of hot water × the fall of temperature is approximately equal to the weight of cold water × the gain of temperature. The difference shows the amount of heat absorbed by the glass of the cold beaker.

Unit quantity of heat.—As in all other cases of measurement, a unit or standard quantity is required with which to compare quantities of heat. The unit quantity of heat generally adopted is the **amount of heat necessary to raise the temperature of one gram of water through one degree Centigrade.** This unit is called a **calorie** or **therm.** The amount of heat required to raise the temperature of 2 grams of water through 1° C. is thus 2 units or 2 calories. Similarly, if 1 gram of water at 0° C. be heated until its temperature is 1° C., it will have received 1 unit of heat, or 1 calorie. When the temperature of this 1 gram of water reaches 3° C. it will have received 3 units of heat. If the temperature of 10 grams of water at 0° C. be raised to 12° C., the water will have received 10 times 12 units of heat, the number of units being equal to weight (in grams) × increase of temperature (in degrees Centigrade). In fact, the number of units of heat taken up by any weight of *water* as its temperature rises, or the amount given out by any weight of *water* as it cools, may be found by multiplying the number of grams of water used by the number of degrees, as measured by a Centigrade thermometer, through which the temperature rises or falls.

Comparison of heat quantities.—It has been seen that the quantity of heat in water depends upon (i) the weight of the water, and (ii) its temperature. It might be supposed, therefore, that as any weight of water at a certain temperature contains a certain quantity of heat, the same weight of another substance at the same temperature contains the same quantity of heat. This, however, is not the case. 100 grams of water at a temperature of 50° C. always contain 5000 units of heat,* but 100 grams of turpentine, mercury, lead, iron, or any other substance at the same temperature as the water, namely 50° C., do *not* contain this number of units of heat. The quantity of heat in a substance thus not only depends upon the weight and the temperature, but also upon the substance itself.

* Assuming for simplicity that water at 0° C. contains no heat.

EXPT. 125.—**The same quantity of heat may produce different changes of temperature.** Weigh out equal quantities of water and turpentine at the same temperature in two beakers of the same size. Pour equal quantities of hot water at the same temperature into the cold water and into the turpentine. Observe the rise of temperature produced in each case. Though the equal amounts of hot water contain the same quantity of heat, the rise of temperature of the turpentine will be found to be more than the rise of temperature of the cold water; in other words, the *capacity of turpentine for heat is less than the capacity of water for heat.*

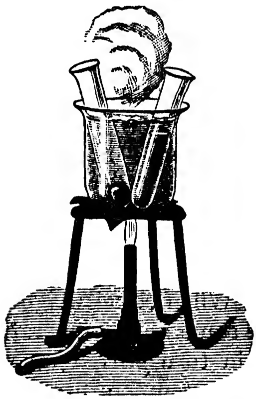


FIG. 119.—Equal weights of water and mercury do not become hot at equal rates, though they both have the same opportunity.

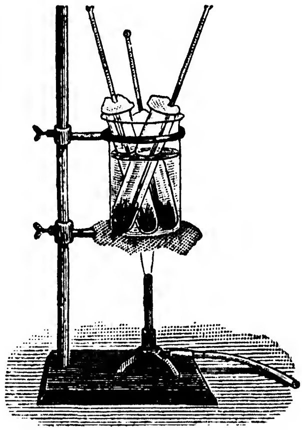


FIG. 120.—Method of heating metals in test-tubes for the determination of their capacities for heat. Each test-tube has a loose plug of cotton wool at the top.

EXPT. 126.—**Comparison of rates at which water and mercury gain heat.** Weigh out equal quantities of cold water and mercury at the same temperature in two test-tubes or flasks. Support the two vessels side by side at the same distance above a flame, or in a large beaker of boiling water. Let them remain for a few minutes; then observe their temperatures. The rise of temperature of the mercury will be found to be greater than the rise of temperature of the water; in other words, mercury gets hot more quickly than water under the same conditions.

EXPT. 127.—**Different quantities of heat in equal weights of different substances at the same temperature.** Place equal weights of lead, iron, and water in different test-tubes standing in the same beaker. Put a Centigrade thermometer into each tube, so that the bulb is immersed

in the liquid or metal; and close the mouth of the tube with a loose plug of cotton wool. Heat the beaker over a laboratory burner until the water boils; and keep the water boiling until the contents of the tubes are at a constant high temperature. Provide three beakers containing equal weights of cold water at the temperature of the room. Put the hot lead into one of these, the hot iron into another, and the hot water into the third. Stir the mixtures and note the temperature in each case.

Equal weights of water at the same temperature are thus shown to be heated to different degrees of temperature by equal weights of lead, of iron, and of water at the same high temperature.

Capacity of water for heat.—Of all known substances, water has the greatest capacity for heat; consequently a larger amount of heat is required to raise the temperature of a given weight of water through a certain number of degrees than is needed by an equal weight of any other substance. Similarly, in cooling through any number of degrees of temperature a definite weight of water gives out a larger amount of heat than an equal weight of any other substance, the temperature of which falls through the same number of degrees.

Comparison of capacities for heat.—When equal weights of water, iron, lead, and mercury at the same high temperature, *e.g.* that of boiling water, are each in turn stirred up with equal weights of cold water at the same temperature and in separate beakers, it is found that the hot water raises the temperature of the cold water in which it is placed through a larger number of degrees than any of the other substances. This is because the capacity for heat of water is greater than that of any of these (or any other) substances.

If the temperature be observed of the mixture formed in each of the cases supposed, namely, iron and water, lead and water, and so on, and then the number of degrees through which each has raised the temperature of the water into which it was put is calculated, a series of numbers is obtained which enables a comparison to be made of the capacities for heat of each of the substances experimented with.

The amount of heat required to raise the temperature of one gram of a substance through 1° C. (or, the amount of heat given out by one gram of a substance the temperature of which falls through 1° C.)

in comparison with the amount of heat taken up (or given out) by an equal weight of water in the same circumstances, **is known as the specific heat of the substance.**

Determination of specific heats.—To obtain the specific heat of a substance, usually a convenient quantity of the substance is heated to a definite temperature and then allowed to give up its heat to a known weight of water, contained in a thin brass or copper vessel known as a **calorimeter.** If losses through radiation and other causes are avoided as much as possible, the heat lost by the substance in cooling may be taken as equal to that gained by the water in having its temperature raised. The weight and rise of temperature of the water having been observed, this gain of heat can be calculated by multiplying the weight of water by its rise of temperature. The heat lost by each gram of the substance, the specific heat of which is being determined, in cooling 1° C., can then be calculated, and the result is the specific heat required.

In an experiment of this kind, however, a certain amount of heat is spent in warming the calorimeter, which may be regarded as equivalent to an extra quantity of water. The amount of water to which the calorimeter is equivalent is called its **water-equivalent or water-value**; and it must be taken into consideration in a determination of the specific heat of a substance. Of course, the water-value of a calorimeter has only to be found once by experiment or calculation, and it can then be used in any determination of specific heat in which the particular calorimeter is employed.

The following experiments illustrate the method of determining the water-equivalent of a calorimeter and the use of this value in determining specific heats.

EXPT. 128.—**The water-equivalent of a calorimeter.** Determine the weight in grams of a copper calorimeter. Observe the temperature of the air and consequently of the calorimeter.

Place the calorimeter in cotton wool in a beaker. Pour into the calorimeter a convenient quantity of warm water at a temperature of from 35° C. to 40° C. Enough to fill the calorimeter to one-third is a good amount. Notice with a thermometer, which you should use carefully as a stirrer, that, on pouring the warm water into the cold calorimeter, its temperature falls. When its temperature becomes

stationary, which it will soon do, record the temperature again. Determine the weight of the calorimeter and water. Subtract the weight of the calorimeter, and so obtain the weight of water used.

Weight of calorimeter, - - - - - -	...... gm.
Temperature of calorimeter, - - - - -	......° C.
Weight of water, - - - - - - -	...... gm.
Temperature of water, - - - - - -	......° C.
Resulting temperature, - - - - -	......° C.

The exchange of heat which takes place may be considered as follows:

Weight of hot water × fall of temperature
...... × ......
...... calories.

This result gives the number of heat units used in increasing the temperature of the calorimeter by an observed number of degrees. Find from the result the number of calories required to raise the temperature of the calorimeter through 1° C., that is, the water-equivalent or water-value of the calorimeter.

EXPT. 129.—**Determination of the specific heat of solids.**—Determine the weight of the copper calorimeter, the water-equivalent of which you have found already. Pour in enough water to fill it to one-third. Again weigh. Put a thermometer into the water and leave it to take the temperature of the water. When the temperature is stationary, record it. Weigh out about 50 grams of iron (short nails are suitable). Heat the iron in a test tube standing in a beaker or can as shown in Fig. 120, and record the temperature shown by a thermometer standing in the iron. Quickly introduce the hot iron into the cold water, stir, note the rise in temperature of the water, and, when constant, record.

Set down your observations thus:

Weight of calorimeter and water, - -	...... gm.
" " alone, - - -	...... "
Weight of water in calorimeter, - - -	...... "
Water-value of calorimeter, - - -	...... "
Total water, - -	...... "
Temperature of mixture, - - - -	......° C.
Initial temperature of water, - - -	...... "
Rise of temperature, - - - - -	...... "
Quantity of heat gained, - - - -	...... calories.

Weight of iron, - - - - - ...... gm.
Temperature of iron before mixing, - ......° C.
,, ,, mixture, - - - ...... ,,
Fall of temperature, - - - - ...... ,,

...... grams of iron the temperature of which fell ...... degrees gave out ...... calories gained by cold water and calorimeter;

therefore

1 gram of iron the temperature of which fell ...... degrees would give out ...... calories;

and

1 gram of iron the temperature of which fell 1° C. would give out ...... calories.

The result thus obtained is the specific heat of iron.

EXPT. 130.—**Specific heats of liquids.** Weigh a calorimeter. Half fill it with turpentine, and find the weight of the turpentine. Observe the temperature of the turpentine. Observe also the temperature of some boiling water. Pour boiling water into the turpentine; keep the two liquids well stirred, and observe the temperature of the mixture. Find the weight of the water added. From these observations calculate the specific heat of turpentine.

Determine in the same way the specific heat of mercury.

(The specific heats of liquids can be determined also by the method of cooling, as described in Expt. 162, p. 225.)

TABLE OF SPECIFIC HEATS.

Aluminium,	0·212	Steel,	0·118
Brass,	0·094	Zinc,	0·094
Copper,	0·093		
Glass, crown,	0·161	Glycerin,	0·576
Glass, flint,	0·117	Mercury,	0·033
Iron,	0·112	Olive oil,	0·471
Lead,	0·032	Turpentine,	0·420
Sulphur,	0·184	Petroleum,	0·511

EXERCISES ON CHAPTER XIV.

1. If a pound of water at 100° C. be mixed with a pound of water at 0° C., the temperature of the mixture is 50° C. How would the result have differed if a pound of oil at 100° C. had been substituted for the hot water? Explain the difference.

2. Explain what is meant by specific heat. How would you show that equal weights of different substances give out different amounts of heat when cooled through the same range of temperature?

3. What is the capacity for heat of a body?

Which has the greater capacity for heat, 5 c.c. of mercury or 2 c.c. of water? (Specific gravity of mercury, 13·6; specific heat, 0·033.)

4. A copper vessel, weighing 125 gm., holds 800 gm. of water at its temperature of maximum density (4° C.). How much heat must be imparted to the vessel before the water begins to boil? Assume that there is no loss of heat by radiation.

5. If 90 gm. of mercury at 100° C. be mixed with 100 gm. of water at 20° C., and if the resulting temperature be 22°·3 C., what is the specific heat of mercury?

6. In order to determine the specific heat of silver, a piece of the metal weighing 10·21 gm. was heated to 101°·9 C. and dropped into a calorimeter containing 81·34 gm. of water, the temperature of which was raised from 11°·1 C. to 11°·7 C. If the water-equivalent of the calorimeter, stirrer, and thermometer was 2·91 gm., find the specific heat of silver.

7. If you had at your command a supply of tap water at 10° C. and of boiling water, what quantities of each would you take in order to prepare a bath containing 20 gallons of water at 35° C.?

8. In order to determine the temperature of a furnace, a platinum ball weighing 80 gm. is introduced into it. When it has acquired the temperature of the furnace it is transferred quickly to a vessel of water at 15° C. The temperature rises to 20° C. If the weight of water, together with the water-equivalent of the calorimeter, be 400 gm., what was the temperature of the furnace? (Specific heat of platinum = 0·0365.)

9. For the purposes of a foot-warmer, which is preferable, a bottle containing 10 lb. of water or a 10-lb. block of iron, both initially at 100° C.? Explain your answer.

10. A silver tea-pot weighs 300 grams. One gram of silver requires as much heat to warm it as would be required by 0·056 gram of water to warm it equally. The tea-pot contains 20 grams of tea-leaves, and each gram of tea-leaves requires as much heat to warm it as would suffice to warm equally 0·5 gram of water. If 600 grams of boiling water be poured into the tea-pot, calculate the highest temperature of the tea, assuming that tea-pot and tea-leaves were originally at a temperature of 15° C.

11. I take 2 ounces of lead and 2 ounces of water, place them in the same beaker, and heat the beaker over a Bunsen flame. I then take two other beakers, each containing 2 ounces of cold water, and

add the hot lead to one and the hot water to the other beaker. After stirring, I note the temperature in each case with a thermometer. State (*a*) how the thermometer readings differ, and (*b*) the cause of this difference.

12. It is required to find the quantity of heat lost by one gram of copper when its temperature falls through one degree. Describe how this can be done.

13. What is a unit of heat? If I pour a kilogram of mercury at 100° C. into a kilogram of water at 0° C., will the result be the same as though the water had been at 100° C. and the mercury at 0° C.? Give reasons for your answer.

14. How many heat units would be required to raise 50 grams of water at 0° C. to the boiling point? If this quantity of heat were added to a litre of water at 15° C., what would be the final temperature?

15. A piece of bronze weighing 67 grams and at a temperature of 100° C. was dropped into 65 grams of water at a temperature of 16°·5 C. The temperature of the water rose to 23°·5 C. Determine the specific heat of the bronze and explain what your result means.

CHAPTER XV.

PROPERTIES OF VAPOURS. EVAPORATION AND BOILING. HYGROMETRY.

The kinetic theory of gases.—It has been explained previously (p. 53) how such properties as pressure, the tendency to indefinite expansion, and diffusion observed in gases, may be attributed to a state of continual and rapid motion of the molecules of which the gas consists.

The pressure which a gas exerts on the walls of a containing vessel is due to the continuous and steady bombardment of the molecules of which it is composed; and the pressure which each molecule exerts is measured by the kinetic energy which it possesses at the moment of impact, and this kinetic energy is equal to $\frac{1}{2}mv^2$, where m is the mass of the molecule and v is its velocity.

Experiments on Boyle's Law (p. 85) have shown that when the volume of a gas is halved (*i.e.* when the density is doubled) the pressure is doubled. This is just what might be expected if the pressure be due to the bombardment of the molecules against the containing walls—for, by doubling the density we double the number of molecules striking in unit time each unit area of the vessel's surface, and therefore the pressure is doubled.

At the same temperature, **the average velocity of the molecules is not the same for all gases.** Thus, at normal temperature and pressure, 1 c.c. of *air* exerts a pressure of 1·033 kilogram per sq. cm., and, under the same conditions, 1 c.c. of *hydrogen* exerts the same pressure. But the latter gas weighs only one-fourteenth as much as the former; hence, to exert the same pressure, the average

velocity of the hydrogen molecules must be much greater than that of the air molecules. The average velocity, at normal temperature, of hydrogen molecules is about 1800 metres per second, and of air molecules about 450 metres per second. In general, we may say that the less dense the gas, the greater is the average velocity of the molecules.

Further evidence of this difference in molecular velocity is obtained in the phenomenon of **diffusion**. Suppose that a cubical metal vessel is divided into two compartments by a partition of porous material, such as unglazed earthenware, and that one compartment is filled with hydrogen and the other with air. If there are as many hydrogen molecules in each c.c. of that gas as there are of air molecules in each c.c. of air, then, since the velocity of the former is four times as great as that of the latter, the hydrogen molecules will strike the partition four times as frequently as the air molecules strike it, and the former will pass through the pores of the partition four times as rapidly as the latter. Hence, there will be an increase in the number of gas molecules, and a consequent increase of pressure, in the compartment which originally contained air, and there will be a diminution of pressure in the other compartment. This phenomenon can be verified by the following experiment.

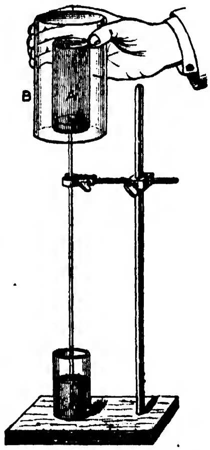


FIG. 121.—Experiment on the diffusion of gases.

EXPT. 131.—**Diffusion apparatus.** Close the open end of a porous cylindrical cell (such, for instance, as is used in fitting up a Daniell voltaic cell) with a rubber stopper perforated with one hole (A, Fig. 121). Fix the end of a long narrow glass tube through the stopper. Clamp the tube in a vertical position with its lower end dipping into a beaker of water. Fill an inverted beaker B with hydrogen, and hold it over the porous cylinder. The hydrogen diffuses inwards more rapidly than the air diffuses outwards from the porous cell, and the increased pressure inside A causes bubbles of gas to escape from the lower end of the tube. Remove the beaker B; the hydrogen *inside* A now diffuses

outwards, and the diminished pressure inside causes the water to rise up the glass tube.

The Kinetic Theory of Gases also affords a simple explanation of the increase of pressure which a gas exerts when its temperature is raised. The additional energy contained in the gas when warmed is represented by an increase in the average velocity of the molecules, and this increase in velocity necessarily implies an increased pressure.

Evaporation and Boiling.

Evaporation of liquids.—It is a common observation that water contained in a dish, and exposed to the air, gradually disappears. This **evaporation**, as it is termed, may be observed with many other liquids. The gradual escape of the molecules can be attributed only to the fact that the molecules of a liquid are in a state of vibratory motion; and this state differs from that which characterises a gas only in that the molecules of a liquid are packed much more closely together, and that mutual collisions are correspondingly more frequent. In these collisions some molecules may acquire occasionally a velocity considerably above the normal; if these, by chance, happen to be near the surface, they *may* escape into the air above the liquid, and there they assume the properties of gas molecules. The term **vapour** is given to the molecules in this condition.

An increase of temperature brings about an increased rate of evaporation. This would be anticipated from the theory that a rise in temperature increases the velocity of the molecules, and therefore the possibility of molecules escaping from the free surface of the liquid.

During the process of evaporation, we can imagine the space above the liquid to be occupied by a mixture of air molecules and the vapour. All these molecules are in a state of motion, and in frequent collision with each other; occasional molecules of the vapour may, as a result of collision, re-enter the liquid, while others escape to a distance and diffuse outwards. **This process of evaporation results in the atmosphere always containing more or less water vapour**; and it can be proved by simple experiments (i) that there is a limit to the quantity of water vapour which can be retained in that form by the atmosphere, and (ii) that warm

air is capable of retaining more water vapour than cold air can retain. The presence of water vapour in the air of a room can be demonstrated by the following experiment:

EXPT. 132.—**Moisture from the air.** Cool some water in a beaker, either by adding ice or crystals of sodium hyposulphite and stirring well. Notice the gradual deposition of moisture on the outside of the beaker.

It is a well-known fact that wet clothes or wet roads dry more rapidly on a warm summer's day than in the winter. Even in the summer, the air, though warm, may be very moist (or 'muggy'), and wet roads will not dry then so rapidly as when the air is fairly dry. Also, the evaporation is aided if the vapour molecules are prevented from re-entering the liquid; and this explains why a windy day is more favourable than a calm day for drying a wet surface, since the vapour molecules as soon as they escape from the liquid are conveyed by the breeze to a distance.

Evaporation takes place at all temperatures: even a lump of ice, or snow, will evaporate gradually.

All liquids do not evaporate with equal readiness; thus ether, gasolene, and alcohol evaporate more rapidly than water, providing that the temperature is the same. This fact can be demonstrated by weighing, at frequent intervals, dishes containing these liquids. Liquids which evaporate readily are termed **volatile.**

Cooling caused by evaporation.—Evaporation necessarily causes a cooling of the liquid; for the more energetic of the molecules are those which are the more likely to escape from the liquid, and this results in the *average* kinetic energy of the molecules which remain being less than before. The diminution of kinetic energy is indicated by a lowering of the temperature of the liquid. This loss of heat is more or less counterbalanced by the heat given up to the liquid from the surrounding air and from neighbouring objects, but the fall of temperature is generally sufficient to be detected readily.

EXPT. 133.—**Evaporation of ether and water.** Allow a bottle of ether and a bottle of water to stand in a room until their temperatures correspond with that of the room. Note the temperature of the room. Pour some of the ether into an empty dish and some of the water into another dish; place a thermometer in each. After a few minutes, note the reading of each thermometer.

EXPT. 134.—**Freezing by evaporation.** Pour a few drops of water upon a dry piece of thin wood, and stand in the water a thin beaker containing a little ether. Blow vigorously down a tube having one end in the ether (Fig. 122) or use a pair of bellows. The ether rapidly evaporates, and *in doing so takes heat from the water* between the beaker and piece of wood. The beaker thus becomes attached to the wood by a layer of ice.

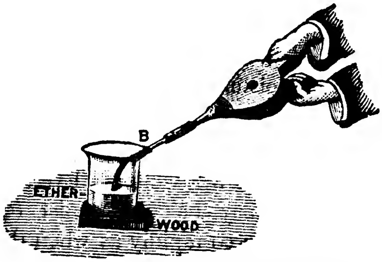


FIG. 122.—Experiment to show that water may be frozen by the rapid evaporation of ether close to it.

Boiling.—At low temperatures evaporation takes place only from the exposed surface of the liquid, and the change from liquid to vapour is invisible. As the temperature is raised the rate of evaporation increases, and a temperature is reached finally when evaporation takes place anywhere within the liquid as well as from the surface. Bubbles of vapour are then formed within the liquid and rise to the surface; the evaporation is then rapid and visible, and the liquid is said to **boil.** If heat be applied to the containing vessel from below, the liquid in contact with the bottom of the vessel is always slightly hotter than the liquid above, hence the bubbles of vapour as a rule appear to form at the bottom of the vessel. For each liquid there is a definite temperature at which visible evaporation takes place, and this temperature is termed the **boiling point** of the liquid.

The temperature at which any liquid boils is influenced, to a slight extent, by the nature of the containing vessel and by the degree of cleanliness of the inner surface of the vessel. It has been known for a long time that, although the temperature *of the boiling liquid* is subject to such variations, the temperature of the *vapour* immediately above the liquid is constant, providing that the pressure of the air is constant: the influence of variations of atmospheric pressure on the boiling point is explained on p. 150. For the above reason, when determining the boiling point *of a pure liquid,* it is necessary to support the thermometer with its bulb in the vapour immediately above the surface of the liquid.

EXPT. 135.—**Boiling point of water.** Fit up the apparatus shown in Fig. 123. Insert the thermometer through the cork so that the bulb is just *above* the surface of some distilled water contained in the flask. Boil the water continuously for about 5 minutes and note the reading of the thermometer. At the same time, read the height of the barometer; the reason for reading the barometer is explained on p. 192.

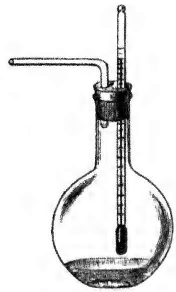


FIG. 123.—Determination of the boiling point of water.

EXPT. 136.—**Boiling point of alcohol.** Fit a boiling-tube A (Fig. 124) with a cork through which pass a thermometer and a long glass tube C. Support A in a beaker B containing water. Pour alcohol into A, and add a few fragments of glass rod so as to ensure steady boiling. The tube C serves to condense the vapour of the alcohol, and it lessens the possibility of the vapour taking fire. Heat the water in the beaker until the alcohol boils, and note the reading of the thermometer. At the same time read the height of the barometer.

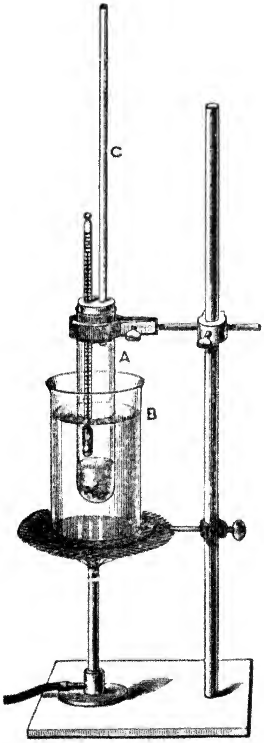


FIG. 124.—Determination of the boiling point of alcohol.

VAPOUR PRESSURE.

Vapour pressure.—In previous paragraphs we have considered the phenomenon of evaporation as taking place when the liquid is exposed to the open air. It is necessary now to consider how the evaporation is influenced when the space above the liquid is limited.

When the liquid is contained in a *closed* vessel, the escape of molecules to a distance is prevented; and after a time, a condition is set up when molecules re-enter the liquid just as frequently as they leave it. The space is then stated to be full

of the **saturated vapour** of the liquid, and the pressure which the vapour exerts on each sq. cm. of the surface of the liquid is termed the **maximum vapour pressure of the liquid at the observed temperature of the experiment.** The maximum vapour pressure is expressed usually in its equivalent height of a mercury column; thus the statement that the maximum vapour pressure of water at 15° C. is 12·7 mm. means that the pressure per sq. cm. of the vapour at 15° C. is equal to the pressure per sq. cm. due to a column of mercury 12·7 mm. high.

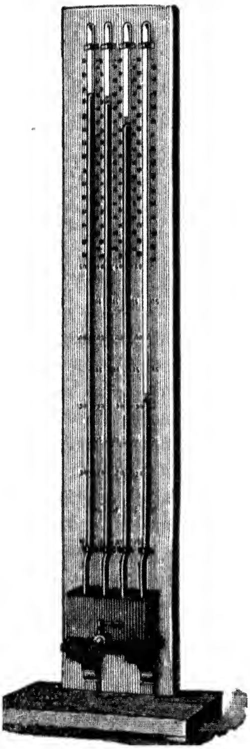


FIG. 125.—Apparatus for vapour pressure experiments.

The pressure of the vapour increases with increase of temperature.—When the temperature of the containing vessel is raised, the escape of molecules from the liquid is more frequent, and more vapour molecules must accumulate above the liquid in order that equilibrium between the liquid and its vapour can be established. Since the number of molecules in the condition of vapour and contained in the same space is increased, and since their velocity of movement is increased by an increased temperature, we should anticipate that the pressure of the vapour would increase.

The density and the pressure of a saturated vapour are independent of the volume occupied by the vapour.—For, supposing that the closed vessel, containing the liquid and its saturated vapour, contract suddenly, the number of vapour molecules in each cubic centimetre of the space above the liquid will be increased, and the number of molecules striking the liquid surface will be increased correspondingly; thus, more molecules will enter the liquid than will leave it, or, in other words, some of the vapour will condense. This condensation will continue until equilibrium is again established, and then the density and pressure of the vapour will be the same as before the contraction of the containing vessel took place.

The density and pressure of the saturated vapour are independent of the presence of air.—Suppose, for example, that the space above a liquid within a closed containing vessel is occupied by air. Just as much vapour will be generated from the liquid surface as would be the case if the space were a vacuum ; for, evaporation will continue until as many vapour molecules re-enter the liquid in a second as leave it in a second, and the number which re-enter the liquid depends solely upon the number contained in each c.c. of the space above. The only effect due to the presence of air is a delay in the final establishment of equilibrium, and this is due to the frequent collisions, between the air molecules and the vapour molecules, retarding the diffusion of the vapour to the upper parts of the containing vessel.

All these points may be demonstrated by means of barometer tubes, fitted up as shown in Fig. 125. The first tube on the left is an ordinary barometer tube, the remaining three have had introduced into them respectively water, alcohol, and ether. The water having evaporated into the Torricellian vacuum, as the space above the mercury in a barometer is called, causes but a slight fall of the mercury column. The alcohol and the ether exert a greater pressure. The depression of each mercury column measures the pressure of each vapour at the temperature of the experiment. If the liquids and vapours in the tubes are warmed their pressures increase, and the mercury level falls.

EXPT. 137.—**Vapour pressure.** Clean and dry the insides of two barometer tubes. Fit them up as barometers, using clean dry mercury. Measure the heights of the mercury columns. By means of a bent pipette (Fig. 126), introduce not more than *two* drops of ether into one tube, and two drops of water into the other tube. Again measure the heights of the mercury columns. Observe that there is no visible layer of either liquid at the top of the columns: the space above the columns is *not* saturated with the vapours. Add more liquid in each case, sufficient to form a layer of liquid above the mercury: the spaces above the columns are now filled with the *saturated* vapours of the liquids. Measure the heights of the columns, and note the temperature of the room. *The depression of the mercury, in each case, represents the pressure of the saturated vapour of the liquid at the observed temperature.*

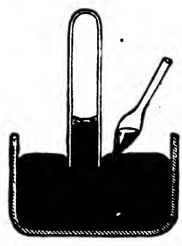


FIG. 126.—Introducing a liquid into a closed tube.

Pass a Bunsen flame rapidly up and down either of the tubes, and notice how the vapour pressure increases.

Close with the thumb the open end of either of the tubes, and transfer it to a deep vessel containing mercury, so that the tube may be depressed to a considerable depth in the mercury. Notice that the height of the mercury column remains constant, thus proving that the vapour pressure is independent of the size of the space above the mercury column.

EXPT. 138.—**Independent vapour pressures.** Fit up a barometer tube, as before. Introduce a little air into the vacuum at the top of the tube, and measure the height of the mercury column. Introduce ether, as before. Note how the column is depressed less rapidly than before. Measure the height of the column, and verify that the vapour pressure is independent of the presence of air.

Vapour pressure of a liquid at its boiling point.—It has been stated (p. 188) that a liquid boils when bubbles of its vapour form within the liquid and rise to the surface. Consider the forces acting on such a bubble when near the surface (Fig. 127): the arrows directed inwards indicate the pressure due to the air and tending to compress the bubble inwards; the arrows directed outwards indicate the pressure due to the vapour within the bubble and tending to expand the walls of the bubble. The fact that the bubble, during its upward movement, maintains a fairly uniform size indicates that the pressure due to the vapour is equal approximately to the atmospheric pressure. Hence, we may say that **when a liquid is heated to its boiling point its vapour pressure is equal to the pressure of the air upon it.** This is an important deduction, since it affords a simple method of determining the boiling point of a liquid; and the method is particularly useful when only a small quantity of the liquid is available. The deduction suggests also how dependent the boiling point of a liquid is upon the pressure of the surrounding air: for, supposing that the atmospheric pressure is reduced, then the vapour pressure of the liquid at a lower temperature will be equal to that of the air,—or, in other words, the liquid will boil at a lower temperature. Similarly, if the atmospheric pressure be increased,

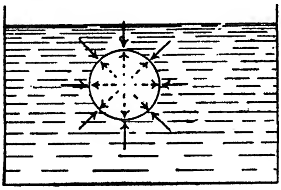


FIG. 127.—Forces acting upon a bubble of vapour in a liquid.

the liquid must be warmed to a higher temperature than before in order that its vapour pressure may equal that of the air,—or, in other words, the liquid will boil at a higher temperature.

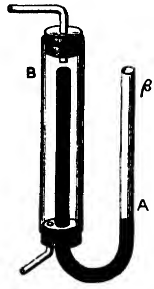


FIG. 128.—Experiment on the vapour pressure of a liquid at the boiling point.

EXPT. 139.—**The vapour pressure of a liquid at its boiling point is equal to the atmospheric pressure.** In Fig. 128, A is a U-tube with limbs about 30 cm. long, and with one limb sealed. The latter limb is surrounded by a wide glass tube B closed at both ends with corks. The upper cork is fitted with a glass tube for the purpose of leading steam into B; the lower cork supports the U-tube, and it has also a side tube for the escape of condensed steam. Fit up the tube A as follows: fill it completely to the point β with clean dry mercury, close the open end with the thumb, tilt the tube so that the air bubble at β travels round to the closed end, and finally bring the bubble back to the open end. The walls of the tube should now be quite free from small air bubbles. Introduce two or three drops of well boiled distilled water into the tube so as to fill it completely; close the end with the thumb, and tilt the tube so that the water travels round to the closed end. Finally, withdraw mercury from the open end, by means of a narrow pipette, until the mercury surface is just above the bend. Pass steam through the steam-jacket B, and notice how the vapour pressure of the water depresses the mercury in contact with it *until the two mercury surfaces are at the same level.*

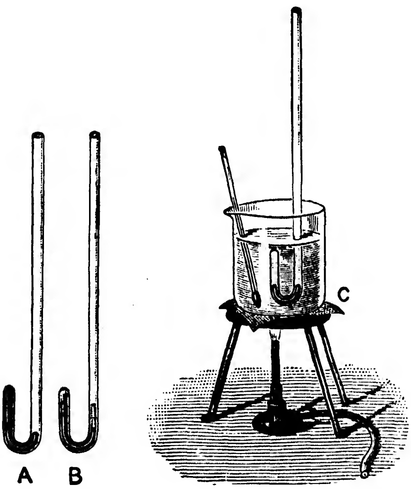


FIG. 129.—Method of determining the boiling point of alcohol.

EXPT. 140.—**Vapour pressure method of determining boiling point.** Make a narrow glass U-tube, similar to Fig. 129, and with a closed

limb at least 10 cm. long. Introduce mercury and alcohol, by the procedure explained in Expt. 139. Support the tube in a beaker of water, and support a thermometer in the water. Gradually warm the beaker, keeping the water well stirred, and note the temperature when the mercury surfaces in the two limbs of the U-tube are at the same level.

Effect of change of pressure on the boiling point.—It has been seen that at sea-level the normal pressure of the atmosphere will support a column of mercury 30 inches in length. At the top of a mountain, the pressure is less because there is less air above the barometer; and at the bottom of a mine it is more for the reverse reason. At any one place, also, the pressure varies from day to day. If we wish to boil a liquid, therefore, where the pressure of the atmosphere is great, the liquid has to be heated to a higher temperature than when the pressure is less before the bubbles of vapour formed can escape at the surface of the liquid. If we heat the liquid more, its temperature gets higher before there is any conversion into vapour, and consequently its boiling point is higher when the pressure is greater. **In finding the boiling point of a liquid we must therefore know the pressure of the atmosphere at that time and place.**

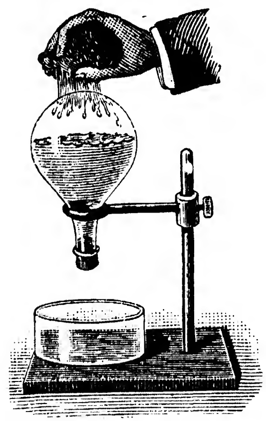


Fig. 130.—Water below 100° C. boiling under diminished pressure.

A simple experiment shows that water may boil at a temperature considerably below 100° C. when the pressure upon its surface is diminished. All it is necessary to do is to take a sound cork which fits tightly the neck of a round-bottomed flask. Water is then boiled in the flask and allowed to continue boiling for some minutes so that all the air in the flask is driven out and its place taken by steam. The burner is then removed and the cork inserted into the neck of the flask as rapidly as possible. After standing to cool for a minute or two, when, owing to cooling, the temperature can no longer be 100° C. the flask is turned over and cold water poured upon its upturned under surface, or a cold wet sponge is squeezed upon it as shown in Fig. 130.

The cold water causes the steam in the flask to condense, and, as no air can get in, the pressure on the surface of the warm water is now less than it was before, and therefore the water is seen to boil quite briskly again.

EXPT. 141.—**Boiling under different pressures.** Fit up the apparatus shown in Fig. 131, in which C is a glass tube bent twice at right angles, tapered slightly at the lower end, and cut off in a slanting direction. D is a narrow cylinder into which mercury may be poured, whereby the pressure is increased when steam is escaping through the mercury. The total pressure is obtained by adding the difference of level between α and β to the height of the barometer. By adding successive quantities of mercury obtain a series of simultaneous readings of boiling point and pressure.

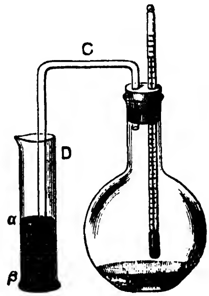


FIG. 131. — Arrangement for determining the effect of increased pressure upon the boiling point.

Obtain a similar series of readings under diminished pressure thus: leave a small quantity of mercury in D, and turn down the Bunsen burner. The mercury now rises up the tube C, and diminishes the pressure. The pressure is obtained by subtracting from the barometer-reading the difference of mercury level in C and in D. More rapid boiling is ensured by wrapping blotting-paper wetted with cold water round the neck of the flask. Verify the results by the data given in footnote, p. 150.

Effect of dissolved solids on the boiling point.—The vapour pressure of a solution is *less* than that of the pure solvent (or fluid in which a substance has been dissolved), if the temperature be the same; hence, a solution of a solid must be raised to a higher temperature than the pure solvent in order that its vapour pressure may be equal to the atmospheric pressure, or, in other words, **the boiling point of a solution is higher than that of the pure solvent.**

EXPT. 142.—**Boiling points of saline solutions.** Measure out 250 c.c. of water into a large flask which has been weighed previously. Weigh the flask and its contents, and support a thermometer with its bulb immersed in the water. Weigh out about six equal quantities of dry salt, each weighing about 5 grams. Heat the water to boiling. Note the temperature of the boiling water, also note the time; add 5 grams of salt, and, at the end of 2 minutes, note the temperature when boiling; add another 5 grams of salt, and again, at the end of

another 2 minutes, note the temperature. Repeat this process until all the salt has been added. Finally, cool the flask as quickly as possible and weigh it. Allowing for the total weight of salt added, find the weight of pure water still in the flask. The loss during the experiment is due to the escape of steam. Assuming that the loss takes place at a constant rate, determine the weight of water present at the moment when each reading of the thermometer is taken. Calculate the percentage of salt present in each case. Plot the observations on squared paper, taking percentages of salt as abscissae and temperatures as ordinates.

HYGROMETRY.

Moisture present in the atmosphere.—Evaporation is constantly going on from all water and wet surfaces exposed to the air, and the air would be saturated with moisture always were if not for the retarding influence (p. 191) of the air. The term **hygrometry** is applied to all those phenomena which result from the moist condition of the atmosphere.

The quantity of water vapour necessary to saturate a given volume of air depends upon the temperature of the air (p. 190). Warm air can retain more water vapour than cold air. When the temperature of moist air is diminished, the air is soon cooled to a temperature at which the moisture present is sufficient to saturate the air; and when the temperature is diminished still further, part of the moisture will condense—either as **dew, hoar-frost, or cloud.**

Absolute amount of moisture present in air.—The quantity of moisture present in 1 cubic metre of the atmosphere, at a particular time, is determined by drawing a measured volume of the air through tubes containing a suitable hygroscopic substance, the tubes being weighed before and after the passage of the air.

EXPT. 142A.—**Chemical hygrometer.** Heat some broken pumice-stone in a dish in a furnace; and, while hot, throw it into a vessel containing concentrated sulphuric acid. Pour off the excess of acid, and transfer the pumice to a glass stoppered store bottle. Fit up two U-tubes with corks and glass tubes, as shown in Fig. 131A. The corks should be previously soaked in melted paraffin-wax. Nearly fill the tubes with the pumice; adjust the corks and wax them carefully to make all joints air-tight. Close the entrance and exit tubes with rubber tubing in

which a short piece of glass rod has been inserted to act as a plug. Weigh the tubes accurately.

Fit up a suitable type of aspirator such as is shown in Fig. 131A. To prevent the possibility of moisture passing from the aspirator to the weighed tubes it is advisable to have a third drying-tube, or a bottle

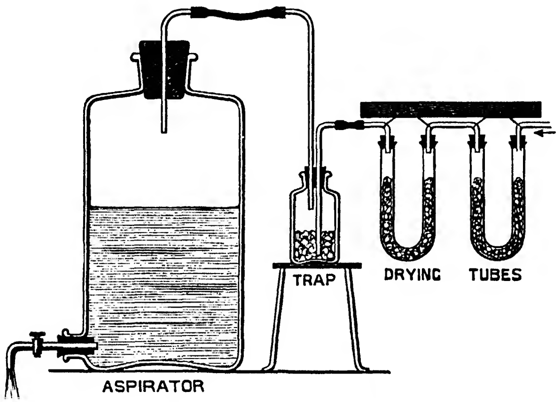


FIG. 131A.—A chemical hygrometer.

containing more pumice which has been soaked in sulphuric acid, between the aspirator and the weighed tubes.

Fill the aspirator with water and connect it with the bottle by means of rubber tubing. Take the plugs out of the two drying-tubes and connect the tubes with the bottle. Turn on the tap of the aspirator and let the water run out gently. When a convenient quantity—four or five litres—has escaped, turn off the tap, disconnect the U-tubes, re-insert the glass plugs and again weigh the tubes. The increase of weight shows the weight of moisture in a volume of air equal to the volume of water run off. From this result, calculate the weight of moisture present in 1 cubic metre of air at the time of the experiment.

When very precise results are required, the temperature of the air in the aspirator and where it enters the U-tubes must be taken into consideration, but the foregoing experiment is sufficiently accurate to illustrate the method of determining the absolute quantity of aqueous vapour in a given volume of air.

Relative humidity.—The term **humidity** means **the degree of dampness of the air**; and the **relative humidity** is defined as **the ratio of the weight of water vapour present in unit volume of the air to the**

weight of water vapour which would be present in the same volume if the air were saturated. At any given temperature the vapour-pressure of the water present in the air is proportional to the weight of the water present in a given volume of the air; and it is more usual to express the relative humidity in terms of the pressure which the vapour exerts. Hence, we may say that

$$\text{Relative humidity} = \frac{\text{pressure actually exerted}}{\text{pressure exerted if the air were saturated}}.$$

The measurement of relative humidity is important, since it affords information as to whether any slight fall in temperature will cause condensation of moisture. The measurement is also useful in order to determine whether the air of rooms is sufficiently moist for healthy conditions.

Relative humidity is sometimes expressed as a fraction, and sometimes as a percentage. Thus, when the air is found to contain one-half as much water vapour as would be necessary to saturate it, the relative humidity may be expressed either as 0·5 or as 50 %.

The relative humidity is determined by finding out to what temperature the air must be cooled in order that the moisture present will suffice to saturate the air; and this temperature is observed by cooling the air until it begins to deposit moisture in the form of dew. This temperature is called the **dew-point.** The various devices for determining the dew-point are termed **hygrometers**; some typical devices are explained in the following paragraphs. Suppose that the temperature of the air is 21° C., and that an experiment shows that the air must be cooled to 15° C. before deposition of dew commences. The pressure exerted by the water vapour is the same in the cooled air as in the warm air, since the air and the water vapour contract in the same ratio on cooling; hence the pressure which the vapour in the uncooled air actually exerts must be equal to the maximum pressure at 15° C. Reference must now be made to the Physical Table No. 16. From such a Table we see that the maximum pressure of water vapour at 15° C. is 12·67 mm., and the maximum pressure at 21° C. is 18·47 mm. Hence, the relative humidity $= 12{\cdot}67/18{\cdot}47 = 0{\cdot}68$ (or 68 %).

The aluminium-cup hygrometer (Fig. 132).—This consists of a small well polished aluminium cup, in which water is cooled by the gradual addition of ice until dew is deposited on the outer surface.

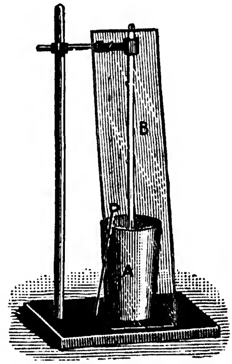


FIG. 132.—Aluminium-cup hygrometer.

EXPT. 143.—**Relative humidity.** About half-fill the aluminium cup A with water. Suspend a thermometer B, graduated to 0°·2 C., in the water. Place a large sheet of glass in front of the apparatus so as to screen it from the warmth and breath of the observer. Add a *small* fragment of ice, and stir continually until the ice is melted. Add another small fragment of ice, and stir until melted. Continue this process until the deposition of dew upon the cup is observed; note the temperature of the water. Continue to stir the water and note the temperature when the deposited dew disappears. The average of these two temperatures is the *dew-point*. Observe the temperature of the room, and, from the data in Physical Table No. 16, calculate the relative humidity of the air in the room.

Regnault's hygrometer.—The principle of this hygrometer differs from that of the aluminium-cup hygrometer only in the method adopted for cooling the air. The lower end of a thin glass test-tube is fitted with a polished silver 'thimble'; ether is poured into the tube, and its temperature is lowered (p. 188) by passing a stream of air bubbles through it until dew is deposited on the silver thimble. Fig. 133 represents a simple device based upon this principle. The upper end of a brightly polished cylinder A (of brass or copper) is closed with a 3-holed cork. Through the cork pass (i) a thermometer, (ii) a glass tube B terminating below the surface of the ether contained in the cylinder, and (iii) a short glass tube for the escape of ether vapour. The ether may be cooled by breathing gently down the tube B.

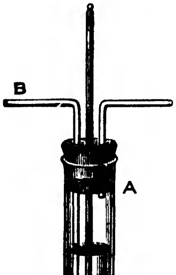


FIG. 133.—Principle of Regnault's hygrometer.

Mason's hygrometer.—Mason's instrument consists of two precisely similar thermometers, suitably attached to a frame, or suspended side by side as in Fig. 134. Round

the bulb of one of the thermometers is tied a piece of muslin, to which cotton threads are attached ; these hang down into water in a glass. The instrument depends for its use upon two facts which have been brought before the student's attention already. The first is that water is vaporised only at the expense of a certain amount of heat ; and, secondly, the quantity of water vapour which air can take up at any temperature depends upon the amount already contained by it. Water rises up the cotton threads by the force known as capillary attraction, and consequently keeps the muslin moist. The water on the muslin evaporates, getting the heat necessary for evaporation from the bulb of the thermometer which it surrounds. The thermometer is cooled thereby, and the column of mercury sinks. This process continues until the air round the bulb is saturated and evaporation ceases. Thus the wet-bulb thermometer records a lower temperature than that with a dry bulb. The difference between the readings is greater the drier the air at the commencement of the observation, and it provides a means of estimating the amount of water vapour present by seeing how much more must be added to saturate the air.

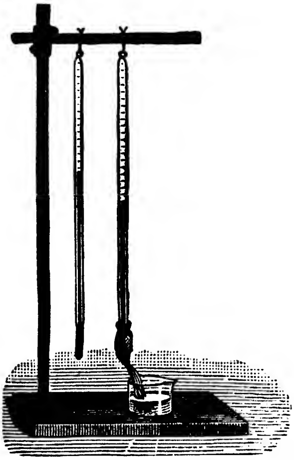


FIG. 134.—Wet- and dry-bulb thermometer.

The **wet- and dry-bulb thermometer**, as Mason's hygrometer is called, is employed usually to indicate the relative amount of moisture in the air, but the readings may also be used to determine the dew-point by a simple calculation in connection with a set of hygrometrical tables prepared for the use of practical meteorologists. When the dew-point has been found, the relative humidity of the air or the percentage of saturation can be determined.

Cloud.—When moist air is warmed, through contact with the earth's surface, it expands and tends to rise vertically upwards. Such ascending columns of air soon become cooled, either by coming into contact with cooler strata of air or by their natural expansion under the diminished pressure of higher altitudes, and the degree of cooling may be sufficient to cause the condensation

in the form of minute drops of some of the moisture. These drops will tend to fall, and the larger the drops the more rapidly will they fall; if, in falling, they pass through strata of comparatively dry air they will be evaporated before reaching the earth; but if they pass through strata of warm air saturated with moisture, they will increase in size—owing to each cold drop condensing more moisture—and in velocity, and the drops may reach the earth's surface as **rain drops.**

If the initial condensation of the water vapour takes place at a temperature below 0° C., the particles assume the solid form, and a fall of **snow** results. **Hail** is believed to be formed by the freezing of rain-drops which have been carried up several times in succession into colder regions of the atmosphere by strong air currents. Hailstones usually have a stratified or concentric structure in which several different layers can be seen when they are cut across. This structure could scarcely be produced if hail consisted merely of rain-drops which had been frozen by passing through a layer of air below a temperature of 0° C. on their way to the earth.

Fog.—Fog closely resembles cloud, except that it is formed near the earth's surface. The condition usually necessary for its formation is that the upper strata of air are warmer than the strata near the earth's surface. The moisture contained in the upper strata diffuses downwards into the cooler strata beneath, and if the latter acquire thereby more than sufficient moisture to produce saturation the excess is condensed in the form of minute drops of water.

Dew.—Dew differs from the forms of condensed moisture seen in mists, clouds, rain, and snow, in being formed *upon* the surface of the earth. After sunset, the surface of the earth, which has been receiving heat throughout the day, begins to lose this heat by **radiation** (p. 221). Different objects and surfaces possess differing powers of radiation; those which during the day absorb heat to the greatest extent radiate it most abundantly after the sun has disappeared, and consequently become cooled before those the radiating power of which is small. Similarly, the air in contact with these bodies also becomes cooled and is

then unable to hold as much water vapour as before, and the surplus is deposited in the form of **dew.**

Experiments have proved, however, that dew is only partly derived from the moisture in the air, much of it having its origin in vapour exhaled from the earth, or from the grass or other plants on which the dew appears. Moisture is being exuded by the leaves of plants continually; and in the absence of sunshine or wind it accumulates on the surface as drops of water or dew instead of being dried up as it is during the daytime. Vapour is arising also constantly from the earth, and this contributes to the formation of dew, so that though the upper surfaces of stones are not bedewed visibly on a clear night, the lower surfaces have often a heavy deposit.

Conditions favourable to the formation of dew.—For an abundant formation of dew several conditions are necessary. First, radiation must go on freely, and this happens on *bright clear evenings* when there are no clouds to obstruct the radiation. The air which is being cooled by contact with the body from which free radiation is taking place must not be disturbed before the dew-point is reached, or no dew will be thrown down, that is, the *evening must be still.* A breeze will renew constantly the air above the body which is being cooled by radiation and will prevent the dew-point being reached. Good radiating surfaces are those of leaves—whether of grass or other plants—also stones.

Hoar-frost—or as it is sometimes called *white-rime* or simply *rime*—is deposited instead of dew on these evenings when the radiation cools the overlying air to the temperature of freezing water before any deposition of moisture takes place. Hoar-frost is not frozen dew. It does not assume the liquid condition first, but is precipitated at once in the solid form. In these circumstances the dew-point is at or below the freezing-point.

EXERCISES ON CHAPTER XV.

1. A flask containing pure water is heated by a single burner, and one thermometer is placed with its bulb below the surface of the water, and another thermometer with its bulb just above the surface. When the water boils the readings of the two thermometers are taken. Will the readings be the same?

What will be the effect on the reading of each thermometer (1) of placing a second burner under the flask, and (2) of dropping some common salt into the flask?

2. Two mercury barometers are set up. Will the heights of the mercury columns be the same when the inside of *one* of the tubes is wet? If the temperature of the room be 17° C., what will be the difference in the heights?

3. Why does a muddy road dry better on a windy and warm day than on a quiet and damp day?

4. Explain why, in order to cook food at a high altitude, it is necessary to adopt a method which differs from that which would be adopted at ordinary levels.

5. Define the *boiling-point* of a liquid. Distinguish between *boiling* and *evaporation*. What condition determines whether a liquid will boil or evaporate?

6. If a narrow-necked flask and a wide dish, both containing ether, are placed side by side on a table, would you expect the ether in both cases to have the same temperature? If not, explain the reason for the difference.

7. Why is an iceberg frequently surrounded by fog? If a breeze be blowing, would the fog be distributed equally on all sides of the iceberg?

8. Why does fanning produce a sense of coolness to the face? Is the effect influenced by the degree of dampness of the air?

9. The temperature of a room is 14° C., and the dew-point is found to be 5° C. By means of Physical Table (16), find the pressure of the water vapour present, and determine the *relative humidity* of the air.

10. The dimensions of a closed room are $10 \times 10 \times 5$ metres. If the temperature of the room be 20° C., calculate, from Physical Table No. 17, the weight of water required to saturate the air.

11. A saucer containing water is left to evaporate on a window sill. Explain the atmospheric conditions which will favour or retard the disappearance of the water.

12. Explain what happens to the steam issuing from the funnel of a steam-engine: (*a*) on a fine warm day; (*b*) on a damp day.

13. How is the reading of a thermometer altered by wrapping a wet rag round the bulb? What will happen if the rag be wetted with (1) ether, (2) oil, instead of water? How do you explain the various results?

14. The windows of a room frequently become dimmed with moisture on dry, cold days. Is the moisture on the outside or the inside, and how do you account for its formation?

15. Describe a simple form of hygrometer, and explain as fully as you can what measurement is made with it.

16. Two simple barometers are set up side by side in the same vessel of mercury, and a little water is introduced into one. Explain why the height of the mercury is different in the two cases. How will change of temperature affect the height in each case?

CHAPTER XVI.

MELTING POINT. LATENT HEAT.

Temperature of melting.—When a solid is heated, the first effect is usually to make it expand. But if the heating is continued long enough, when the solid reaches a certain temperature, which differs for different solids, melting begins. The solid changes into a liquid. The temperature at which the melting takes place is called the **melting point.** Thus, when a piece of lead is heated its temperature rises, it gets larger, and as the heating is continued it is converted into a silvery-looking liquid. Wax, ice, and iron are other examples of solids which melt. But ice, wax, lead, and steel differ very widely in the temperatures at which they begin to melt, as the following table shows:

Ice	melts at - - - -	0° C.
Bees-wax	" - - - -	65° C.
Lead	" - - - -	326° C.
Steel	" - - - -	1360° C.

So long as any of the solid remains unmelted, the temperature does not rise above the melting point. It can be shown easily by experiment that this is true in the case of ice.

EXPT. 144.—**Melting point of ice.** Put some small pieces or shavings of clean ice into a beaker and insert a thermometer into them. Record the temperature indicated. Pour in a little water, stir the mixture, and again record the temperature. Place the beaker on a sand-bath and warm it gently. Notice the reading of the thermometer *so long as there is any ice unmelted.*

EXPT. 145.—**Melting point of wax.** Melt a little paraffin wax in a test-tube placed in a beaker of boiling water, and immerse the bulb of a thermometer in the liquid. When the thermometer is taken out, a thin film of liquid paraffin will be seen upon it. Let the

bulb cool, and notice the temperature when the wax assumes a frosted appearance, which shows that it is solidifying. When the wax on the bulb has become solid, place the thermometer in a beaker of water and gently heat the water. Observe the temperature at which the wax becomes transparent again. The average of this result and the preceding one is the *melting point* of paraffin wax.

Latent heat.—The experiments which have just been described are of the very greatest importance, and should be clearly understood. It is certain that when a mixture of ice and water is heated over a laboratory burner heat is being given to the mixture continually. Yet the temperature as recorded by the thermometer gets no higher. The question arises, what becomes of this heat, as it has no effect upon the temperature of the mixture? The ice is melted gradually, and if the heating be continued long enough it is all changed into water. As soon as this has happened, every further addition of heat raises the temperature of the water. These considerations lead to the conclusion that the heat previously given to the mixture is all used up in bringing about the change of ice into water. Further, it is found that not only in the case of ice, but also when any solid is turned into a liquid, there is no increase in temperature, even while heat is being added, until the whole of the solid has been changed to a liquid.

This amount of heat which is necessary to change a solid into a liquid is spoken of as **latent heat.** The word latent comes from a Latin word, meaning 'lying hidden,' and refers to the fact that the heat used up in changing a solid to the liquid condition causes no increase of temperature, but appears to be hidden away in the liquid.

EXPT. 146.—**Heat required to melt ice.** Let a few lumps of ice stand in a beaker until some of them have melted. Notice that the temperature is 0° C. Counterpoise two empty beakers of the same size in the pans of a balance, and put a small lump of the ice into one, and the same weight of water from the melted ice in the other. You have thus equal weights of ice and water at 0° C. Pour equal weights of hot water into the two beakers. When the ice is melted, observe the temperature of the water in each beaker. The temperature of the water in the beaker in which the ice was placed will be found much lower than that of the water in the other beaker, owing to the ice using up a large quantity of the heat in melting into water.

EXPT. 147.—**Ice and ice-cold water.** Take equal weights of hot water in two large beakers of the same size. Place a piece of ice in one of the beakers, and observe the temperature of the water when it has melted. Pour ice-cold water into the other beaker until the same temperature is reached. Find, by weighing, the weights of ice and ice-cold water which have been added. It will be found that a small weight of ice has as much cooling effect as a large weight of ice-cold water.

Latent heat of water.—The number of units of heat which are required to change the state of a gram of ice, converting it from the solid to the liquid condition without raising its temperature, is called the latent heat of water or the **latent heat of fusion of ice.** Thus, to melt 1 gram of ice requires 80 heat-units; that is to say, as much heat as would raise the temperature of a gram of water through 80° C., or would raise that of 80 grams of water through 1° C., is used up in changing a gram of ice into a gram of water at the same temperature. Similarly, to melt 1 lb. of ice requires as many heat-units as are necessary to raise the temperature of a pound of water from 0° C. to 80° C., or, as much heat as is wanted to raise the temperature of 80 lb. of water through one degree Centigrade.

EXPT. 148.—**Latent heat of fusion of ice.** Weigh a metal calorimeter. About half fill the calorimeter with water, previously warmed to about 35° C., and again weigh. Break some ice into small pieces, and place within the folds of some blotting-paper, in order to dry the ice, a quantity sufficient to weigh about one-fifth as much as the warm water. Stir the water with a thermometer, and when the temperature has fallen to about 30° C., note the exact temperature, and transfer the dry ice into the calorimeter. Keep the contents continually stirred until all the ice is melted, and at once note the temperature. In order to find the weight of ice added, take a final weighing of the calorimeter and its contents. Enter the observations thus:

Weight of calorimeter $= w_1$ gm.
" " warm water $= w_2$ "
" " ice $= w_3$ "
Initial temp. of water $= T_1°$ C.
Final " " $= T_2°$ C.
Latent heat of water $= L$ heat units.

Heat lost by warm water $= w_2 \times (T_1 - T_2)$ units.

$$\text{Heat gained by the ice} = \left.\begin{matrix}\text{Heat required to}\\ \text{melt the ice}\end{matrix}\right\} + \left\{\begin{matrix}\text{Heat required to warm}\\ \text{melted ice up to } T_2°\end{matrix}\right\}$$
$$= (w_3 \times L) + (w_3 \times T_2).$$

But, Heat lost by warm water = Heat gained by the ice.

Hence, $$w_2 \times (T_1 - T_2) = (w_3 \times L) + (w_3 \times T_2),$$

or $$L = \frac{w_2(T_1 - T_2) - w_3 T_2}{w_3}.$$

A more accurate result is obtained when allowance is made for the fact that the calorimeter cools through the same range of temperature as the warm water contained in it. If the calorimeter be made of copper, of which the specific heat is 0·09, it will give up ($w_1 \times 0{\cdot}09$) heat units in cooling through 1° C.; and, in cooling through $(T_1 - T_2)$ degrees it will give up $w_1 \times 0{\cdot}09 \times (T_1 - T_2)$ heat units. Hence,

$$\text{Heat lost by warm water and calorimeter} = (T_1 - T_2)\{w_2 + (w_1 + 0{\cdot}09)\}.$$

Melting point, by means of a cooling curve.—If a solid be melted and allowed to cool, and the temperature observed at

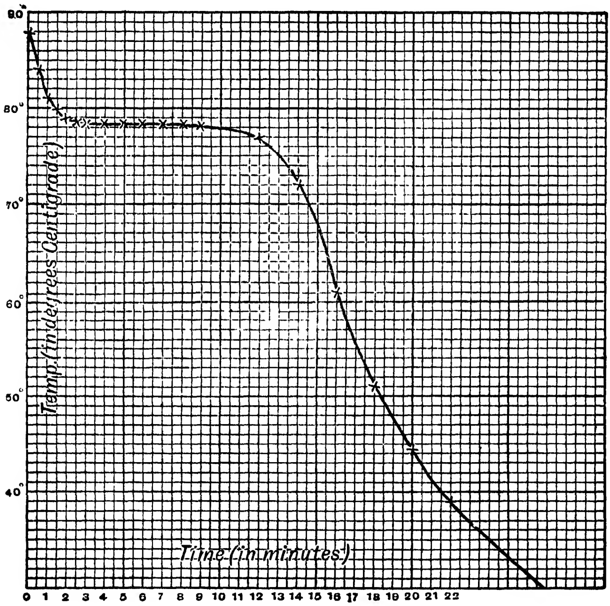


FIG. 135.—Melting-point curve of naphthalene.

equal time intervals, it will be noticed that during the period of solidification the temperature will remain more or less constant.

If the readings be plotted on squared paper, this period will be indicated by part of the curve being practically horizontal; and the position on the temperature scale of this horizontal portion will indicate the temperature of solidification, which is the same as the temperature of melting (Fig. 135). The constancy of temperature during solidification is due to the fact that the latent heat given up by the liquid on solidifying more or less counteracts the loss of heat by cooling from the walls of the containing vessel.

EXPT. 149.—**Determination of melting-point.** Fit a small test-tube (5 cm. × 1·5 cm.) with a cork and thermometer (Fig. 136). Cut a

FIG. 136.—Expt. 149.

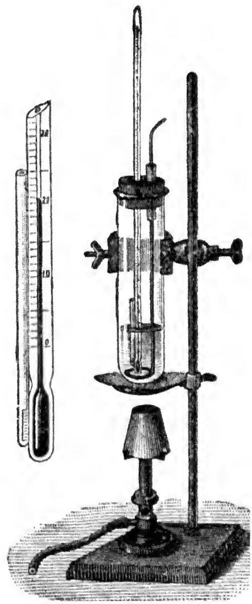


FIG. 137.—Determination of the melting point of a small quantity of a substance.

small groove in the side of the cork so that it shall not fit air-tight. Melt some paraffin wax in the tube. Clamp the thermometer in a vertical position and so that the tube is not touching the table. Take readings every half-minute until the wax has cooled to about 40° C. Plot the readings on squared paper, and state the temperature of solidification.

EXPT. 150.—**Alternative method.** A short piece of thin glass tubing of narrow bore is dipped into some melted wax, naphthalene or butter, of which the melting-point is required, and in this way some of the substance, which soon solidifies, enters the tube. The tube is placed by the side of a thermometer, as indicated in the illustration, and fixed there. The combination is then introduced into a beaker or test-tube containing a curved stirrer which can be used to keep the water or other liquid in the vessel at a uniform temperature. When the substance in the tube is seen to melt, the temperature is recorded. The flame is then removed and the temperature noted at which the substance solidifies. Several experiments should be made and the mean temperature taken as the melting point. This method is convenient to use when only a small quantity of a substance is available for the determination of the melting point. By using a liquid like castor oil or sulphuric acid in the test tube, instead of water, the melting point of sulphur or other substance which melts below the temperatures at which these liquids boil can be determined.

Expansion during solidification.—The fact that ice floats on water is convincing evidence that ice is less dense than water, and therefore that water expands on solidifying. In fact, 100 c.c. of water expand to about 109 c.c. on solidifying.

Nearly all substances contract when they solidify, but water and a few others are exceptions. The reason that sharp castings can be made from molten cast iron is that iron contracts scarcely at all on solidifying; also, the alloy of antimony, lead, and copper used in making printers' type behaves in the same way.

The force with which water expands on freezing is extremely great: strong steel shells filled with water will burst when exposed to extreme cold. It is a common experience for water pipes in houses to be burst during cold weather; as a general rule the ice inside the pipe prevents the escape of water, and the damage becomes evident only when the ice begins to melt.

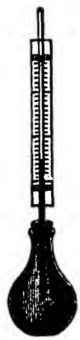


FIG. 138.—Expt. 151

EXPT. 151.—**Contraction of ice when melted.** Fit up a flask, similar to Fig. 138. Introduce some broken ice into the flask, and fill it completely with tap water. Insert the cork until the surface of the water is high up the narrow

glass tube. Notice how, as the ice melts, the surface of the water falls.

EXPT. 152.—**Fracture by formation of ice.** Using a blowpipe, seal up one end of a piece of wide glass tubing; at a few centimetres from this end melt the glass and draw off the tube to capillary dimensions. Fill the tube with water up to the constriction, and seal it with a blowpipe. Place the tube in a freezing mixture of ice and salt, and cover the vessel with a cloth. In a few minutes the tube will burst.

Fig. 139 is an instructive representation of the changes in volume observed when cold ice is warmed gradually until it is converted finally into vapour. In warming up to 0° C. the ice expands, like any ordinary solid; it then contracts considerably on melting; the cold water contracts gradually until its temperature is 4° C., and then it expands until its temperature is 100° C.

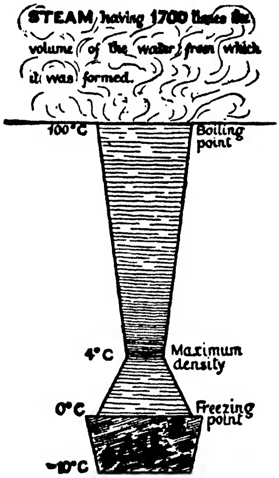


FIG. 139.—Changes in volume of ice, water, and steam.

Regelation.—The increase in volume of water on freezing leads to an important influence of pressure on the temperature at which the freezing takes place; and it has been proved experimentally that the freezing-point is reduced by 0°·0072 C. for each increase of 1 atmosphere in pressure (*i.e.* 14·7 lbs. per sq. inch).

The effect may be explained thus: Suppose some water at 0° C. to be enclosed in a metal cylinder by means of a freely moving piston (Fig. 139A). If the water is frozen the expansion raises the piston, and work has to be done against the pressure of the air acting on the upper surface of the piston. If, previously to the freezing, the pressure on the piston were increased by adding weights, more work would have to be done by the expansion on freezing.

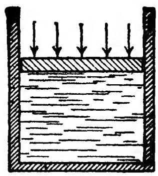


FIG. 139A.

This additional work delays the solidification, which will occur only if the water is cooled below its former freezing-point;

in other words, **the freezing-point of water is reduced when the pressure is increased.**

This influence of pressure explains several natural phenomena. Thus, when two lumps of ice at 0° C. are pressed together they become firmly attached. Where the surfaces are in contact the increased pressure reduces the melting point; and, if the ice is at 0° C., some of it melts; this absorbs heat, so that the water formed and its surroundings are slightly below 0° C.; hence, as soon as the pressure is removed, this water solidifies and firmly unites the lumps of ice. This phenomenon is known as **regelation.**

Similarly, in making a snowball, the hands press the snow crystals together, and wherever these are in contact, slight melting takes place. On removing the pressure the crystals adhere together. A snowball cannot be made from snow which is much below 0° C.; in this case the pressure of the hands is quite insufficient to reduce the melting-point to the temperature of the snow, which will still retain its powdery form.

Regelation is well demonstrated by supporting a block of ice at its two ends and suspending a heavy weight from the ice by means of a loop of wire (Fig. 139B). Where the wire exerts pressure, the ice is melted; the water formed is squeezed to the back of the wire and there resolidifies. This process continues until the wire has passed completely through the ice; but the block of ice remains entire.

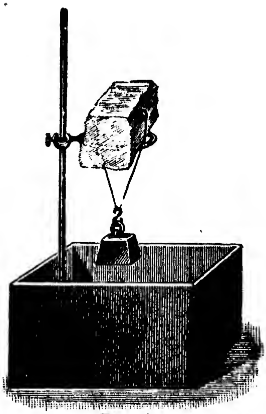


FIG. 139B.—Experiment on regelation.

Latent heat of steam.—As previously explained (p. 187), when a liquid is changed into vapour a certain amount of heat is used up. It does not matter whether the liquid evaporates or boils; every gram of it requires a certain amount of heat before it becomes converted into vapour. When once water has started to boil, its temperature gets no higher than the boiling point. So long as there is any water left, no matter how much it is heated, the

temperature remains the same. All the heat is absorbed, or used up, in bringing about the change from the liquid state to that of vapour. It requires a great many more heat-units to convert one gram of water at a temperature of 100° C. into steam at the same temperature, than it does to change a gram of ice at 0° C. into a gram of water at 0° C. Whereas to bring about the latter change requires an expenditure of 80 heat-units, to convert a gram of water at 100° C. into a gram of steam without changing its temperature requires no fewer than 538 heat-units. Thus, the **latent heat of steam**, or, as it is sometimes called, the **latent heat of vaporisation of water**, is 538.

Just as a large quantity of heat is required to convert water into steam, so a large quantity is given up when steam becomes water. It is for this reason that a scald from the steam of boiling water is worse than the scald from the boiling water itself.

A rough method of determining the latent heat of steam consists in comparing the time required for a uniform source of heat to raise ice-cold water to 100° C. with the time required to convert the same weight of boiling water into steam. Thus, if the latent heat of steam were 200 heat-units, the period of time required by a steady source of heat to raise 1 gram of ice-cold water to the boiling point would be one-half the time required to convert 1 gram of boiling water into steam, since the quantities of heat required in the two processes would be 100 units and 200 units respectively.

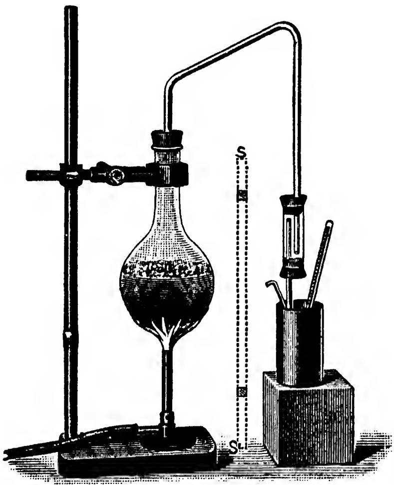


FIG. 140.—Determination of the latent heat of vaporisation of water.

EXPT. 153.—**Conversion of water into steam** Pour about 50 c.c. of ice-cold water into a metal calorimeter which has been cooled by surrounding it with ice. Quickly dry the outside of the calorimeter, and place it on gauze heated by a steady flame. Note the time (i) at the instant when

heating commences, (ii) when the water commences to boil, and (iii) when all the water has evaporated. By comparing the time intervals (i-ii) and (ii-iii), calculate the latent heat of steam. The error in the result will probably amount to about 10 %.

EXPT. 154.—**Determination of latent heat.** Arrange a flask with the connections shown in Fig. 140. The short length of wider glass tube is a trap to catch condensed steam. Put some water into the flask and boil it. While the water is getting hot, weigh out about 300 grams of water in a beaker or a thin metal vessel, and observe its temperature. After steam has been issuing from the glass tube for a few minutes, place the vessel so that the end of the tube is well under the water, and let it stay there until the thermometer records a temperature of about 40° C. Then weigh the water again to find the weight of steam condensed. Enter the observations as follows:

Weight of calorimeter $= w_1$ gm.	Initial temp. of water $= T_1°$ C.
„ „ cold water $= w_2$ „	Final „ „ $= T_2°$ C.
„ „ steam $= w_3$ „	Latent heat of steam $=$ L heat units.

Heat gained by cold water $= w_2 \times (T_2 - T_1)$ units.

$$\text{Heat lost by steam} = \left.\begin{matrix}\text{Latent heat of}\\ w_3 \text{ gms. steam}\end{matrix}\right\} + \left.\begin{matrix}\text{Heat given by } w_3 \text{ gms. water}\\ \text{in cooling from } 100° \text{ to } T_2° \text{ C.}\end{matrix}\right\}$$

$$= w_3 L + w_3(100 - T_2).$$

But, Heat gained by cold water $=$ Heat lost by the steam.

Hence, $$w_2(T_2 - T_1) = w_3 L + w_3(100 - T_2),$$

or $$L = \frac{w_2(T_2 - T_1) - w_3(100 - T_2)}{w_3}.$$

A more accurate result is obtained if allowance is made for the fact that the calorimeter is warmed through the same range of temperature as the cold water contained in it. The student should re-calculate the result, taking this into account.

EXERCISES ON CHAPTER XVI

1. Suppose that it requires 80 times as much heat to melt one ton of ice as would be required to warm one ton of water one degree of temperature on the Centigrade scale, how much of the ton of ice would be melted by pouring into a cavity in its surface a gallon of boiling water? A gallon of water weighs 10 lb.

2. How would you propose to prove by experiment that to boil away a gallon of water requires about five times as much heat as is needed to raise its temperature from the freezing to the boiling point?

3. Four ounces of hot lead filings and four ounces of water at the same temperature are poured upon separate slabs of ice. Will the lead or the water melt more ice? Give reasons for your answer.

4. An ounce of water at 0° C. is mixed with ten ounces of water at 70° C. What is the temperature of the mixture?

An ounce of ice is dissolved in ten ounces of water at 70° C., and the temperature of the mixture is found to be something over 56° C. What can be learnt from this experiment?

5. One hundred grams of boiling water are poured upon one hundred grams of ice. What results may be observed?

6. (i) How many grams of ice must be put into 100 gm. of water at 40° C. to lower the temperature to 5° C.?

(ii) How many grams of steam at 100° C. must be put into 100 gm. of water at 5° C. in order to raise the temperature to 40° C.?

7. (i) Why does steam cause much more severe burns than are caused by water at the same temperature?

(ii) When the temperature on a winter's day rises above 0° C., why does not the snow all melt at once?

8. Into a calorimeter containing 120 gm. of water at 10° C. steam at 100° C. is passed. The total weight of steam condensed is 5 gm., and the final temperature is 34°·5 C. What value does this give for the latent heat of steam?

In this experiment the calorimeter weighed 25 gm. and the specific heat of the metal of which it is made is 0·1. Re-calculate the above result, allowing for the heat absorbed by the calorimeter.

9. It is often stated that water freezes at 0° C. and boils at 100° C. Explain why this statement is not exactly true (*a*) if sea-water is used, (*b*) if the water is boiled at the top of a high mountain.

10. How would you determine the temperatures at which (*a*) butter melts, (*b*) water boils?

11. A piece of ice is placed in an evaporating basin under which a small Bunsen flame is kept steadily burning until there is nothing left in the basin. What can be learnt from this experiment?

12. Describe experiments to prove that heat is absorbed without raising the temperature when a solid is changed into a liquid and a liquid into a gas.

The temperature of some water is raised from 0° C. to 100° C. in half an hour. Assuming the supply of heat to be constant, about how long would it take for all the water at 100° C. to be converted into steam? (The latent heat of steam may be taken as 536.)

13. What is a thermometer? What temperature would you expect to be indicated by a Centigrade or a Fahrenheit thermometer when the bulb is placed in (*a*) melting ice, (*b*) boiling water, (*c*) your mouth, (*d*) ordinary drinking water?

14. Why is it that (*a*) water pipes sometimes burst during frosty weather, (*b*) the crack is not discovered until after the thaw sets in?

15. Describe two methods of determining the melting point of beeswax, and name any precautions you would take to secure accurate results.

CHAPTER XVII.

TRANSFERENCE OF HEAT.

Conduction, convection and radiation.—The transmission of heat from one point to another may be effected in three ways, viz.:

1. Heat may pass from one particle of a body to the next, travelling from the hotter to the colder parts, and causing no motion that can be seen of the particles of the body. This mode of transference is called **Conduction**, and is the process by which solids are heated.

2. When the heated particles actually move from one part of the body to another, causing it to become warmer throughout, the process is known as **Convection.** Liquids and gases become heated in this way.

3. When the heat passes from one point to another in straight lines with great speed, without heating the medium through which it passes, it is said to be transmitted by **Radiation.** The heat of the sun is transmitted through space in this way.

CONDUCTION.

Conduction of heat.—If you place one end of a short metal rod in a fire and hold the other, the rod soon begins to feel warm, and as time goes on it gets warmer and warmer, until at last it can be held no longer. Heat has passed from the fire along the rod, or has been **conducted** from the fire by the rod. **The process by which heat passes from one particle of a body to the next is called conduction, and the body along which it passes is known as a conductor of heat.**

Those substances which easily transmit heat in this way are called **good conductors**, while those which offer a considerable amount

of resistance to the passage of heat are called **bad conductors.** Metals are, as a rule, good conductors of heat, but some metals conduct heat better than others.

EXPT. 155.—**Difference of conductivity.** In Fig. 141, A, B, and C are rods of copper, brass, and iron, each 4 mm. in diameter, and about 40 cm. long. A short length at the end of one of the rods is bent at right angles and the three rods are bound together at D with several layers of thick copper wire.

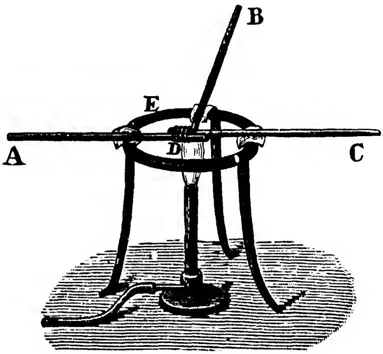


FIG. 141.—Conduction of metals.

Apply, by means of a brush, a coating of melted paraffin wax to the surface of the rods. Support the rods on a tripod E, placing pads of asbestos, or other bad conductor, between the rods and the top of the tripod. Heat the junction D with a Bunsen flame, and note how the wax gradually melts at a distance from the junction. In a short time the melting ceases to extend; and this condition is arrived at when the heat is lost from the surface of the rods just as rapidly as it is conveyed along the rods by conduction. Notice that the melting has not extended equally far along all the rods: the melting will be most extended in the case of the best conductor. Measure, and note, the distance along each rod to which the melting has extended.


FIG. 142.—Effect of different conductivities of wood and metal.

The fact that glass is a bad conductor is demonstrated when the end of a glass rod is fused in a blow-pipe flame although the rod is held in the hand only a short distance from the heated end. Fig. 142 represents an experiment which shows that wood is a bad conductor as compared with metals. A single layer of paper is wrapped tightly round a rod

consisting partly of metal and partly of wood. If this be moved to and fro in a flame the paper touching the wood is scorched much sooner than that touching the metal. This is due to the heat transmitted through the paper being conveyed away by the metal more readily than by the wood; consequently the paper touching the wood is soon heated to the temperature at which it scorches.

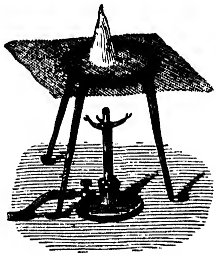


FIG. 143.—Principle of the Davy safety lamp.

Fig. 143 represents how the combustible mixture of coal gas and air rising from a Bunsen burner may be ignited above a piece of wire gauze held over the burner, and without the ignition proceeding downwards through the gauze. The mixture of gas and air, like all combustible substances, will not commence to burn unless it is heated up to its 'temperature of ignition.' In this experiment, the gauze conducts away the heat of the flame, and prevents the mixed gases passing through its meshes from being heated to the temperature of ignition. In a short time the gauze may become heated to dull redness, and the flame will then pass through to the top of the burner.

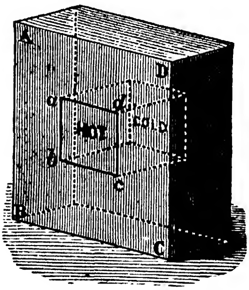


FIG. 144.—Conductivity.

The principle illustrated by this experiment is utilized in the Davy safety lamp used by miners. The flame of the lamp is surrounded by wire gauze which effectually prevents it from igniting any combustible gases in its neighbourhood, so long as the gauze remains relatively cool.

Coefficient of thermal conductivity.—ABCD (Fig. 144) represents a slab of metal, 1 cm. thick, the front face of which is kept 1° C. hotter than the back. This difference of temperature will cause a steady flow of heat to pass through the slab. Suppose *abcd* to be a 1 cm. cube situated in the centre of the slab; the quantity of heat conveyed through it in one second from the hot face to the cold face is termed the **coefficient of conductivity** of the metal. Hence, this coefficient may be defined as **the quantity of**

heat which passes in one second through a 1 cm. cube of the metal between opposite faces which are maintained at a difference of temperature of 1° C.

Consider the case of any slab of metal, of area s sq. cm., and of uniform thickness d cm. Then, if a temperature difference of $\theta°$ C. is maintained between the opposite faces, the quantity Q of heat transmitted through the slab will be proportional to (i) the coefficient of thermal conductivity k of the metal, (ii) to the fall of temperature per unit thickness, *i.e.* to $\frac{\theta}{d}$, (iii) to the area s, and (iv) to the time t. Hence,

$$Q = k \times \frac{\theta s t}{d}.$$

The following table gives the coefficient of thermal conductivity of different substances :

THERMAL CONDUCTIVITY.

Brass,	at 0° C., 0·204; at 100° C., 0·254	Ice,	0·005
Copper,	at 18° C., 0·918; at 100° C., 0·908	Glass,	0·0011–0·0023
Iron (wrought),	at 18° C., 0·144; at 100° C., 0·142	Asbestos,	0·00043
		Flannel,	0·00023

EXAMPLE.—Water contained in an open tank is covered with ice 3 cm. thick. If the area of the tank be 1 square metre, and if the coefficient of thermal conductivity of ice be 0·005, calculate the amount of heat transmitted in 30 minutes through the ice when the temperature of the air is $-15°$ C. Also, if the latent heat of water be 80 units, calculate what increase in the thickness of the ice will take place during this interval.

The fall of temperature per unit thickness, $\frac{\theta}{d}$, $= \frac{15°}{3} = 5°$ C.

Area, s, $= 10^4$ sq. cm.

Time, t, $= 30 \times 60 = 1800$ sec.

Hence, $Q = 0{\cdot}005 \times (5 \times 10^4 \times 1800) = 45 \times 10^4$ heat units.

(ii) Weight of ice formed $= \frac{45 \times 10^4}{80} = 5{\cdot}6 \times 10^3$ gm.

$$\text{Volume of ice} = \frac{\text{Weight}}{\text{Density}} = \frac{5{\cdot}6 \times 10^3}{0{\cdot}92} = 6{\cdot}1 \times 10^3 \text{ c.c.}$$

$$\text{Thickness} = \frac{\text{Volume}}{\text{Area}} = \frac{6{\cdot}1 \times 10^3}{10^4} = 0{\cdot}61 \text{ cm. approximately.}$$

Bad conductors.—Most liquids are bad conductors of heat, though quicksilver, being a metal, is an exception. If liquids were

heated only by conduction, water would boil throughout just as quickly when the source of heat was placed in contact with the top layer of liquid as it does when the heating takes place from below.

EXPT. 156.—**Water as a poor conductor.** Fill a test-tube three-quarters full with cold water, and having weighted a small piece of ice by winding wire round it, or in some other way, drop it gently into the test-tube. Hold the test-tube near the bottom where the piece of ice is, and warm the top of the water in a flame, as shown in Fig. 145. The water at the top can be heated until it boils vigorously and yet the ice is not melted.

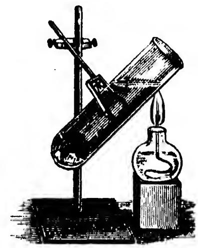


FIG. 145.—Experiment to show that water is a bad conductor of heat.

Experiments have shown that gases are very bad conductors, the conductivity of air being only about one ten-thousandth that of copper. In reckoning the conductivities of solids the proportion of heat conducted away by the air may, therefore, be neglected.

To keep ice in the warm days of summer the custom is to wrap it up in flannel and put it into a refrigerator. The flannel, because of its loose texture, encloses a quantity of air, which, being a bad conductor of heat, prevents the passage of heat from the warm outside air to the cold ice inside. Similarly, ice which has to be conveyed by rail or boat is packed in sawdust.

The refrigerator itself, too, depends upon much the same facts. The common form consists of a double-walled box with a space between the walls. This is either left 'empty,' as it is called when it is full of air; or, it is filled with some other bad conductor, such as the mineral substance *asbestos*.

If we wish to lift a hot plate we hold it with a folded cloth which does not conduct heat readily. Cylinders of engines are sometimes encased in a packing of some badly conducting material in order to prevent loss of heat.

CONVECTION.

Convection in liquids and gases.—The process by which water and other liquids are heated may be studied easily by heating

water into which some solid colouring matter has been thrown, in a round-bottomed flask over a small flame as in Fig. 146. The water nearest the flame gets heated, consequently expands, and gets lighter. It therefore rises, and causes a warm ascending current of coloured water. But something must take the place of this water which rises, and the cold water at the top, being heavier than the warm water, sinks to the bottom and occupies the space of the water which has risen. This water in its turn gets heated and rises, and more cold water from the surface sinks. Upward currents of heated water and downward currents of cool water are thus formed, until the whole of the water is heated. These currents are known as **convection currents**, and the process of heating in this manner is called **convection.**

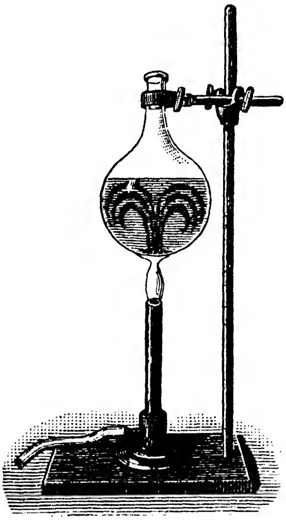


FIG. 146.—Convection in a liquid.

Gases similarly are heated by the process of convection, which may be thus defined: **Convection is the process by which fluids (liquids and gases) become heated by the actual movement of their particles due to difference of density.**

EXPT. 157.—**Circulation of water.** Fit up apparatus as shown in Fig. 147. A is a 6-oz. wide-mouthed, corked bottle, with the bottom knocked out (or an ordinary lamp glass may be used). A well-fitting cork with two holes is inserted, through which the bent glass tubes B, B′ pass, as shown. They are united at the bottom by a short piece of india-rubber tubing, C. Pour water into A until it covers the open ends of the tubes. Now pour in about a teaspoonful of ink. Apply a small flame at B. Notice what happens.

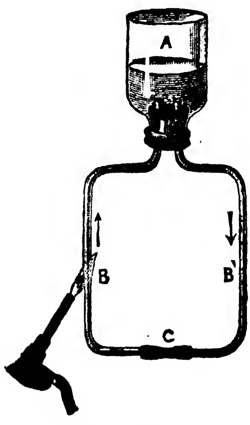


FIG. 147.—Expt. 157.

EXPT. 158.—**Convection currents in liquid.** Heat over a small flame a round-bottomed flask containing water (Fig. 146). Drop into the water some solid colouring matter, like cochineal, anilin dye or litmus. Notice how the hot, coloured water ascends.

EXPT. 159.—**Convection currents in air.** Place a short piece of candle in a saucer, light it, put a lamp glass over it, and pour sufficient water into the saucer to cover the bottom of the lamp glass (Fig. 148). Watch how the light of the candle is affected. Next cut a strip of card less than half the height of the lamp glass, and nearly as wide as the internal diameter of the top. Insert the card into the lamp glass so as to divide the upper part into halves. Now light the candle again, and see whether it will burn with the divided chimney over it. Test the direction of the currents of air at the top of the chimney by means of smouldering brown paper.

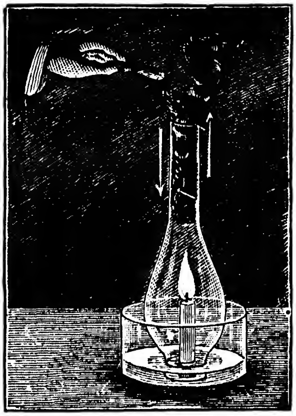


FIG. 148.—Expt. 159.

Ventilation.—The ventilation of ordinary dwelling rooms is rendered possible by the way in which gases become heated by convection. The air in a room becomes warmed and rendered impure at the same time. Consequently there is a tendency for the impure air to rise, and if a suitable place near the ceiling be made for it to get out, as well as a place near the floor for the colder, purer air from outside to enter, a continuous circulation of air is set up which will keep the atmosphere of the room pure and sweet.

RADIATION.

Radiation of heat.—The fact that we feel warm in a summer sun, or that bread can be 'toasted' by holding it near the fire, is sufficient to show that heat can travel from one place to another in a third way which is neither conduction nor convection. The respects in which **radiation** differs from the other ways in which heat moves from one place to another are:

(1) it does not require a material substance for its transmission,

(2) it travels in straight lines.

Curtains have sometimes been burnt by the sun's rays being concentrated upon them by a bottle of water, though the water is not warmed much by the passage through it of the radiations from the sun. Evidently, then, the water in such a case does not pass on its heat after first becoming warmed itself—that is, it does not act as a conductor. Yet something must pass through it which can make bodies hot. This something is called **radiation.** Its nature is simply a wave motion in the medium through which the rays pass. This medium is known as luminiferous ether, or shortly as the 'ether,' but it is in no way connected with the liquid ether used for scientific purposes. The ether is little more than a name, for, though something must exist to transmit waves of light and heat, nothing is certain as to its constitution.

The student will understand the phenomena of heat radiation more clearly after a short study of subsequent chapters on visible radiations (known as Light). There is indeed a close relationship between the two classes of radiation, and their chief difference may be compared to the difference between ocean waves and the small ripples on the surface of a pond, the long waves representing relatively slow rates of vibration and the short waves quicker vibrations. This difference is more apparent than real, and it is accentuated by the fact that the human eye is sensitive only to a very limited range of radiation. The close relationship between **radiant heat** and **light** is shown by the following facts:

(i) At the time of an eclipse of the sun, caused by the moon coming directly between the sun and the earth, it is observed that the heat from the sun is cut off at the same instant as the light. Hence radiant heat must travel through space with the same velocity as light, viz.: 186,000 miles per second. Compared with this, the transference of heat by conduction and convection are extremely slow processes.

(ii) Radiant heat may be *reflected* by exactly the same processes as light; and the laws of reflection are the same in both cases.

(iii) When passing from one medium into another, radiant heat is bent or *refracted* in the same manner as light. Thus, the rays of light from the sun may be refracted and focussed by means of a lens, and the fact that a piece of paper is ignited

when held at the focus proves that the radiant heat is refracted in the same manner as the rays of light.

Radiant heat, like light, is a wave motion: the difference between them is simply due to the difference in the length of their waves. When a piece of metal is heated in a dark room it gives out long waves of radiant heat produced by relatively slow vibrations in the ether which can be detected by the surface of the body and by instruments of special construction; but the eye is not sensitive to these slow vibrations, and the metal is therefore invisible. As the temperature rises, the metal begins to give out shorter waves in addition to the longer ones, and the eye may perceive a radiation which it interprets as a 'dull-red' colour. At a higher temperature still shorter waves or quicker vibrations are added to the radiation, and the metal appears to be bright red.

Radiating power.—The rate at which heat is radiated outwards from a hot body depends upon (i) the difference of temperature between the hot body and the surrounding space, and (ii) the nature of the hot body's surface. Thus, a dull black surface radiates heat more rapidly than a bright metallic surface.

The quality of the surface affects in a similar manner the rate at which heat is **absorbed** by a cold body; thus, a dull black surface absorbs heat more rapidly than a bright metallic surface. In fact we may say briefly that **good radiators are good absorbers.** That this must be the case is evident if we consider two similar vessels, one having a dull black surface and the other a bright surface, containing equal quantities of warm water, and both supported near together inside a hollow vessel made of non-conducting material. Each of the vessels is radiating heat—the black one more rapidly than the bright one—and we should anticipate at first that the former would cool more rapidly than the latter. But it would be found that the temperatures of both vessels remained constant; and this is because the more rapid radiation from the black surface is counter-balanced by the fact that it absorbs far more of the heat rays falling upon it than in the case of the bright surface.

EXPT. 160.—**Radiation.** Obtain two small bright tin cans or canisters, and fit into each a cork having a hole through which a

thermometer will pass.* Cover the outside of one of the vessels with lamp-black by holding it over a candle or luminous gas flame, or over burning camphor. Put the same quantity of hot water at the same temperature in each, and then cork up the vessels, each cork having a thermometer through it so that the bulb is well immersed in the water. Observe the temperature of each vessel of water, and if the temperature of one is higher than that of the other, cool the vessel until the temperatures are equal.

Place the vessels a short distance apart, with a screen of cardboard between them. Note the reading of each thermometer, at intervals of half a minute; and continue the readings until the temperatures are at about 30° C. Plot the readings on squared paper, and deduce, from the curves obtained, which of the vessels is radiating heat more rapidly.

EXPT. 161.—**Absorption.** Similarly pour equal amounts of cold water of the same temperature into a blackened and a bright vessel, and hang them for 20-30 minutes before an even fire or closed stove; or at the same distance above an iron plate, supported on a tripod stand and heated by two laboratory burners placed so that the vessels may be in a position to receive heat equally. At the end of this time observe their temperatures. The blackened vessel will be found at a higher temperature than the bright one.

Law of cooling.—Newton's Law of Cooling states that **the amount of heat lost in a given interval of time by a particular vessel, when filled with any liquid, is proportional to the mean difference of temperature between the vessel and the surrounding air.** Hence, if when filled with warm water a given vessel takes 2 minutes to cool from $T_2°$ to $T_1°$, and if with *the same volume* of another liquid it only requires 1 minute to cool through the same range of temperature, it follows that the number of calories of heat lost in the first case must be twice as great as the number lost in the second case, for the rate of loss of heat at corresponding temperatures must be the same in both cases. In other words, the quantities of heat lost are proportional to the time-intervals required.

If w_1 be the mass of water, the heat lost is $w_1(T_2 - T_1)$ calories. If w_2 be the mass of the other liquid, of which the specific heat is s, the

* Two circular cigarette tins are suitable for this experiment. A circular hole should be filed through the centre of each lid, the hole being just large enough to admit the thermometer; and it is an advantage to solder the lid to the body of the tin.

quantity of heat lost is $w_2 s(T_2 - T_1)$ calories. If t_1 and t_2 be the time-intervals required in the two cases, then

$$\frac{w_1(T_2 - T_1)}{w_2 s(T_2 - T_1)} = \frac{t_1}{t_2},$$

or

$$s = \frac{w_1}{w_2} \cdot \frac{t_2}{t_1}.$$

EXPT. 162.—**Specific heat by method of cooling.** Weigh an empty vessel, as used in Expt. 160, and nearly fill it with a measured volume of water. Weigh the vessel and its contents. Warm the vessel until its temperature is about 50° C. Support it on a non-conducting surface and screen it from air currents. Allow it to cool, and note the time required to cool (i) from 45° C. to 35° C. and (ii) from 35° C. to 25° C. Empty and dry the vessel. Pour into it the same volume of the liquid of unknown specific heat, and weigh the vessel and its contents. Raise the temperature to about 50° C., and observe the time-intervals required for it to cool through the same ranges of temperature. Calculate the specific heat of the second liquid from each range of cooling.

Heat as a form of energy.—Heat was believed formerly to be a fluid called **Caloric**, and it was supposed that a piece of hot iron differed from a cold piece in having entered into some sort of union with this fluid, which was considered to be indestructible. But in the year 1798 Rumford boiled water by the heat developed by the friction between two metal surfaces which he rubbed together; and he found that the amount of water he could bring to the boiling temperature depended only on the amount of work he expended in rubbing. Since he could obtain an indefinite amount of heat from two definite masses of metal, it was quite clear that heat could not be matter, which cannot be created. Davy made the truth even clearer by obtaining heat enough to melt ice by simply rubbing two pieces of ice together; they were both cold or without caloric; and since heat could be obtained by rubbing them together, it was quite certain that heat could not be a fluid. Joule went a step further and measured the amount of work which must be done to obtain a given quantity of heat; that is, he measured the **mechanical equivalent of heat.**

His apparatus consists essentially of a large calorimeter containing water which is set in motion by paddles on a spindle turned by falling weights. The water is prevented from moving

as a whole in the calorimeter by vanes projecting radially inside this vessel. When the weights are allowed to fall, the spindle turns, the paddles move the water, the motion is stopped almost immediately by the vanes and is converted into heat, and a rise of temperature is observed. The work performed can be found from the masses of the weights and the distance through which they fall; and the heat produced can be determined in calories, knowing the water-equivalent of the calorimeter, the weight of water in the calorimeter, and the rise of temperature. Results of experiment show that **to raise the temperature of one gram of water one degree Centigrade requires about 42 million ergs of energy**, or the mechanical equivalent of heat is 42,000,000 ergs per gram-calorie.

The average of a large number of experiments made by Joule gave the value of the mechanical equivalent of heat in British units as 1,390 foot-pounds. Or, **to raise the temperature of one pound of water through one degree Centigrade requires an expenditure of 1,390 foot-pounds of work.** To raise the temperature of a pound of water through 1° F. requires 772 foot-pounds.

Conversion of motion into heat.—Some examples which will be familiar to the student will provide proofs that heat and work are convertible. When a brake is applied to the wheels of a train as it stops at a station, it is a common thing to see sparks fly. The resistance of friction which overcomes the motion of the train causes a sufficient amount of heat to be developed to raise the temperature of the particles of steel, which get rubbed off, to a red heat. By continually hammering a piece of iron on an anvil it can be made too hot to hold in the hand.

The following experiments show that heat appears when motion is destroyed:

EXPT. 163.—i. **Heat from motion.** Hammer a piece of lead, or saw wood, and test the temperature of the lead or saw before and after the experiment.

ii. Rub a brass nail or button on a wooden seat, and notice its increase of temperature.

When a lucifer match is rubbed along a rough surface, the heat into which the work is converted is enough to ignite the match. In all such cases mechanical work is converted into heat. The converse is true also; heat is convertible into work. In the

steam-engine the heat of the furnace changes the water in the boiler into steam. The steam forces the piston along the cylinder, and this movement of the piston in a straight line is converted into the circular motion of a fly-wheel; and is used, through the intervention of suitable mechanism, in pumping water or performing some other kind of work. The steam which enters the cylinder is hotter than that which passes from it into the condenser. Thus, part of the heat of the steam has been converted into useful work and parts of it have been lost to the condenser, the air, etc.

Determination of the mechanical equivalent of heat.—Mechanical work is done when a body is raised from the ground. When the body is then allowed to fall freely, the work done in raising it is converted gradually into kinetic energy; and when the body reaches the ground all this kinetic energy is converted into heat. When the weight of the body and the distance through which it is raised are known, the work done in lifting it can be calculated; and when the specific heat of the material is known, and the rise in temperature after falling is observed, the quantity of heat developed can be calculated. Such data serve as a means of determining approximately the mechanical equivalent of heat.

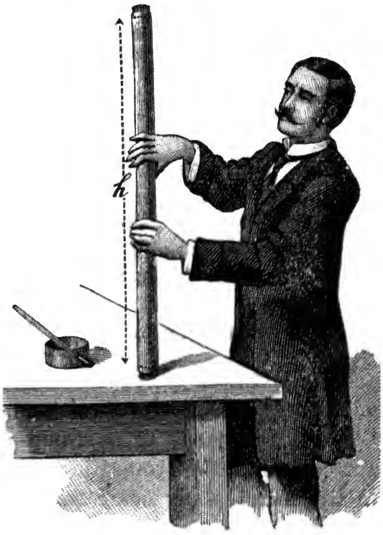


FIG. 149.—Experiment to determine the mechanical equivalent of heat.

EXPT. 164.—Obtain a card-board tube, about 1 metre long and 5 cm. diameter, fitted with a cork at each end. Weigh out about 500 gm. of small lead shot contained in a dish or beaker, and observe the temperature (t_1) of the shot, using a thermometer reading to 0°·2 C. Remove one of the corks; and, holding the tube almost horizontal, transfer the

shot to the tube. Hold the tube vertically, and measure by means of a metre scale the distance from the top of the lead shot to the top of the tube. Subtract from this the length of the tube occupied by the upper cork when firmly inserted. This gives the distance h through which the shot are raised when the tube is inverted. Insert the upper cork, and, while holding the middle of the tube firmly, rapidly invert it to a vertical position. Repeat this movement at least 50 times, counting the number n of times the inverting has been repeated. Transfer the shot back to the dish, at once insert the thermometer into the shot, and note the final temperature (t_2).

If w gm. = weight of shot,
h cm. = vertical distance within the tube,
n = number of times the tube is inverted, then
mechanical work done $= n \times wh$ gm.-cm. units.

If s = specific heat of lead,
$(t_2 - t_1)^\circ$ C. = rise in temperature, then
heat developed $= ws(t_2 - t_1)$ calories.

Hence, **mechanical work equivalent to one calorie** $= \dfrac{n \times wh}{ws(t_2 - t_1)}$ gm.-cm.

The following is a typical result of an experiment with this apparatus :

Weight (w) of lead = 434 gm.
Specific heat (s) „ = 0·0315.
Vertical height (h) of fall = 67·7 cm.
Rise in temperature $(t_2 - t_1) = 16^\circ{\cdot}5 - 14^\circ{\cdot}5 = 2^\circ$ C.
Number (n) of inversion = 40.
Mechanical work done $= 40 \times (434 \times 67{\cdot}7)$ gm.-cm. units.
Heat developed $= 434 \times 0{\cdot}0315 \times 2$ calories.

Hence,

$$\text{work equivalent to 1 calorie} = \frac{40 \times 434 \times 67{\cdot}7}{434 \times 0{\cdot}0315 \times 2} = (4{\cdot}3 \times 10^4) \text{ gm.-cm. units.}$$

Determination of the mechanical equivalent of heat by means of magneto-electricity. If a metal disc be rotated rapidly between the poles of a strong electro-magnet electric currents are caused to flow round it. These currents which are induced in the metal plate, cease to flow when its rotation is stopped; moreover, these induced currents flow in such a direction that they tend to stop the rotation of the disc. The first transformation of energy is the conversion of the energy of rotation into that of the electric currents flowing round the metal plate. But this is followed by the transformation of the

energy of the electric currents into that of heat. Such a plate is rotated between the poles of a very powerful electro-magnet only by the expenditure of a great amount of mechanical work, and this is eventually converted into sufficient heat in the plate to make it too hot to touch. By measuring the amount of work expended in rotating the disc, and also the quantity of heat developed in it, it is easy to calculate the mechanical equivalent of heat.

Compression and expansion of a gas.—When a gas is compressed, work is done upon it and this energy is converted into heat. Dalton showed that when air is compressed to half its volume, a rise of temperature of 50° F. (27·8° C.) is produced. Conversely, when a compressed gas is allowed to expand there is a fall of temperature. This fact is utilised in apparatus for cooling air and other gases until they are converted into the liquid form.

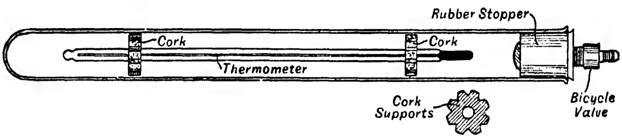


FIG. 149A.—Apparatus for showing heating by compression and cooling by expansion.

EXPT. 165.—**Heating by compression.** Observe the temperature of the thermometer in the tube A (Fig. 149A). Pump air into the tube with a cycle pump. Note the rise of temperature.

EXPT. 166.—**Cooling by expansion.** After forcing air into the tube, let the apparatus stand until the temperature is that of the room. Release the valve. Note the fall of temperature due to the sudden expansion of the compressed air.

Radiation and work.—Radiation can be converted into work, but in a less direct manner than is the case with ordinary heat; it must first be absorbed and heat some material body thus causing its molecules to oscillate in the manner described already. This form of heat has a mechanical equivalent; and it is fair to conclude that, if the whole radiation be absorbed, the mechanical equivalent of the absorbed heat is an exact measure of the energy of the radiation.

EXERCISES ON CHAPTER XVII.

1. On a cold morning a gardener grasps the iron part of his spade with one hand and the wooden part with the other. Explain why one hand feels colder than the other.

2. If a spoon made of solid silver and one made of brass and only silver plated are placed in some boiling water, the handle of the silver spoon becomes much hotter than that of the plated one. Why is this?

Describe an experiment by which you would show that your explanation is correct.

3. Why is a vessel of water heated more quickly when heat is applied at the bottom than when it is heated at the top?

Draw a diagram to illustrate the movements of a liquid heated from below.

4. Two test-tubes A and B are filled with water. A small piece of ice is allowed to swim in A, and a similar piece of ice is sunk by a weight to the bottom of B. Heat is applied to the closed end of A and to the open end of B. In which test-tube may we expect the ice first to melt? and in which may we expect the water first to boil? Give reasons for your answer.

5. State which modes of heat transference are involved in warming a room by means of an open fire-grate. Which mode is the more important?

6. Explain why a thick glass vessel usually cracks when hot water is poured into it.

7. A metal plate, 1 square decimetre in area and 0·5 cm. thick, has the whole of one face covered with melting ice, while the other face is in contact with boiling water. If the coefficient of conductivity of the metal be 0·14, how many kilograms of ice will be melted in an hour?

8. Two similar thermometers are taken, and the bulb of one is blackened. They are both (i) placed in sunlight, (ii) exposed on a clear night. How will the readings of the thermometers differ in each case?

9. A silver cup containing boiling water is placed on a silver tray in a room. Describe the various ways in which the water loses heat.

10. Illustrate the various ways in which heat can be transmitted from one body to another. What conditions determine the rate at which the transmission is effected in each case?

11. What experiments would you perform to show that:

(*a*) Silver is a good conductor of heat;

(*b*) Water is a bad conductor of heat?

12. In the case of a shot fired at a target, state (*a*) why the velocity of the shot changes when it strikes the target; and (*b*) why the target is made hot where the shot strikes it.

13. When a brake is applied to a wheel of a moving train, red-hot sparks are seen. What are these sparks, what is the source of their heat, and why do they soon disappear?

14. The point of a gimlet with which a hole has been bored is found to be hot. How do you explain this? Give other instances of the same sort.

15. Describe experiments to show that energy of motion can be converted into heat.

16. In a shipbuilding yard a machine pierces holes in iron plates by punching out circular fragments. Before beginning work the machinery and iron plates are quite cold. After the operation the circular fragments are too hot to hold in the hand. Why is this?

17. A brass button, when rubbed on a school-form, becomes hot. (*a*) What is the source of this heat?

The button is placed on a table, and in a few minutes it becomes cool. (*b*) What has become of the heat?

18. Describe the different ways in which heat may be conveyed from one place to another. Which mode of transmission is utilised in the ventilation of a sitting room?

19. It is often stated that heat and work are equivalent. Explain this in the case of a steam engine, and when the brake is applied in a railway train. How may the mechanical equivalent of heat be determined?

20. What happens when a gas is compressed considerably in a water-cooled cylinder, and then is allowed to expand? Mention any practical use of this that you know.

21. Heat may be described as a form of energy. Explain precisely what the statement means. Describe one method by which the amount of energy corresponding to the unit quantity of heat has been determined.

22. If a hot lamp-chimney be touched with a cold knife-blade it will probably crack. If a tightly-corked bottle full of water be put out of doors on a frosty night it will burst. Explain as fully as you can the reasons for these two results.

23. What is meant by convection? Illustrate your answer by sketches, taking the case of a vessel filled with water and heated from below, and explain why it is that convection is set up.

24. Explain why the polished fire-irons in front of a fire are sometimes found to be only slightly warm, while the blackened fender is too hot to be touched.

25. Describe experiments to show that good radiators are good absorbers of heat.

26. What relation would you expect to find between the reflecting and emissive powers of a substance? Give reasons for your answer.

Describe a method of comparing the emissive powers of different substances at the same temperature (e.g. 100° C.).

27. Describe and explain the phenomenon of the convection of heat.

Two test tubes are partly filled with liquid air; the lower part of one is surrounded by a vacuous space, the other by air. State and explain what you would observe.

28. Describe experiments illustrating the relation between heat and work, and explain one method of determining the mechanical equivalent of heat.

29. Describe experiments by which it has been shown that when work is done against friction, the quantity of heat produced is proportional to the amount of work so done.

30. Define the mechanical equivalent of heat.

Describe an experimental method of determining it, pointing out the quantities which must be measured for this purpose.

PART IV.

LIGHT.

CHAPTER XVIII.

PROPAGATION OF LIGHT. SHADOWS. PHOTOMETRY.

Light is a form of radiation.—In considering, in a part of the previous chapter, the ways in which heat can be transferred from one place to another, it was seen that the heat of the sun reaches the earth by radiation. These solar radiations comprise what collectively is called sunlight; they are conveyed in the form of waves through the medium ether, which is believed to pervade all space, and may be referred to conveniently as **ether-waves**. These ether-waves are of various lengths and can produce different effects. If they fall upon our bodies the longer waves may be absorbed, and the energy of the wave-motion become converted into **heat**; if they fall upon the retina of an eye, the shorter waves may produce a sensation of light, and the waves are then spoken of as **light**; falling upon a photographic plate or upon a green leaf, the shortest ether-waves may produce chemical effects, and these waves are referred to as **actinic**. But, in their passage through the ether, ether-waves do not give rise to any of these effects; they are simply waves transferring energy by wave-motion.

Opaque and transparent media.—Ether-waves are not transmitted through all media with equal facility. Substances such as iron and other metals, wood, or stone, through which the waves which produce the sensation of light cannot be transmitted, are termed **opaque**; others, such as glass, water, or air through

through a uniform medium in lines that were sometimes bent, there is no reason why we should not. Or, again, everyone knows that it is only necessary to put a small obstacle in the path of the light from a luminous body to shut out completely our view of it.

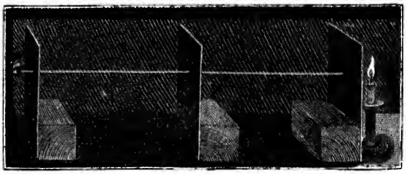


FIG. 150.—Experiment to show that light travels in straight line.

EXPT. 167. — **Rectilinear propagation.** Place three cards together and pierce a small hole through them. Support the cards as in Fig. 150. The light of the candle can be seen only when the holes are in a straight line. What is true of light applies equally to all other kinds of radiation.

Inverted images produced by a pin-hole.—When an object is viewed through a pin-hole camera, it is seen to be upside down upon the screen. Similarly, all images produced by a small aperture are inverted. This inversion is a direct consequence of the fact that light travels in straight lines. That this is really the case can be understood fully by the following simple considerations.

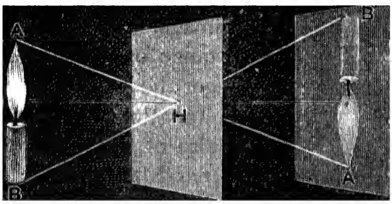


FIG. 151.—Explanation of the inversion of images seen through a pin-hole.

Let H, in Fig. 151, be the pin-hole, and AB the candle. Rays are sent out in all directions by every point of the candle, but of all the rays from one point, such as A, only that in the direction AH can pass through the hole and form an image A′. Similarly, the only ray from B which can get through the hole is BH, so an image of B is formed at B′. The same reasoning applies to any part of the candle, hence a complete inverted image is produced.

If the light which passes through a small hole in the shutter of an otherwise dark room be caught by a screen of cardboard, a coloured, inverted image of the sky and landscape will be seen.

By using a pin-hole camera, a photograph of the view can be obtained (Fig. 152). A pin-hole camera can be made from a light proof box having a minute hole at one end, or one can be purchased for a few pence. The bright circles of light seen under trees in summer are really images of the sun formed by the small spaces between the leaves.

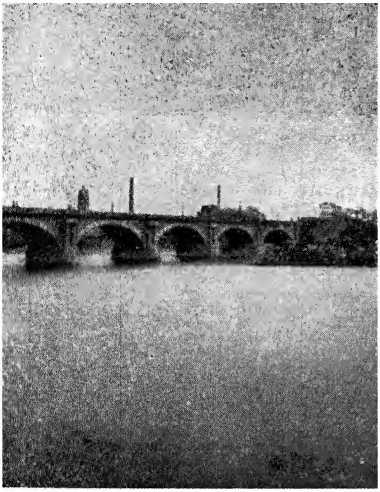


FIG. 152.—Photograph obtained with a pin-hole camera, by Mr. F. Butterworth.

Size of image produced by a pin-hole.—That the size of the image depends upon the distance of the screen from the pin-hole is proved practically by varying the distance of the screen from the pin-hole and measuring the length of the image. The greater the distance of the screen the longer the image. The reason for this alteration in the size of the image is a simple one. The rays of light from the top and bottom of the object travel through the pin-hole, and since one is travelling upwards and the other downwards, they will be farther apart the greater the distance they travel. Consequently, the image is longer the more the screen is moved from the pin-hole.

The relation between the sizes of the object and image according to their distances from the aperture is

$$\frac{\text{Length of object}}{\text{Length of image}} = \frac{\text{distance of object from aperture}}{\text{distance of image from aperture}}.$$

The larger the image the less bright it is, because the small amount of light which passes through the aperture is spread over a greater area in the case of the enlarged image.

Illumination due to overlapping of images.—When a pin-hole is made in the front face of a pin-hole camera, an image of the

bright object looked at is formed on the screen in the manner described in the preceding paragraphs. If a second hole be pierced, a second image is obtained. When the number of holes is increased steadily one at a time, the images, it is observed, start overlapping, at the same time becoming blurred. When the number of images has become considerable, no separate image can be distinguished; **diffused light**, as it is called, is produced, and the screen is illuminated in the ordinary way.

Intensity of light.—In proceeding from the source of illumination, light spreads out as indicated in Fig. 153, so that though each ray retains its original intensity the number of rays which illuminate a given area depends upon the distance of that area from the luminous source S. At twice the distance the rays are spread over four times the area, so their illuminating effect, as at M, is only one-fourth of what it is at *m*. The amount of light received from a luminous source is thus inversely proportional to the square of the distance from the source.

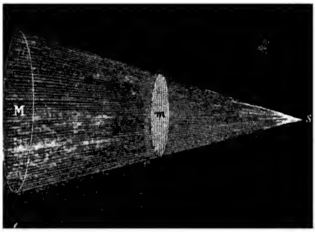


FIG. 153.—Intensity of light at different distances from the source.

Suppose L to be the rate per second at which luminous radiation is emitted uniformly in all directions from a point situated at the centre of a sphere of radius r cm. Since the area of the sphere is $4\pi r^2$, the quantity of radiation falling on each square centimetre of the sphere will be $L/4\pi r^2$. This quantity is termed the **intensity of illumination** of the surface; and it is evident that the illumination varies inversely as the square of the distance.

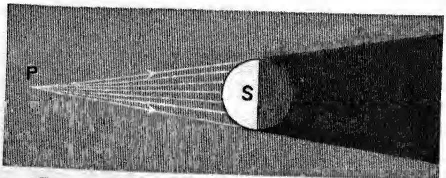


FIG. 154.—Shadow thrown by a luminous point.

Shadows. — Let P (Fig. 154) be a luminous point and S an opaque sphere intercepting the light which falls upon it. A screen held beyond S and at right angles to

the axis of the cone of rays will indicate a well-defined circular shadow of the sphere, all points of the screen within the shadow being protected totally from the rays.

If the source of light be large, as in S (Fig. 155), a more complicated shadow is obtained. Each point of S throws its own shadow cone; and by drawing the cones due to two points at the upper and lower edges of S, it is seen that only the space common to both these cones is protected completely from light; this space is indicated by the black cone terminating at the point P. When a screen AB is held between E and P, so that it is normal to the line joining the centres of S and E, there will be visible on its surface a black central circle called the **umbra**, surrounded by a ring of partial shadow, termed the **penumbra**. The appearance of the shadow is shown at *ab*.

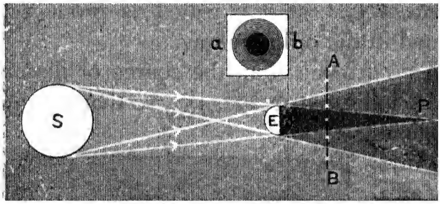


FIG. 155.—Umbra and penumbra, due to an extended source of light.

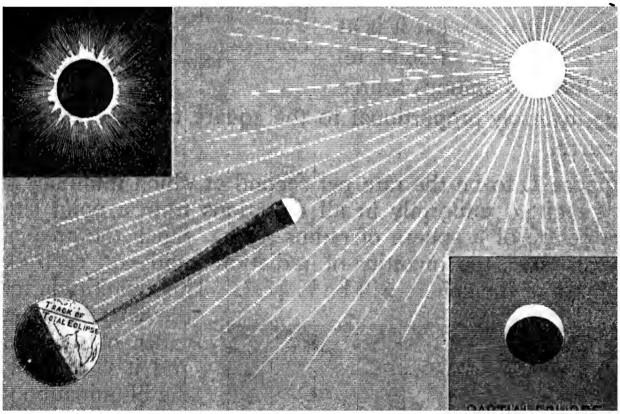


FIG. 156.—Eclipse of the sun, caused by the moon coming between the sun and the earth.

A shadow of this type is thrown by the light of the sun falling upon the moon. When this shadow falls upon the earth's surface, a *total* eclipse of the sun is seen from any point within the umbra, and a *partial* eclipse is seen from any point within the penumbra (Fig. 156). Similarly, the sun throws a shadow of the earth, and the moon is partially (or totally) eclipsed when it passes into the shadow.

Dimensions of shadow-cones.—The length of a shadow-cone, or the diameter of the cone at any point, can be determined by graphical construction or by a simple calculation. For instance,

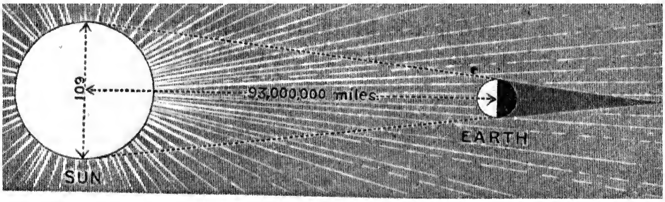


FIG. 157.—Shadow-cone of the earth.

Fig. 157 represents the shadow-cone caused by the sun shining upon the earth. The diameter of a cone at any point is directly proportional to the distance from the apex. Hence we have the relation

$$\frac{\text{Earth's diameter}}{\text{Sun's diameter}} = \frac{\text{Earth's distance from apex}}{\text{Sun's distance from apex}}.$$

If the earth's diameter be taken as unity, the sun's diameter is about 109 times greater. The earth's distance from the sun is 93,000,000 miles; and the distance (x) of the earth from the apex of the shadow may be found from the simple proportion:

$$\frac{1}{109} = \frac{x}{93{,}000{,}000 + x}.$$

The length of the earth's shadow is thus found to be 861,111 miles. Knowing the distance of the sun, the diameter can be calculated by the same principle based upon the fact that a half-penny, which is one inch in diameter, exactly covers up the sun's

disc when held at a distance of nine feet from the eye. For we have the proportion:

$$\frac{\text{Diameter of halfpenny}}{\text{Diameter of sun}} = \frac{\text{Distance of halfpenny}}{\text{Distance of sun}},$$

or, reducing inches to decimals of a mile,

$$\frac{0{\cdot}000019}{x} = \frac{0{\cdot}000019 \times 12 \times 9}{93{,}000{,}000}.$$

From this relation the sun's diameter is found to be about 860,000 miles. It is of interest to notice that the earth's shadow-cone has about the same length as the sun's diameter.

It is often easy to determine the size and shape of a shadow by a graphical construction as in the following example:

Example. A man 6 feet in height is standing 15 feet from a lamp 12 feet high: what is the length of the man's shadow upon the ground?

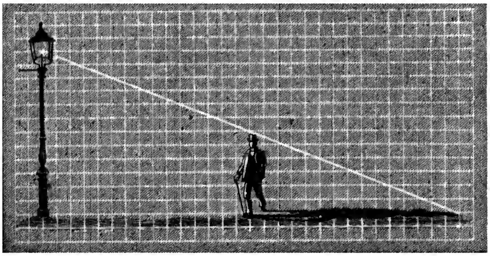


FIG. 158.—Length of shadow.

By drawing the given dimensions to scale, or on squared paper, as in Fig. 158, it will be found that the required length is 15 feet. The same result is obtained by calculation as in previous examples.

Velocity of light.—The ether-waves travel with a velocity of 186,000 miles per second. This rate of propagation has been determined in the case of light in several different ways, two of which will here be described briefly.

1. **By observations on Jupiter's satellites.**—The planet Jupiter, represented by J in Fig. 159, has several satellites or moons which revolve round Jupiter in a plane nearly coincident with that of the planet's orbit round the sun, and consequently one or other of these satellites frequently passes into the shadow of the planet thrown by the sun, and so becomes invisible to us. When the

earth E, and Jupiter J, are on the same side of the sun, let the time of disappearance of one of the moons into the planet's shadow be observed. Suppose the time at which the phenomenon should happen six months hence to be calculated by considering the period which the satellite takes to revolve around the planet. It is found that when the half-year has elapsed the

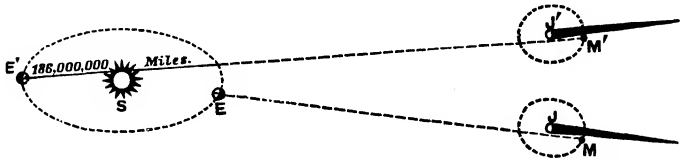


FIG. 159.—Romer's method of determining the velocity of light from observations of eclipses of Jupiter's satellites.

observed time of disappearance and the calculated time do not agree. The eclipse occurs about 1000 seconds late. This is because the earth's position relatively to Jupiter has undergone a change; it is now at E′ on the other side of its orbit, and the light-message has to travel across the orbit before reaching us. This extra journey takes 1000 seconds. The distance across the earth's orbit, that is, from E to E′, may be taken as 186,000,000 miles; and as light takes 1000 seconds to traverse this, the velocity is 186,000,000 divided by 1000 or about 186,000 miles per second.

2. **By terrestrial experiments.**—The velocity of light has been determined by various experimenters on the earth itself. We shall only describe the principle of the method employed by M. Fizeau in 1849 (Fig. 160). Two places about five miles apart were chosen and light sent from one station was reflected back to its starting point by a mirror which it struck normally at the more distant station. There was then interposed between the mirror and the source of light a toothed wheel, carefully constructed so that the width of the teeth and spaces between them were equal. This was placed near the source of light. It is clear that if the reflected light be received by a tooth of the wheel it will not reach an eye suitably placed on the same side of the wheel as the source of light. Moreover, if the wheel be rotated it can be given such a speed that there is always a tooth in the way to meet the light which has travelled to the mirror and back again. Similarly

the speed can be made of such a value that one of the spaces shall always fall in the path of light. When this is the condition of things it is easy to see that the wheel rotates through an angular distance equal to the width of a tooth in a time that the

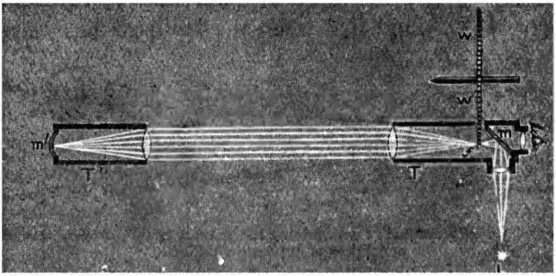


FIG. 160.—Fizeau's apparatus for determining the velocity of light. L, source of light; *w*, *w*, toothed wheel; *m*, plain glass mirror; *m'*, silvered mirror; *f*, place where the light strikes the toothed wheel.

light travels from the wheel to the mirror and back again to the wheel. The time occupied by the wheel in rotating the angular distance can be calculated at once from the rate of rotation, while the distance from the wheel to the mirror can be measured directly.

PHOTOMETRY.

Illumination of a surface.—When light is emitted from a point at the rate L, the illumination on a surface situated at a distance d, and at right angles to the path of the rays, is equal to $L/4\pi d^2$. Suppose two luminous points, emitting light at the rates L_1 and L_2 respectively, to be situated at a distance apart; if an opaque white screen be held between the points, at right angles to the line joining them, and if the screen be at distances d_1 and d_2 from the points, the illumination on the two sides of the screen will be represented by $L_1/4\pi d_1^2$ and $L_2/4\pi d_2^2$. If the screen be moved to and fro until the two sides are equally illuminated, then

$$\frac{L_1}{4\pi d_1^2} = \frac{L_2}{4\pi d_2^2},$$

or

$$\frac{L_1}{L_2} = \frac{d_1^2}{d_2^2}.$$

This principle serves as an accurate method for comparing the luminosity of different sources of light; and this comparison is termed **Photometry**.

Candle power.—In order to give a numerical value to the illuminating power of any source of light, it is necessary to have some standard source which can be taken as a unit. It is sufficient to mention here the simplest unit available, viz. the **standard candle.** This is defined as a sperm candle, weighing six to the pound, and burning 120 grains of wax per hour. The luminosity of a candle flame is influenced by the temperature and purity of the air, and therefore this unit is not absolutely constant. The ratio between the illuminating power of any source of light and that of a standard candle is termed the **candle power** of that source.

Rumford's shadow photometer.—In this photometer (Fig. 161) an opaque rod is placed in front of a vertical screen of unglazed paper (or of ground glass) and the two sources of light are placed so as to throw separate shadows of the rod upon the screen. The shadow due to one of the sources represents a strip of the screen which is illuminated by light from the other source only. The distances of the sources from the screen are varied until the shadows are equally dark, and then the above equation can be applied for comparing the luminosity. It is not necessary to have a completely dark room for experiments with this photometer.

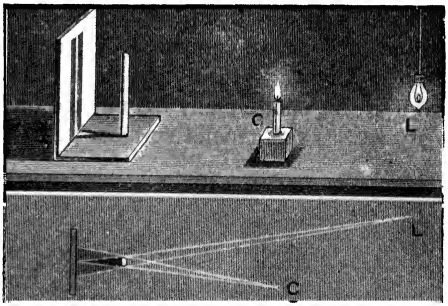


FIG. 161.—Rumford's shadow photometer.

EXPT. 168.—**Law of inverse squares.** Pin a piece of white paper upon a drawing board to act as a screen. Fix the drawing board at right angles to a table in a darkened room. In front of the screen place a vertical rod about 1 to 2 cm. in diameter (a retort stand will do). Beyond this, place to one side a candle fixed on a block of

wood, and to the other side four candles, fixed close together on a second block of wood. Notice that two shadows of the upright rod appear on the screen. Move the candles near each other so that the two shadows of the rod touch but do not overlap. Notice that one shadow, that cast by the four candles, is darker than the other. The latter shadow is illuminated by one candle, the other is illuminated by four candles. Now, move back the four candles until the shadows appear equally dark, that is, in equal contrast with the bright part of the screen. Then the four candles give just as much light to the screen as the single candle. Measure the distance of the single candle, and the mean distance of the four.

It will be found that the latter distance is twice the former, thus showing that at double the distance of a single candle four candles are required to give the same illumination.

EXPT. 169.—**Candle-power.** Using a standard candle, determine the candle-power of (i) an ordinary wax candle, (ii) a paraffin lamp, and (iii) a fish-tail gas burner. For this comparison, use the relation $L_1/L_2 = d_1^2/d_2^2$.

Bunsen's grease-spot photometer.—In this photometer the two sources of light are compared by placing them on either side of a screen having a grease spot in it. Its use depends on the fact that a grease spot equally illuminated on either side has the same brightness as the general surface. The light lost by transmission

FIG. 162.—Bunsen's grease-spot photometer.

through the translucent or semi-transparent spot in one direction is compensated for by the equal transmission in the opposite direction. The eye is indifferent whether the light it receives is due to a reflection from white paper or to transmission through a translucent spot. It observes that the brightness of the grease

spot is equal to that of the rest of the surface. The intensities of the two lights are again proportional to the squares of their distances from the screen.

EXPT. 170.—**Unequal illumination.** Obtain a piece of white paper Make a grease spot in the centre. Allow a light to shine on the paper. Observe that the grease spot is darker than the surrounding surface. Observe the paper by transmitted light. Notice that the grease spot is now brighter than the general surface.

EXPT. 171.—**Equal illumination.** Use the paper as a screen and illuminate one side of it by means of a candle, and the other with a lamp. Move the candle and lamp until the grease spot is barely distinguishable in point of brightness from the white surface near it. Measure the distance of the candle and lamp from the grease spot. Using the law of inverse squares, calculate the luminosity of the lamp in terms of the candle.

EXERCISES ON CHAPTER XVIII.

1. Describe a pin-hole camera, and explain, illustrating your answer by a diagram, how the image of a luminous object is formed by it.

What experiment would you perform to show why it is that the image first becomes blurred and then disappears when the size of the hole is increased gradually?

2. Three candles are placed quite close together in a row at the centre of a room, and a wooden rod is held in a vertical position at a distance of about a foot from the candles. Explain, giving diagrams, why it is that as the rod is moved in a circle round the candles the shadow cast on the walls is in some positions sharp and in others very ill defined.

3. The sun shines through a crack in the shutter of a darkened room. A person inside the room says that he sees a ray of light entering the room. Put his statement in a more accurate form. What can he really see?

4. A small opaque sphere is placed between a gas burner and a white screen, and when the gas is turned down so that the flame is very small it is found that the shadow cast on the screen is quite sharp, but on turning up the gas so that the flame is large, that the edge of the shadow is blurred. Explain the reason for this change, illustrating your answer by means of diagrams.

5. A man, $5\frac{1}{2}$ ft. high, is standing at a distance of 5 ft. from a street lamp, the flame of which is 9 ft. above a horizontal roadway. Find the length of the man's shadow.

6. Draw a diagram of the sun, moon, and earth during an eclipse of the moon. How far from the earth does its umbra extend if the diameters of the sun and earth are 800,000 miles and 7900 miles respectively, and if the distance of the earth from the sun is 92,000,000 miles?

7. The sun subtends the same angle as a halfpenny at a distance of 10 feet. Give a diagram showing the size and nature of the shadow of a halfpenny cast by the sun on a surface perpendicular to the rays at a distance of 5 feet from the halfpenny.

8. A shadow of a circular table 3 feet high and 3 feet in diameter, is cast on the ceiling of a room 10 feet high by a night light on the floor and vertically beneath one edge of the table. Give a diagram illustrating the formation of the shadow, and find its size and shape.

9. In a comparison of the luminous intensity of an incandescent electric lamp and of a standard candle, it is observed that the shadows were of equal intensity when the distances of these sources from the screen were 42 cm. and 15 cm. respectively. What was the 'candle-power' of the lamp?

10. The shadows of a vertical rod on a wall, at equal distances of 2 feet each from the rod, cast by two gas flames are observed to be equally dark when the flames are at distances of 6 and 4 feet respectively from the rod. Compare the illuminating powers of the flames.

11. Compare the intensities of the illumination produced on the floor of a room (i) when lit by a gas-lamp of 400 candle-power at a height of 16 ft., and (ii) when lit by an arc lamp of 1000 candle-power at a height of 40 ft.

12. (i) A standard candle is 210 cm. from a 16 candle-power electric lamp. Where should a screen be placed between them in order that its two sides may be illuminated equally?
(ii) Where else, along the line joining the lights, might the screen be placed and yet be illuminated equally by each source of light?

13. What is meant by the intensity of illumination on a surface? Which would give the greater intensity of illumination on the ground immediately beneath the lamp, (i) a 100 candle-power lamp at a height of 12 feet, or (ii) a similar lamp of 1200 candle-power 45 feet high?

14. How would you arrange an experiment to determine the percentage of light that is transmitted through a neutral tinted glass plate? If a plate of such glass allowed 40 per cent. of the light incident upon it to pass through, how much light would be transmitted by a plate of the same glass of four times the thickness, assuming no light to be lost by reflection at the surfaces in either case?

15. Two lamps are placed on opposite sides of a screen, and their distances from the screen so adjusted that the two faces of it are

illuminated equally. A semi-transparent sheet is then placed between one of the lamps and the screen, and it is found that the other lamp must be moved to twice its original distance from the screen in order that the two faces may be illuminated equally again. What fraction of the light falling upon it is cut off by the sheet?

16. Describe some way of comparing the candle-powers of two different sources of light.

How would you estimate by experiment the percentage of the light available which is lost by a person working in a room with a dirty window?

17. A candle is placed inside a box in a dark room. A small hole is cut in one side of the box, and a sheet of paper is held a short distance in front of the hole. Describe and explain the appearance seen on the paper.

18. A candle flame is placed on one side of an opaque screen in which there is a small hole. Explain—and illustrate your explanation by a diagram—why it is that if a piece of white paper be held on the other side of the screen an image of the candle is seen on it.

Is the result affected by (i) the shape, (ii) the size of the hole?

19. Describe a method by means of which the velocity of light has been determined.

CHAPTER XIX.

REFLECTION AT PLANE SURFACES.

Reflection and refraction.—When rays of light fall upon the surface of any medium, some may be reflected back from the surface, and others may be transmitted through the medium. If transmitted, they may be more or less absorbed, as in the case of a translucent medium, or as in the case of coloured water or glass, which rapidly absorb light of certain colours but readily transmit those of other colours. As a rule, the transmitted rays are **bent**, or **refracted**, away from their original path.

Reflection of light rays may happen in two ways, either **regularly** or **irregularly**. In the first case, they are turned back according to simple rules, while in the second there is no uniformity about the direction of reflection. In Fig. 163 the surface AB of a medium which is denser than air is supposed to be a perfect plane, and it gives rise to regular reflection; the surface BC is uneven, and the reflection from it is irregular. The diagram shows also how some of the rays may be transmitted through the denser medium.

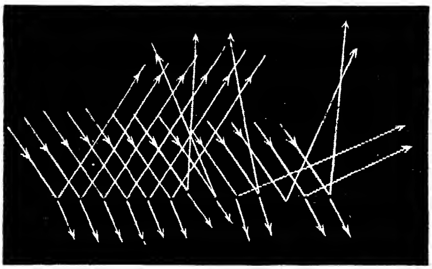


FIG. 163.—Regular and irregular reflection.

The page on which this explanation is printed appears to be white because—owing to the roughness of the paper—of the

irregular reflection of the light which falls upon it. Or, if we powder a sheet of glass, the powder seems to be white for a similar reason; there are, in these and similar cases, many surfaces formed from which irregular reflection takes place.

Laws of reflection of light.—Light is reflected regularly from a plane mirror—that is, a flat reflecting surface. Such a mirror can be made from a variety of substances, but the most common is bright metal or silvered glass.

The angle between the path of the incident ray and the normal to the mirror (that is a perpendicular to the mirror at the point where the ray strikes it) at the point of incidence is termed the **angle of incidence.** The angle between the normal and the path of the reflected ray is termed the **angle of reflection.**

There is a definite connection between the angles of incidence and reflection, and it can be expressed as follows:

The paths of the incident and reflected rays are (i) **in the same plane as the normal at the point of incidence,** (ii) **at equal angles on opposite sides of the normal.**

It can be proved by experiment also that when a wave strikes a reflecting surface *normally*, *i.e.* having travelled along the normal, it is reflected back upon the same line.

EXPT. 172.—**Pin method of proving laws of reflection.** Fasten a sheet of white paper on a drawing board, and support in a vertical position on the paper a strip (about 2 inches by 1 inch) of good plane mirror AB (Fig. 164). (The mirror may be supported by fixing a small wooden cube to the back of the mirror with wax.) Fix two pins, P and Q, vertically in the board and approximately in the positions shown. View the images of these pins by looking in the direction SR, and move the eye until the image of Q overlaps that of P. Keep the eye in this position, and insert two other pins at points such as R and S, and so that these pins together with the images of P and Q all appear to be in one straight line.

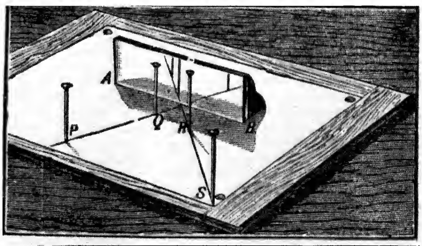


FIG. 164.—Pin method of determining the laws of reflection of light.

Draw a pencil line at the back of the mirror. Remove the mirror; draw the lines PQ and SR and produce them until they meet (Fig. 165). They should meet at a point O approximately on the line of the silvered surface. Draw a line ON normal to the mirror at O. Measure the angle of incidence PON, and the angle of reflection SON, and compare them. Repeat the experiment two or three times, with different angle of incidence in each case.

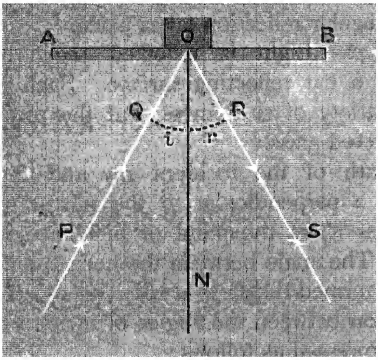


FIG. 165.—Construction to illustrate laws of reflection of light.

Equality of distances of object and of image from a plane reflecting surface.—The two rules of reflection enable the formation of an image by a plane mirror to be understood easily. By a simple geometrical construction it can be shown that the image of a point is at the same distance behind a plane mirror that the point itself is in front of the reflecting surface.

Let MM′ (Fig. 166) be a horizontal section through a plane mirror, and O a bright point like the head of a pin. Let O*r* and O*p* be any two rays from O which are reflected by the mirror along the paths *pq* and *rs*. When an eye is situated at e_1 these rays give rise to the impression that the source of the rays is at some point I behind the mirror. Draw *pn*, a normal to the mirror at the point *p*.

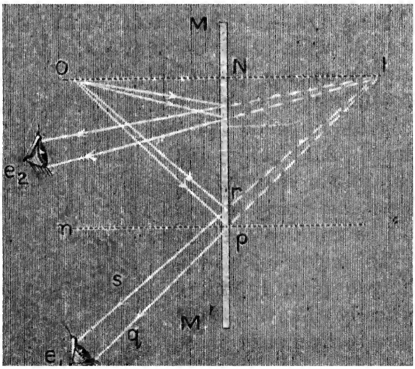


FIG. 166.—Reflection by a plane mirror.

Since $\angle Opn = \angle qpn$,
then $\angle Opr = \angle qpM' = \angle Ipr$.
Similarly $\angle OrM = \angle IrM$;
therefore $\angle Orp = \angle Irp$.

Hence, the triangles Orp and Irp are equal; and $Or = Ir$. Join the points O and I by a line passing through the mirror at N. Then, in the triangles OrN and IrN, the sides Or and Ir are equal, the base Nr is common, and $\angle OrN = \angle IrN$. Hence, the triangles are equal in all respects (*Euc.* I. iv.). Therefore,

$$ON = IN; \text{ and } \angle ONr = \angle INr.$$

Hence, **the image is situated on the normal to the mirror drawn from the source of the rays; and its distance behind the mirror is equal to that of the object in front of the mirror.**

Fig. 166 represents also how the image will appear to be situated at the same point I when the eye is in any other position, such as e_2.

The above result applies equally to the formation of the images of objects, which can be regarded as accumulations of small material particles to which the construction given already for a point may be applied.

EXPT. 173.—**Image and object.** Support a strip of plane mirror AB (Fig. 167) in a vertical position on a sheet of paper, and fix a pin vertically at O. View the image of the pin in the direction PQ, and insert pins at P and Q. Similarly, view the image in the direction RS, and insert pins at R and S. The image is situated somewhere along the line PQ produced, and also somewhere along the line SR produced. Hence it can only be at the point where these lines intersect. Remove the mirror, and produce the lines PQ and SR to meet at I. Draw normals to the mirror from I and O. These normals should be in the same straight line, and equal in length.

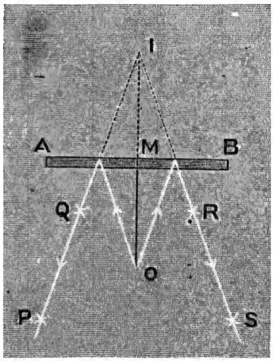


FIG. 167.—Expt. 173.

EXPT. 174.—**Second method of finding an image of an object.** Use the same mirror as before. If it be 1 inch high, use pins about 2 inches long. Insert a pin at O (Fig. 167), and view its image just to one side

of the normal. Fix a pin O′ in the paper behind the mirror so that the upper part of this pin may appear to be a continuation of the image of the pin at O. Move the eye slightly to the right and then to the left; if the upper part of O′ appears to move relatively to the image of O in the *same* direction as the eye, then O′ is *too far* from the mirror. Adjust the position of O′ until, from whatever direction it is viewed, it always appears to be continuous with the image of O. *The pin* O′ *then occupies the position of the image of* O.

This second method of determining the position of an image of an object is known as the **Parallax method.** Parallax may be defined as the apparent change in the position of an object due to a change in the position of the observer. It may be demonstrated thus: Place two rods, A and B, vertically, in line with the eye, and with B just behind A. If the eye is moved to the *right*, then B appears to move to the *right* of A (or, it may be said that A appears to move to the *left* of B). If the eye is moved to the *left*, then B appears to move to the left. **The further object appears to move in the same direction as the observer's eye.** If B is vertically over A, then they appear to be in the same straight line, whatever the direction of view may be.

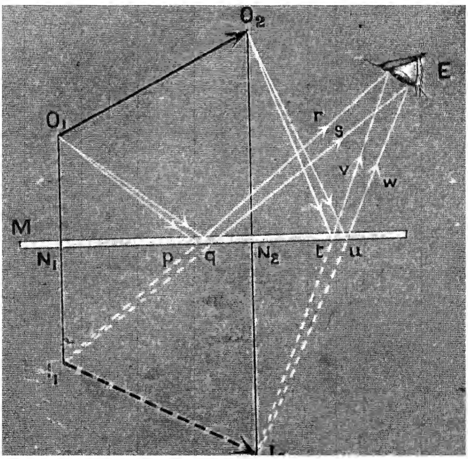


Fig. 168.—Image of an extended object, viewed by means of a plane mirror.

Image of an object in front of a plane mirror.—Let M (Fig. 168) represent a plane mirror, and O_1O_2 an object placed in front of the mirror. From O_1 and O_2 draw normals, O_1N_1

and O_2N_2, to the reflecting surface; and produce these to points I_1 and I_2, such that $N_1I_1 = N_1O_1$, and $N_2I_2 = N_2O_2$. Then I_1I_2 is the image of the object O_1O_2. The path of the pencil of rays which originates from O_1 and enters an eye situated at E is obtained by drawing the pencil (I_1pr and I_1qs) which *appears* to originate from the point I_1, thus locating the points p and q on the reflecting surface at which the rays from O_1 are reflected. Join O_1p and O_1q. The rays O_1pr and O_1qs represent the pencil of rays by which the image of O_1 is seen. In a similar manner, the rays O_2tw and O_2uw represent how the rays from O_2 are reflected so as to give the image at I_2.

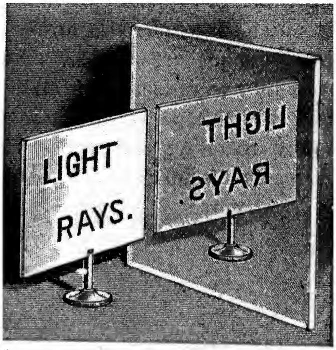


FIG. 169.—Lateral inversion, due to a plane mirror.

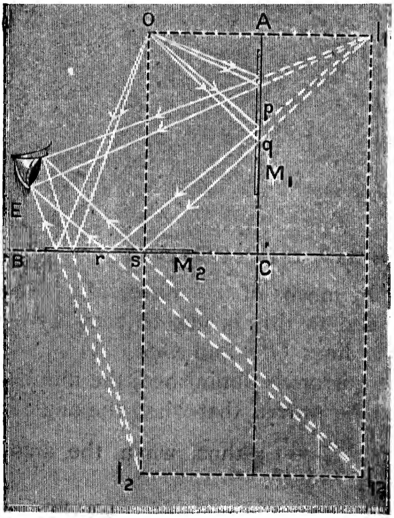


FIG. 170.—Multiple images, due to two mirrors at 90°.

It is evident from this diagram, that an object, when viewed by means of a plane mirror, undergoes **lateral inversion**: the left-hand side of the object when viewed directly, appears to be the right-hand side when viewed by means of the mirror. This result is rendered more apparent when a page of printed matter is viewed in a mirror; each letter, and the sequence of the letters, is reversed sideways (Fig. 169).

Images formed by inclined mirrors.—Fig. 170 represents two plane mirrors, M_1 and M_2, placed vertically with their reflecting

surfaces coinciding respectively with two lines, AC and BC, which are at right angles to each other. When a luminous point O, situated within the angle ACB, is viewed by an eye at E, three images are visible. The images I_1 and I_2 are obtained each by *one* reflection, from the mirrors M_1 and M_2 respectively. The third image I_{12} is obtained by rays which are reflected *twice* before reaching the eye. This third image may be regarded either as an image of I_1 obtained by means of the mirror M_2,

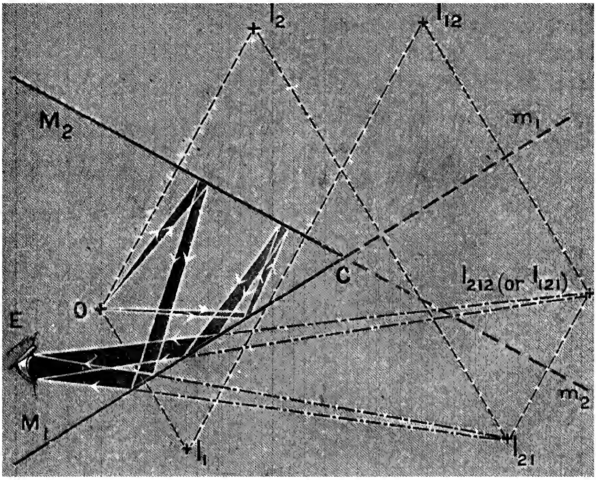


FIG. 171.—Multiple images in two mirrors inclined at 60°.

or as an image of I_2 obtained by means of the mirror M_1. The paths of the rays which give rise to this image, as seen by an eye at E, are indicated by the lines O*pr* and O*qs*.

When the angle between the mirrors is diminished, the number of images is increased. It can be proved that when the angle is $\theta°$, the number of images is $\frac{360°}{\theta°} - 1$; thus, when the angle is 60°, the number of images is 5.

Fig. 171 indicates the position of the five images, of a luminous point O, obtained by means of two mirrors CM_1 and CM_2 inclined at 60°. It is important to remember that any image gives rise to a second image when it is situated in *front* of a reflecting surface or in front of the surface produced. For this

reason the reflecting surfaces are produced in the directions Cm_1 and Cm_2. The images I_1 and I_2 are given by single reflections from the mirrors CM_1 and CM_2 respectively. The image I_1 gives rise to a second image I_{12} by reflection from the mirror CM_2, and the image I_2 gives rise to a second image I_{21} by reflection from the mirror CM_1; in both these cases, the rays of light undergo *two* reflections before reaching the eye E, and the diagram indicates the path of the rays which give rise to the image I_{21}. Finally, the images I_{21} and I_{12} give coincident images at I_{212}, the former by means of the mirror CM_2, and the latter by means of the mirror CM_1; the path of the rays which give rise to this image are shown in the diagram, and it will be noticed that the rays undergo *three* reflections before reaching the eye. No further images are generated, since I_{212} is situated *behind* the reflecting surfaces of both mirrors. It is interesting to bear in mind that the object and all the images are situated on the circumference of a circle described round C as a centre.

EXPT. 175.—**Positions of images formed by inclined mirrors.** Fasten a sheet of paper on a drawing board, and draw two lines inclined at 60°. Place two strips of mirror, similar to those used in Expt. 172, so that their reflecting surfaces coincide with these lines. Fix a pin vertically at any point such as O (Fig. 171). In the first instance, count the number of images. Determine the position of each image by the method of parallax. With centre C, and radius CO, describe a circle; observe whether each image is situated on the circumference. Make a careful diagram showing the paths of the rays giving rise to at least two of the images, assuming the eye to be placed in any given position.

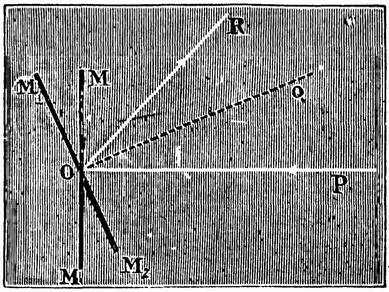


FIG. 172.—Effect of rotating a mirror.

Rotation of a mirror.—**When a mirror rotates, the angle through which the reflected ray moves is twice the angle through which the mirror moves.**—This fact is utilised in the **Sextant,** an instrument used by surveyors and navigators to measure the angle subtended by two distant objects.

Knowing the laws of reflection, the effect of rotating a mirror can be arrived at very easily by simple geometry. Thus, let MM

(Fig. 172) be a plane mirror, capable of rotation round an axis at O, and let PO be a ray of light incident on the mirror at O and normal to its surface. The ray is reflected back along its previous path.

If the mirror be rotated round O into the position M_1M_2, the angle through which it is rotated is equal to POQ, since this is the angle between the normals to the mirror in its initial and final positions.

When the mirror is at M_1M_2, the reflected ray is represented by OR; and, by the rotation of the mirror, the reflected ray has been rotated through the angle POR.

By the fundamental law of reflection,

$$\angle QOR = \angle POQ,$$

or

$$\angle POR = 2\angle POQ.$$

Hence, the angle through which the reflected ray moves is twice the angle through which the mirror moves.

EXERCISES ON CHAPTER XIX.

1. State the two laws in accordance with which a ray of light is reflected by a smooth surface, and describe experiments by which you would demonstrate the truth of each of these laws.

2. What is an inverted image? If the capital letter F were drawn on paper and held in front of a mirror, how would you have to draw the letter on the paper and how hold the paper, in order that the image of the letter in the mirror should present its ordinary aspect?

3. By moving a fragment of looking-glass, a boy finds that he can throw images of the sun up and down the walls and ceiling of a room. Where must he stand to be able to do this? Show by a diagram that the angle through which the image moves is twice as large as the angle through which the boy moves the glass.

4. What deviation is produced by reflection at a plane surface when the angle of incidence is 60°?

5. Make a measured drawing showing the positions of all the images of a luminous object placed between two plane mirrors inclined at 45°.

6. Two plane mirrors are inclined at an angle of 60°; give a carefully drawn diagram showing the position of the images of a luminous object placed so that the plane through it and the line of intersection of the mirrors makes an angle of 45° with one of the mirrors.

7. A mirror hangs on one of the walls of a room; show by means of carefully drawn diagrams that an observer will see by reflection more and more of the room behind him as he approaches the mirror.

8. State the laws of reflection.

Prove that a man can see the whole of his person in a mirror the length of which is half his own height.

9. Draw a diagram to show what images of an object, placed between two plane mirrors that make an angle of 50° with one another, are formed by reflection.

10. How would you show by experiment that the rays from a luminous point proceed after reflection by a plane mirror as if they come from a point as far behind the mirror as the luminous point is in front?

11. A horizontal beam of parallel light is reflected by a vertical plane mirror. The mirror, remaining vertical, is turned through a small angle. What is the relation between the angle through which the mirror is turned and the angle between the initial and final directions of the reflected beam? Give reasons.

12. Two mirrors are placed at right angles to one another. In a plane at right angles to the mirrors an arrow is placed, lying on the bisector of the angle between the mirrors. Draw a diagram to show the positions of the images of the arrow formed by the mirrors.

CHAPTER XX.

REFRACTION AT PLANE SURFACES.

Refraction of light.—Up to the present, rays of light have been supposed to be moving through a uniform medium. When this is so, as has been seen, light travels in straight lines, and, if it meets a reflecting surface, it is turned back, according to the laws described in the last chapter. When, however, the light passes from one medium into another of a different optical density, the propagation of the wave is usually no longer rectilinear: except in those cases where the incident ray is normal to the surface of separation between the two media, the passage from one medium into the other is accompanied by a bending of its path. This bending is known as **refraction**, and the ray is said to be **refracted**.

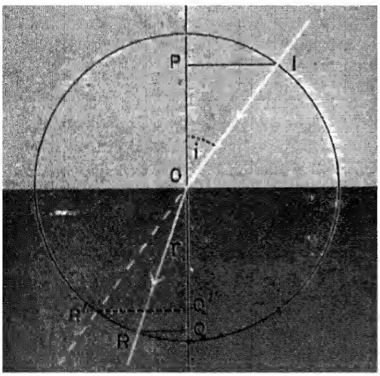


FIG. 173.—Law of refraction.

Laws of refraction.—In Fig. 173 the shaded lower part of the diagram represents a denser medium than the unshaded upper portion. The word denser is used here, and in similar connections, to mean optically denser, and must not be confused with

IO represent a ray passing from the rarer to the denser medium, or the ray incident on the surface of the denser medium at O. The angle IO makes with the normal at O is the **angle of incidence.** The ray is bent; instead of continuing its course in a straight line along OR′, it is refracted and travels in the direction of OR, which represents the refracted ray, the angle ROQ being the **angle of refraction.** The angle ROR′, which represents the amount the ray has been turned out of its original direction, is termed the **angle of deviation.** Let a circle be described with the centre O and any convenient radius, and from the points where it cuts the incident and refracted rays, let perpendiculars be drawn to the normal as in Fig. 173. A perpendicular is also dropped from the point R′. It is clear from geometry that the perpendicular R′Q′ is equal to the perpendicular IP. The ratio between the lengths of R′Q′ and RQ is constant for the same two media, *e.g.* air and water, whatever the angle of incidence. This ratio is called the **index of refraction** of the denser medium, or the **refractive index,** and it is usually denoted by the symbol μ. Its value for air and water is about $\frac{4}{3}$; for air and glass approximately $\frac{3}{2}$, depending upon the kind of glass.

The laws of refraction may be expressed thus:

1. **The incident and refracted rays are on opposite sides of the normal at the point of incidence, and in the same plane as the normal.**

2. **If a circle be described about the point of incidence, and perpendiculars be dropped upon the normal from the intersections of this circle with the incident and refracted rays, the ratio of the lengths of these perpendiculars is constant for any two given media.**

The student who is familiar with the elements of trigonometry will find the following statement of the second law to be more convenient: **The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for any two given media.** This may be written $\mu = \sin i / \sin r$, where i and r are the angles of incidence and of refraction respectively, and μ is the refractive index. It can be proved experimentally that if the direction in which the light is travelling be reversed, the path of the ray remains unaltered. Consequently, Fig. 173 may be used to represent the refraction of a ray proceeding from a dense into a rarer medium; and it is evident that the refracted ray is bent away from the normal. If, in this case,

i' and r' represent the angles of incidence and refraction respectively, then

$$\frac{\sin i'}{\sin r'} = \frac{\sin r}{\sin i} = \frac{1}{\mu}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

or

$$\mu = \sin r'/\sin i'.$$

EXPT. 176.—**The refractive index of glass.** Spread a sheet of white paper upon a drawing board, and draw a straight line on the paper. Lay a thick glass slab (about $10 \times 8 \times 1{\cdot}5$ cm.; *rectangular edges*) on the paper with its edge AB (Fig. 174) upon this line. Close to the edges CD and AB insert pins P and Q, so that the line PQ is oblique to AB. Look at P *through the block*, and move the eye until Q covers the image of P. Insert a third pin, R, in such a position that it appears to be in line with P and Q. Remove the block; draw the normal MM'; with Q as centre, describe a circle, 4-5 cm. radius. Drop perpendiculars LM and KM', and measure their lengths. The ratio LM/KM' is the *refractive index* (μ) of glass.

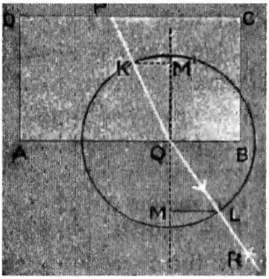
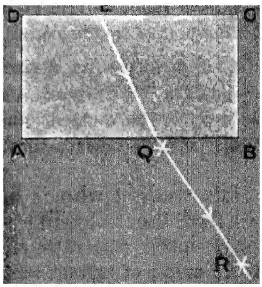


FIG. 174.—Refraction by glass.

Replace the block, alter the position of the pin P, and make another determination of μ.

EXPT. 177.—**The refractive index of water.** In Fig. 175 E is the vertical edge of a strip of paper fastened outside a glass trough ABCD, containing water. Look at this edge, through the trough of water, and trace the path of the rays, outside the trough, by means of two pins, Q and R. Determine the refractive index by the same construction as used in previous experiment.



In speaking of the *refractive index* of any material it is always assumed that the ray of light proceeds *from* air *into* the material, and not in the reverse direction.

It will be shown subsequently that when light is proceeding from a dense into a rarer medium there is no emergent ray if the

angle of incidence exceeds a certain limit; in such a case all the light is reflected back into the denser medium.

Geometrical construction for refraction at a plane surface.—Let IO (Fig. 176) be an incident ray at O, passing into an optically denser medium of which the refractive index is $\frac{4}{3}$. With O as centre describe a circle, of any radius, cutting the incident ray at I. From I draw IN perpendicular to the refracting surface. Divide ON into *four* equal parts, and measure off ON′ equal to *three* of these parts. Draw N′R perpendicular to the refracting surface, and cutting the circle at R. OR is the direction of the refracted ray.

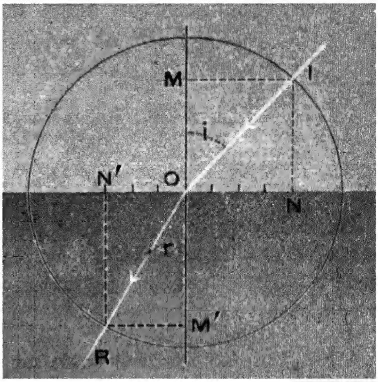


FIG. 176.—Refraction of incident ray passing into an optically denser medium.

The correctness of this construction is evident, since, by definition, the refractive index is equal to the ratio of the perpendiculars IM and RM′; and

$$\frac{IM}{RM'} = \frac{ON}{ON'} = \frac{4}{3}.$$

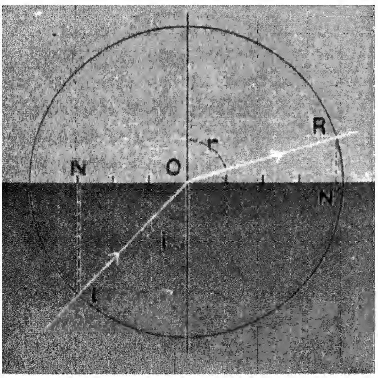


FIG. 177.—Refraction of incident ray passing into a less dense medium.

Fig. 177 represents the construction when the ray passes from the denser into the less dense medium. In this case ON is divided into three equal parts, and ON′ is measured off equal to four of these parts.

When the angle *i* is increased slightly, so that N′ coincides with the circumference of the circle, the refracted ray will coincide with the surface of separation. Any further increase in the angle of incidence will result in all the light being reflected back into the denser medium. This

limiting value for the angle of incidence is termed the **critical angle** for the denser medium.

The critical angle.—When a ray of light is proceeding from a dense into a rarer medium, the maximum value which the angle of refraction can have is 90°. Since $\sin 90° = 1$, the greatest possible value of the angle of incidence such that a refracted ray may be formed is given by the equation

$$\frac{\sin i'}{\sin 90°} = \frac{1}{\mu},$$

or

$$\sin i' = \frac{1}{\mu}.$$

Hence, for any dense medium **the critical angle is the angle of which the sine is equal to the reciprocal of the refractive index of the medium.** The critical angle for water is about 49°, and for crown glass 42°.

EXPT. 178.—**The critical angle for glass.** Fasten a sheet of paper on a drawing-board, and place a glass prism ABC (Fig. 178) on the paper. Trace the outline of the prism on the paper. Fix a pin O vertically in the board and touching the face AC of the prism. Place the eye to the right of B and look along the edge BA; move the eye gradually towards C and look for an image of the pin O reflected from the surface AB. When the line of sight corresponds with a direction $p_1 p_2$ the image will become dim; and if the eye be moved slightly towards C the image will disappear. Mark with pins, p_1 and p_2, the direction in which the image is barely visible. Remove the prism; join the points p_1 and p_2, and produce the lines so as to cut BC at Q. Since AB acts as a mirror, the rays after reflection will proceed as though coming from a point i which is behind AB and at a distance equal to that of O in front of AB. Join iQ. The intersection of this line with AB gives the point P from which the rays are reflected. Join OP. Thus

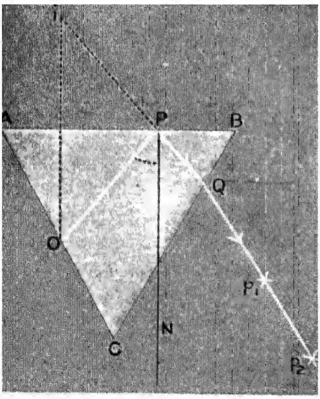


FIG. 178.—Determination of the critical angle for glass.

OPQ, the path of the rays within the glass, is obtained. Draw a normal PN at P. Then OPN is the critical angle.

In the preceding experiment, the rays of light by means of which the pin is seen when viewed from the direction p_1p_2 are said to undergo **total internal reflection**. This phenomenon suggests how a glass prism, the angles of which are 45°, 45°, and 90°, may be used as a mirror (Fig. 179). The prism is placed so that the incident rays are normal to one of the mutually perpendicular faces. The angle of incidence on the hypotenuse is therefore 45°; and, as this is greater than the critical angle, the rays are totally reflected and emerge from the prism normally to the third face. This device serves as a more perfect mirror than a glass plate which is silvered at the back, since, in the latter case, some of the light is reflected from the front surface, and some from the silvered surface, resulting in the image being made more or less indistinct.

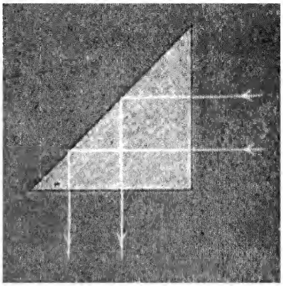


FIG. 179.—Total internal reflection by a prism.

Fig. 180 represents rays of light proceeding from a luminous point A beneath the surface of water. Only those rays the

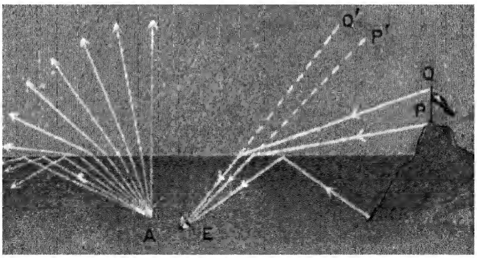


FIG. 180.—Effects due to total internal reflection in water.

angle of incidence of which is less than the critical angle emerge above the surface of the water; all others are totally reflected into the water. Similarly, to an eye situated at E below the

surface, objects above the water appear to be concentrated within a cone, of which the angle is equal to twice the critical angle; thus, the point P will appear to be situated at P′, and the point Q at Q′. Outside the boundaries of this cone objects beneath the surface of the water will be visible.

Refraction through a transparent plate.—The term plate is used to describe a slab of material with parallel faces. In Fig. 181, SP represents a ray incident at P on the side of a plate ABCD. Within the plate the ray is deviated along the path PT. On emerging from the plate the ray is deviated along the path QR; the deviation in this case being equal in amount, but opposite in direction, to the first deviation at P. Hence **in a transparent plate there is no final angular divergence, but the path of the ray is laterally displaced.** It is evident that the angle *e* of emergence must be equal to the angle *i* of incidence, since the angles *r* and *r′* are equal.

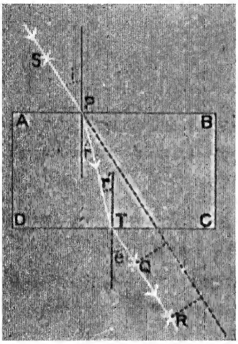


FIG. 181.—Refraction through a transparent slab.

EXPT. 179.—**Parallelism of emergent and incident rays.** Place a slab of glass ABCD (Fig. 181) upon a sheet of paper and trace its outline with a pencil. Insert a pin P near the edge AB. View the pin in an oblique direction, such as QR, and insert pins Q and R in line with the image of P. Finally, insert a pin S in line with R, Q, and P. Remove the slab; draw a line through S and P, also through R and Q cutting the slab at T. Join PT. Draw perpendiculars from Q and R on to SP produced. Measure these perpendiculars and note whether they have the same length.

The image of a point by refraction at a plane surface.—So far we have considered the refraction of a single ray only. In order to find the relative positions of an object and of its image formed by refraction at a plane surface we must consider the refraction of a small *pencil*, or assemblage, of rays.

Suppose O (Fig. 182) to be a luminous point situated in the denser of two media (X and Y), and let OCD be a narrow pencil of rays with its axis ON normal to the surface separating the two

media. The paths of the rays after deviation appear to an eye looking downwards along the normal to diverge from a point I. The paths of the rays can be determined by geometry, if the refractive index between the two media is known.

If μ be the refractive index when the rays pass from X into Y, then $1/\mu$ is the refractive index when the rays pass from Y into X. By drawing a normal at C, it is evident that the angle CON is the angle of incidence of the ray OC, and that CIN is the angle of refraction. Then

$$\frac{1}{\mu}=\frac{\sin \mathrm{CON}}{\sin \mathrm{CIN}}=\frac{\mathrm{CN}}{\mathrm{CO}}\Big/\frac{\mathrm{CN}}{\mathrm{CI}}=\frac{\mathrm{CI}}{\mathrm{CO}},$$

or

$$\mu=\frac{\mathrm{CO}}{\mathrm{CI}}.$$

But when the pencil is *very narrow*, then CO and CI are approximately equal to NO and NI respectively. Hence

$$\mu=\frac{\mathrm{NO}}{\mathrm{NI}}.$$

Or, **the ratio between the true distance and the apparent distance of the point below the surface of separation is equal to the refractive index.**

Hence, as the refractive index of water is 4/3, an observer looking straight down upon the surface of water sees an object in the water at three-quarters its actual distance from the surface.

This result is only true for small pencils of rays which are nearly *normal* to the surface. If the pencil entering the eye strikes the separating surface obliquely, the apparent position of the point is altered considerably; thus, in Fig. 182, the rays O*c* and O*d* are refracted so as to appear to diverge from a point I[1].

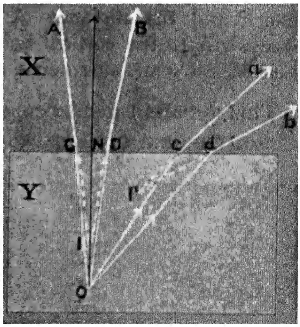


FIG. 182.—Apparent change of position caused by refraction.

EXPT. 180.—**Refractive index of water, by measurement of apparent depth.** Fill a deep glass cylinder with water, and drop to the bottom of the cylinder a small opaque object (*e.g.* a short piece of thick copper wire). Clamp a narrow

glass tube, drawn out to a jet, vertically above the surface of the water. Connect the tube to the gas supply, fix it so that the jet is horizontal, and light the gas at the jet adjusting the supply so that a *small* yellow flame is obtained. View the arrangement vertically downwards and observe whether there is any parallax between the immersed object and the image of the flame reflected from the

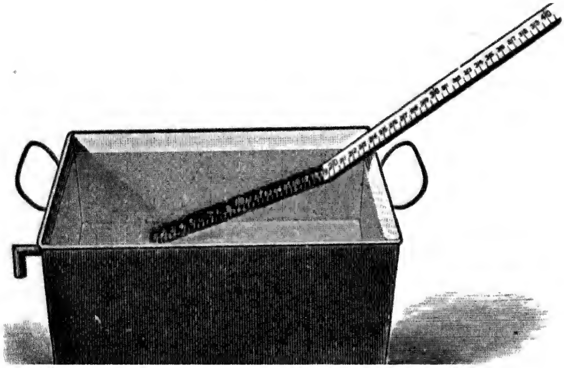


FIG. 183.—Apparent bending and shortening of an immersed rod.

surface of the water. Adjust the distance of the jet above the water until there is no parallax. In this position the apparent depth of immersion of the object must coincide with the position of the flame's image, and the distance of the latter below the surface is equal necessarily to the vertical height of the jet above the surface. Take the necessary measurements, and calculate the refractive index.

This apparent displacement of a source of light immersed in a denser medium explains why a straight rod partly immersed slantwise in water (Fig. 183) appears to be bent and shortened just at the surface of the water, when the stick is viewed from one side.

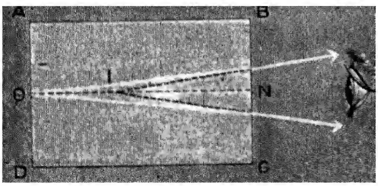


FIG. 184.—Expt. 181.

EXPT. 181.—**Refractive index of glass, by locating the image due to refraction at a plane surface.** Lay a slab of glass ABCD (Fig. 184) on a sheet of paper, and fix a pin vertically at O in contact with one edge of the

glass. View the pin through the glass from a position along the normal ON and beyond the opposite edge. That portion of the pin which is seen through the glass will appear to be at some point I. Determine the position of this point by holding another pin vertically with its point touching the glass and moving the point to-and-fro along ON until a position is found where the movable pin appears to remain continuous with the image of O when the eye is moved slightly to right and left.* Measure the distance IN and ON, and calculate the refractive index of glass.

Refraction through a prism.—When a wedge-shaped piece of glass, or a prism, as it is called in optics, is interposed in the path of a ray of light from a small hole in the cap of a lantern, it is easy to see, by watching the image of the hole on a screen, that the image moves in a direction towards the base of the prism. This is because the ray is bent by its passage through the prism, so that on its emergence from the glass it continues in a new path inclined towards the base of the prism. The amount of bending experienced by the ray of light depends, among other conditions, upon the angle between the inclined sides of the prism meeting in its edge, or **the angle of the prism**, as it is called.

In Fig. 185 let the triangle ABC represent a section of the prism at right angles to its faces, such as we should see by looking at the

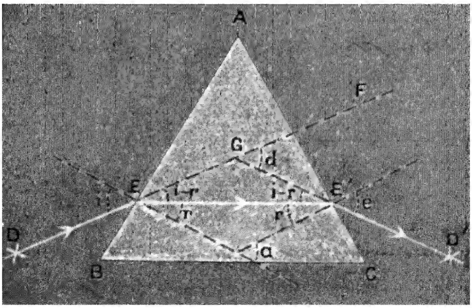


Fig. 185.—Refraction of a ray of light through a prism.

end of it. Suppose DE is a ray of light striking the face AB of the prism. The light on entering the prism passes from the air

* For description of this parallax method, see p. 252.

into the glass, or *from a rarer into a denser medium*, and is bent *towards* a line drawn perpendicular to the face of the prism at the point where the ray of light strikes it. It consequently travels along the line EE′ until it reaches the face AC of the prism. Here it passes from the glass into the air, *i.e. from a denser into a rarer medium*, and is, in such circumstances, bent *from* the perpendicular, and travels along the line E′D′. In every such passage through a prism it is noticed that the light is always bent or refracted towards the thick part of the prism.

EXPT. 182.—**Pin method of tracing deviation by a prism.** Stand a prism upright, that is, upon one of its ends, upon a piece of white paper. Stick two pins into the paper in positions such as D and E (Fig. 185), E being as close as possible to the face of the prism. Place two more, E′, D′, on the opposite side of the prism, so that the four appear in a straight line when looking through the prism. Draw the outline of the prism ABC, and then take away the prism and the pins and connect the pin-holes as shown in the diagram. It will be found that the ray is bent towards the base of the prism both when it enters and emerges. Measure the angle FGD′, which is termed the *angle of deviation* (*d*); also measure the angle of the prism at A.

Repeat the experiment, using a different angle of incidence. Note whether the deviation is the same as before; and note also that the path of the ray within the prism is not necessarily parallel to the base BC of the prism.

Minimum deviation.—When experiments similar to Expt. 182 are carried out with prisms of different angles and of different kinds of glass, it can be proved that the deviation of a ray passing through a prism depends upon

(i) the refracting angle of the prism,
(ii) the material of the prism,
(iii) the angle of incidence of the ray entering the prism, and
(iv) the nature of the incident light.

With a given prism there is one angle of incidence for which the angle of deviation is a minimum; and it can be proved, both theoretically and by experiment, that the deviation has a minimum value when the angle of emergence is equal to the angle of incidence (or, in other words, if the two sides of the prism are equal in length, when the path of the ray within the prism is parallel to the base of the prism).

EXPT. 183.—**Minimum deviation and calculation of the refractive index of a prism.** Place a prism ABC (Fig. 185) upon a sheet of paper, and fix two pins in positions corresponding to D and E. View the two pins through the prism in the direction D′E′. Slowly rotate the prism round the point A, and in either direction: notice that the line D′E′ of the image varies, and that there is *one position* of the prism in which D′E′ approaches most nearly to the direction GF of the incident ray. Mark the emergent ray by means of pins D′ and E′, and trace the outline of the prism on the paper. Remove the prism and pins.

Draw the incident ray DE and the emergent ray E′D′. Join EE′. The path of the ray is represented by DEE′D′. Note whether EE′ is parallel to the base BC. Produce DE to any point F, and D′E′ to G. Measure the angle of deviation (d). Draw normals at the points E and E′, and measure the angles of incidence (i) and of emergence (e). The angle A of the prism is equal, necessarily, to the angle (a) between the normals at E and E′.

Notice that, from Fig. 185, the angle a is equal to twice the angle r of refraction; hence

$$r=\frac{a}{2}.$$

Also, since the angle e of emergence is equal to the angle i of incidence,

$$\angle GEE' = \angle GE'E$$

and $$d=2(i-r)=2i-a;$$

or $$i=\tfrac{1}{2}(d+a).$$

It has been explained previously that the refractive index (μ) is equal to the ratio $\sin i/\sin r$; hence

$$\mu=\frac{\sin i}{\sin r}=\sin\frac{d+a}{2}\Big/\sin\frac{a}{2}.$$

From the measurements obtained in the experiment calculate, by means of this formula, the refractive index of the glass prism.

Graphical diagrams.—The effect of refraction upon the course of a ray can be determined easily by the construction of graphical diagrams based upon the principles described already. It is only necessary to bear in mind that when a ray passes from one medium to another of different optical density, perpendiculars should be drawn to the normal at equal distances on the rays from the point of incidence and their lengths should be in the ratio of the refractive index. In the denser medium the normal

must represent always the smaller number of the ratio given as the refractive index. The following examples illustrate the construction of two diagrams of this kind:

EXAMPLE.—1. A ray of light is incident at an angle of 50° to the surface of a glass plate which is 5 cm. thick. The refractive index of the glass is 1·5. Determine graphically the path of the emergent ray, and measure the lateral displacement which the ray undergoes.

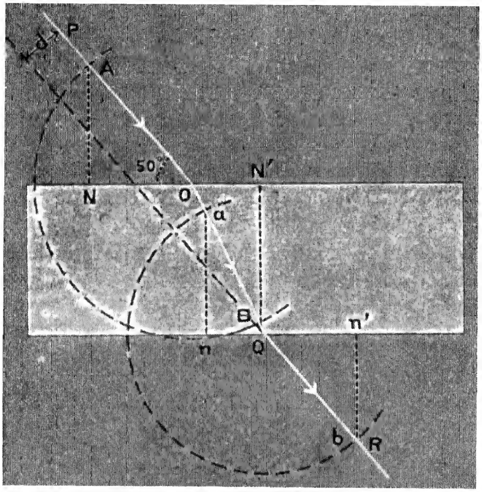


FIG. 186.—Geometrical construction for the path of a ray through a transparent slab.

Let PO (Fig. 186) be the incident ray, and O the point of incidence. With centre O, and with any convenient radius, describe a circle which cuts the incident ray at A.

From A draw a normal AN to the surface of the glass. Measure ON, and mark off a distance ON′ such that ON=(1·5×ON′). From N′ draw the normal N′B cutting the circle previously described at B. Join OB, and produce it if necessary to meet the lower surface of the plate at Q. OQ is the *refracted ray*.

With centre Q, and with any convenient radius, describe a circle which cuts the ray OQ at *a*. From *a* draw the normal *an*. Measure Q*n*, and mark off a distance Q*n*′ such that Q*n*′=(1·5×Q*n*). From *n* draw the normal *n*′*b* cutting the circle at *b*. Join Q*b*. Then Q*b* is the direction of the *emergent ray*.

The lateral displacement is determined by producing the ray QR backwards and measuring the perpendicular distance d between the rays QR and PO.

EXAMPLE.—2. An equilateral hollow glass prism (with very thin walls) is filled with carbon bisulphide. Trace the path of a ray of light which falls on the prism making an angle of incidence of 60° on the side of the normal away from the apex of the prism. Measure the deviation. [Refractive index of carbon bisulphide = 1·7.]

In Fig. 187 let PO be the incident ray. Measure off, along the face AB, distances OD and OE such that OD = 1·7 × OE. From D and E draw normals to the face of the prism.

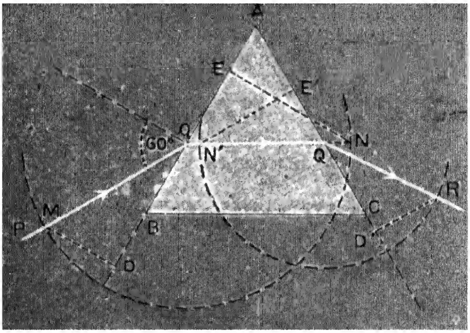


FIG. 187.—Geometrical construction for the path of a ray through a prism.

Let the normal from D cut the incident ray at the point M. With centre O, and radius OM, describe a circle. If this circle cuts the normal, drawn from E, at the point N, then the line ON is the path of the ray through the prism.

Let Q be the point of emergence. Measure off, along the face AC, distances QD′ and QE′, such that QD′ = 1·7 × QE′. From E′ draw a normal, intersecting the path OQ at N′. With centre Q, and radius QN′, describe a circle, cutting the normal from D′ at the point R. The line QR is the path of the emergent ray.

The angle of deviation is found to be 57° approximately.

EXERCISES ON CHAPTER XX.

1. A bright bead is placed at the bottom of a basin of water, and a person stands in such a position that he can just see it over the edge of the basin. While he is looking, the water is drawn off. How will this affect his view?

Draw a diagram showing the direction of a ray of light passing from the bead through the water and the air in each case.

2. A thick layer of transparent liquid floats on the surface of water. Trace the course of a ray of light from an object immersed in the water through the floating liquid to the air.

3. Describe an experiment to show the path of a ray of light which passes obliquely through a thick plate of glass. Illustrate your answer by a sketch in which you indicate clearly the path of the ray in the air before it enters the glass, in the glass, and in the air beyond the glass.

4. An upright post is fixed in the bottom of a pond which is three feet deep; the top of the post is three feet above the water. How will the post appear to an eye about the level of the top of the post and four or five feet away from it?

Draw a figure to illustrate your answer.

What will be seen as the eye moves further and further back from the post?

5. A fish swims in a glass tank; a person whose eye is above the level of the water seems to see two fish. Draw a diagram to illustrate this, and give any explanations you think necessary.

6. A vertical straight wire is viewed so that part is seen directly and part through a thick plate of glass held vertically. Describe the apparent changes in the position of the portion seen through the glass when this is slowly turned round a vertical axis.

7. A glass cube is placed over a pencil mark on a sheet of paper. The mark is viewed through the cube. Explain, by means of a diagram, the apparent position of the pencil mark.

8. What is meant by the refractive index of a substance? Being provided with a thick piece of glass from a box of weights, some pins, a sheet of white paper, a pencil and graduated ruler, how would you determine the index of refraction of the glass?

9. State the laws of the refraction of light. Explain why a tank of water viewed from above appears shallower than it really is.

10. Explain, by the aid of a diagram, why a pole appears to be bent when it is thrust into water in a slanting position.

11. A ray of light is incident at an angle of 30° on one face of a glass plate, 3 inches thick, the index of refraction being 1·5. Give a diagram to scale showing the refracted and emergent rays. Show that the emergent ray is parallel to the incident ray, and measure the perpendicular distance between them.

12. Explain the terms *critical angle* and *total reflection.* A glass cube of two inch edge stands upon a horizontal table; represent in plan, in a diagram drawn approximately to scale, the path of a horizontal ray of light which falls upon a vertical face of the cube and is afterwards totally reflected at a surface of contact of glass

and air. Assume the refractive index of the glass with respect to air to be $\frac{3}{2}$.

13. Two faces of a glass block are parallel and 3 inches apart; and a ray of light strikes one of them at an angle of incidence of 60°. The refractive index of the glass is 1·5. Show in a diagram the directions of the incident ray, refracted ray, and emergent ray. Measure the amount of displacement caused by the glass and record it.

14. The critical angle for a certain medium is 45°. What is its refractive index?

15. Trace the path of a ray of light which falls upon one of the perpendicular faces of an isosceles right-angled glass prism ($\mu=3/2$) at an angle of 70° with that surface. Trace the ray until it emerges from that surface.

16. A layer of water, 2 inches deep, lies upon a slab of glass which is 1·5 inches thick and of which the lower surface is silvered. Trace the path of a ray which strikes the surface of the water at an angle of 45° ($_{\text{air}}\mu_{\text{water}}=4/3$; $_{\text{water}}\mu_{\text{glass}}=9/8$).

17. Find, by geometry, the critical angles for ice, glass and carbon bisulphide, having given that the refractive indices of these substances are 1·3, 1·5, and 1·7 respectively.

18. Show, by geometry, why a ray of light striking one of the perpendicular faces of a right-angled isosceles glass prism along a normal cannot emerge from the hypotenuse face. Trace its path. ($\mu=3/2$.)

19. ABCD is the square section of a glass cube ($\mu=3/2$). P is a luminous point in AC produced, and 1 inch distant from C. PQ is a ray of light striking the side BC at R, so that $\angle$APR is 30°. Trace the path of the ray until it emerges from the glass. [Make AB$=3\frac{1}{2}$ inches.]

20. An equilateral hollow glass prism is filled with carbon bisulphide ($\mu=1·7$). Trace the path of a ray which is incident upon the prism at 30° with the surface.

21. A ray of light is incident at an angle of 30° on one face of an equilateral prism. If the path of the light through the prism is parallel to the base, find the direction of the emergent ray, and the total deviation of the ray after passing through the prism.

22. A dot is made on a piece of paper, and a prism is laid on the paper over the dot. An eye in certain positions now seems to see two dots. Draw a diagram to explain this.

23. A hollow glass prism, and full of air, is immersed in a glass tank full of water. Make a diagram showing rays of light passing through both the water and the prism.

24. The minimum deviation produced by a hollow prism filled with a certain liquid is 30°. If the refracting angle of the prism is 60°, what is the index of refraction of the liquid?

25. Explain clearly how you would determine by experiment the index of refraction of a glass plate. Why is the apparent depth of a pool less than its actual depth?

26. State the laws of the refraction of light.

A ray of light, falling on one face of a plate of glass 10 cm. in thickness, makes an angle of 60° with the normal. Determine the point on the other face at which the ray will emerge from the glass. [The index of refraction of the glass is 1·5.]

27. State the laws of refraction of light.

A ray of light is incident on the surface of water at an angle of 45°. Find by a diagram the direction of the refracted ray. [The index of refraction of water is 1·33.]

28. A ray of light in a prism, which has an angle of 6c°, makes an angle of 30° with the face of the prism. Find the direction of the emergent ray. [The index of refraction of the glass is 1·4.]

29. State the laws of refraction of light.

A ray of light is incident at 60° to the normal upon a polished glass surface. The refracted ray makes an angle of 90° with the reflected ray. Find, graphically or by calculation, the refractive index of the glass.

30. A plane mirror lies horizontally. The image of a point lying above the mirror is viewed by an eye looking at an angle of 45° with the mirror, the height of the eye above the mirror being twice the height of the point. Draw a diagram to indicate the path of two rays proceeding from the point to the eye.

Explain whether the eye would still see the image of the point in the same direction if the mirror lay at the bottom of a tank, and if water were poured into the tank (*a*) so as to just cover the point, and (*b*) so as to reach up to the eye.

31. Draw a diagram of a prism with an angle of 45°, and trace completely the course of a ray of light, which, lying in a plane at right angles to the axis of the prism, falls on the prism making an angle of incidence of 30° on the side of the normal away from the apex of the prism.

From your diagram measure the deviation. [Refractive index of the glass = 1·5.]

32. An equilateral hollow glass prism is filled with glycerine. Determine graphically the minimum deviation of a ray passing through the prism. [Refractive index of glycerine = 1·47.]

33. The refractive index of crown glass is 1·53, and its critical angle is 41° approximately. Determine geometrically the minimum angle of incidence of a ray of light which falls upon one face of an equilateral glass prism and undergoes total internal reflection from the other face.

CHAPTER XXI.

REFLECTION AT SPHERICAL SURFACES.

Spherical mirrors.—A spherical mirror is a mirror having the form of part of the surface of a sphere. It may be either **concave** or **convex**, the former if the reflection takes place from the hollow side, the latter if from the bulging side.

Spherical mirrors may be considered as made up of an infinite number of very small plane facets having negligible curvature, and each such element may be regarded as a small plane mirror. It is known, from geometry, that every radius of a circle is at right angles to the tangent at the point where it cuts the circle. Hence, the normals to all these plane facets will pass, like the radii of a circle, through one point which is the centre of the sphere from which the mirror has been derived. This point is called the **centre of curvature** of the mirror.

Figs. 188 and 189 indicate how a concave mirror and a convex mirror respectively, each consisting of a number of small plane facets, will affect the path of the rays of a diverging pencil of rays originating from a luminous point O. For the sake of clearness each facet is represented with only one ray reflected from its middle point. The path of each ray after reflection is obtained by drawing the normal to the reflecting surface, and by making the angle of reflection r equal to the angle of incidence i. In the case of the concave mirror (Fig. 188) all the rays, after reflection, pass through a point I, which is called the **image** of the luminous point O; and the position of this image could be determined by holding a piece of white paper, so as to intercept the reflected rays, and altering its position until the brightly illuminated portion of its surface is restricted to a point. In the case of the convex

mirror (Fig. 189), the rays after reflection *appear* to proceed from a point I on the further side of the mirror. To an eye situated on the left-hand side of the mirror the image of the luminous point O will appear to be situated at I.

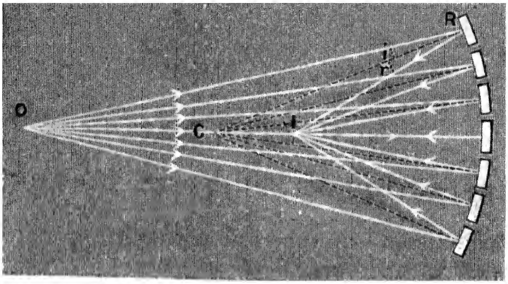


FIG. 188.—The principle of a concave mirror.

It is evident that **when a luminous point is situated at the centre of curvature of a concave mirror all the rays of light from it are reflected back along the lines of incidence, and the image is formed in the same position as the luminous point.**

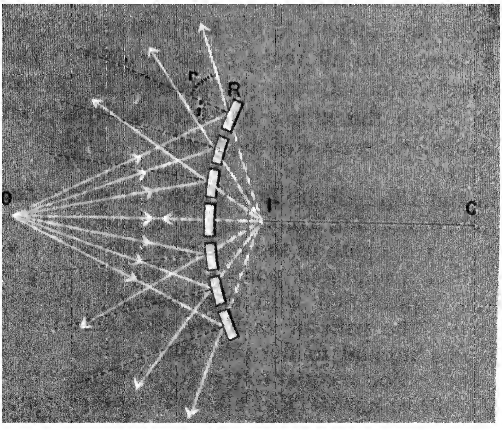


FIG. 189.—The principle of a convex mirror.

In any spherical mirror, the distance from the centre of curvature to the reflecting surface is the **radius of curvature**. Thus,

in Fig. 190, C is the centre of curvature and CM, CR, CM′, are all radii of curvature. MM′ is called the diameter or aperture of the mirror; and P, the centre of the reflecting surface, is often called the **pole** or **apex** of the mirror. A line going through the pole and centre of curvature is the **principal or optic axis** of the mirror, any other radius produced being a **secondary axis**.

Principal focus, and focal length.—(i) Let MPM′ (Fig. 190) be a concave mirror, of which C is the centre of curvature. Let IR be a single ray of a very narrow pencil of rays parallel to the principal axis of the mirror; and let R be the point of incidence on the reflecting surface. Join CR. The line CR is normal to the surface of the mirror at R. Make the angle CR*i* equal to the angle IRC. Then, by the law of reflection, R*i* is the path of the reflected ray. Let R*i* cut the principal axis at F. Then,

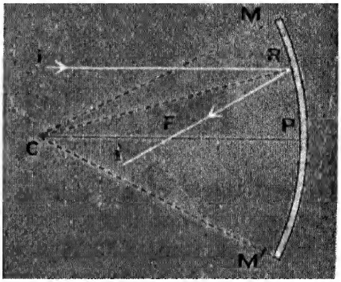


FIG. 190.—Principal focus of a concave mirror.

since $\angle CRI = \angle CRF$,

and $\angle CRI = \angle RCF$,

$\angle CRF = \angle RCF$.

Hence, $FR = FC$.

But, if the pencil of rays be very narrow, the point R coincides approximately with the point P; and, therefore, $FR = FP$ approximately.

Hence, $$FP = FC.$$

The same reasoning will apply to all other rays in the pencil; and, therefore, **all the rays after reflection will pass through a point F which is midway between the centre of curvature and the pole of the mirror.**

(ii) Fig. 191 represents the corresponding case for a convex mirror, where IR is a single ray of a pencil parallel to the principal axis. If C*n* is the normal to the reflecting surface at R, then $\angle IRn = \angle nRi$. But, by geometry, $\angle IRn = \angle FCR$, and $\angle NRi = \angle FRC$.

Hence, $$\angle FCR = \angle FRC,$$

and $$FC = FR.$$

But, since the pencil of rays is supposed to be very narrow, $FR = FP$.

$$\therefore\ FC = FP.$$

Hence, **all the rays after reflection from a convex mirror appear to pass through a point F which is midway between the centre of curvature and the pole of the mirror.**

In each case the point F is termed the **principal focus** of the mirror, and the distance of this point from the mirror is termed

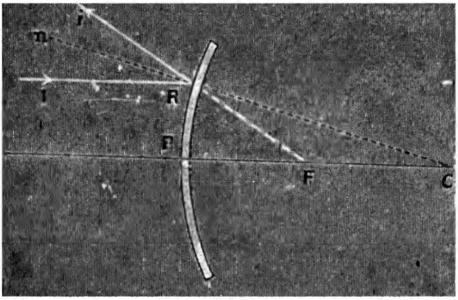


FIG. 191.—Principal focus of a convex mirror.

the **focal length** of the mirror. The following definitions are of fundamental importance :

Principal focus.—When a narrow beam of rays parallel to the principal axis of a concave mirror is reflected from the surface of the mirror, the rays converge to, or diverge from, a point on the principal axis. This point is termed the principal focus.

Focal length.—The distance of the principal focus from the pole of the mirror is termed the focal length of the mirror.

It is evident, from the preceding paragraphs, that **the focal length of a spherical mirror is equal numerically to one half the radius of curvature.**

Real and virtual foci.—In the case of a convex mirror (Fig. 191) the reflected rays only *appear* to pass through the principal focus, and, in order to distinguish this case from that in which the rays actually pass through the principal focus, it is usual to term the former a **virtual** focus, and the latter a **real** focus.

The same terms are used in all such cases, whether the incident pencil of rays is parallel, or converging. or diverging.

Position and nature of the image of an object, due to reflection from a spherical mirror.—In determining by a graphical method the position of the image of an object, use is made of the fact that the paths of certain rays after reflection from the mirror are known,

providing that the positions of the centre of curvature and of the principal focus are given Thus

(i) **A ray which passes through the centre of curvature of a mirror returns by the same path after reflection.**

(ii) **Rays parallel to the principal axis of a mirror pass through the principal focus after reflection.**

And, since the path traversed remains unaltered when the direction of the ray is reversed,

(iii) **Rays which pass through the principal focus assume a direction parallel to the principal axis after reflection.**

In practice, it is sufficient to make use of two only of these general principles, since the point of intersection of the paths of two rays after reflection suffice to determine the position of the image. The diagrams obtained by such methods afford information as to (i) the *position* of the image, (ii) its *nature*, whether real or virtual, and (iii) its *size*.

In calculating the position of the image, by means of the **general equation for mirrors**, which will be deduced in a subsequent paragraph, it is necessary to apply always the following rule as to the signs *plus* and *minus*: Distances measured from the mirror *towards* the source of light are **positive**; but when measured from the mirror and *away from* the source of light they are to be considered **negative**. As a result of this rule, the focal length of a concave mirror is always positive, and that of a convex mirror is always negative (Figs. 190 and 191).

Real images due to a concave mirror.—In Fig. 192 let OB be an object situated further from a concave mirror than its centre

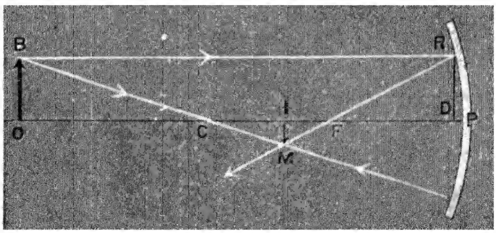


Fig. 192.—Real image, by means of a concave mirror.

of curvature C. A ray BR parallel to the principal axis will be reflected so as to pass through the principal focus F; and a ray

BC, passing through the centre of curvature, will be reflected back along its own path. The point M, where these reflected rays intersect, is the position of the image of the point B. It may be proved, in a similar manner, that the image of any other point on OB is situated along the line IM. Hence IM is the image of the object OB.

Since the reflected rays actually pass through the image, the image is **real**; and it is evident from Fig. 192 that the image is **inverted**.

Conjugate foci.—The nature and position of an image formed by a mirror depends upon the position of the object. Consider a point in any given position upon the principal axis. Rays diverge from it to the mirror and converge to form an image which may be real or virtual, but in either case the object and image may be regarded as interchangeable. Points at which an object and image may thus change places are known as **conjugate foci**. When the image is real, as for instance, when the object is beyond the centre of curvature of a concave mirror, the places actually can be changed, so that the image is formed where the object was previously.

Equation for mirrors.—In determining, by calculation, the relationship between the positions of the object and image, and the focal length of the mirror, it is necessary to remember that the mirror is supposed to be very narrow, and that the pole P of the mirror coincides with the point D, which is the base of the normal drawn from R to the principal axis.

If u = the distance, PO, of the object from the mirror,
v = „ PI, „ image „
r = the radius of curvature, CP,
and f = the focal length, FP, of the mirror;
then OB/IM = CO/CI, and RD/IM = FD/FI.
But, since OB = RD, OB/IM = RD/IM;
hence CO/CI = FD/FI = FP/FI.
But CO = $(u - 2f)$; CI = $(2f - v)$; FP = f; and FI = $(v - f)$;
hence
$$\frac{u-2f}{2f-v} = \frac{f}{v-f},$$
or
$$uv - uf - 2fv + 2f^2 = 2f^2 - fv,$$
or
$$uv = uf + vf.$$

From which the **general equation for mirrors** is derived, namely,

$$\frac{1}{\text{focal length}}=\frac{1}{\text{distance of image}}+\frac{1}{\text{distance of object}},$$

or

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

Virtual images, due to concave mirrors. In Fig. 193 let OB be an object situated nearer to a concave mirror than its principal focus. Consider the two rays BR and BR′ proceeding from the point B. Since the path of the former passes through the

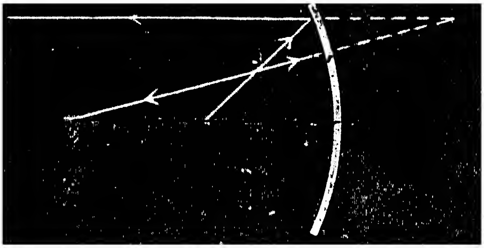


FIG. 193.—Virtual image, by means of a concave mirror.

principal focus F, its direction after reflection will be parallel to the principal axis; and, since the path of the ray BR′ passes through the centre of curvature C, it will be reflected back along its previous path. The point M where these reflected rays intersect is behind the mirror. Hence IM is the image of the object. Since the reflected rays only *appear* to originate from M, the image is **virtual**; it is evident, also, from the diagram that the image is **upright** and **enlarged**.

It is evident, from geometry, that

$$OB/IM = OC/IC, \quad \text{and} \quad OB/RD = OF/DF.$$

But

$$IM = RD;$$

$$\therefore \; OC/IC = OF/DF = OF/PF \text{ (approximately).}$$

If the symbols u, v, and f have the same meaning as on p. 280, then $OC=(2f-u)$; and, since PI is measured in the negative direction, $IC=2f+(-v)=(2f-v)$; also, $OF=(f-u)$ and $PF=f$.

Hence, $$\frac{2f-u}{2f-v}=\frac{f-u}{f},$$

or $$uv=uf+vf,$$

or $$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

This equation is identical with the general equation deduced in the previous paragraph.

Virtual images, due to convex mirrors. In Fig. 194 the object OB is situated in front of a convex mirror; and, as in the previous

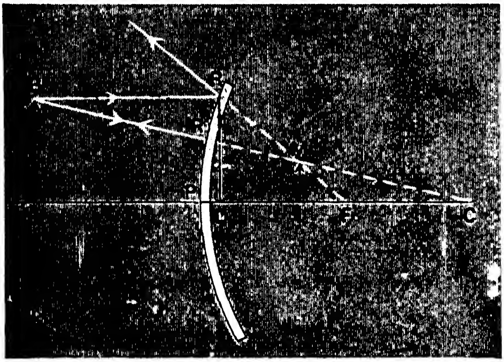


FIG. 194.—Virtual image, by means of a convex mirror.

cases, the position of the image IM is determined by the point of intersection of the paths, after reflection, of the two rays BR and BR′. It is evident that the image is **virtual**, **erect**, and **diminished**.

$$\left.\begin{aligned}\text{Since}\quad OB/IM&=OC/IC,\\ \text{and}\quad RD/IM&=FD/FI;\end{aligned}\right\}\quad \therefore\ OC/IC=FD/FI=FP/FI \text{ (approx.)}.$$

Hence, $$\frac{u+(-2f)}{-2f-(-v)}=\frac{-f}{-f-(-v)},$$

or $$\frac{u-2f}{-2f+v}=\frac{-f}{-f+v};$$

$$\therefore\ uv=fv+uf,$$

or $$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

Summary.—The following deductions are obtained from the preceding paragraphs:

(i) **Concave mirrors.**

(*a*) When the object is beyond C, the image is real, inverted, and *diminished.*

(*b*) When the object is between C and F, the image is real, inverted, and *enlarged.*

(*c*) When the object is nearer than F, the image is *virtual, upright*, and enlarged.

(ii) **Convex mirrors.**

In all positions of the object, the image is *virtual, upright*, and *diminished.*

(iii) When the proper algebraic signs are given to the numerical values of the quantities u, v, and f, the general equation $1/f = 1/v + 1/u$ may be used in all problems relating to spherical mirrors.

The conditions under which an image has any of the above characters may be summarised thus:

Real	(i)	Concave mirror:	object beyond C.
	(ii)	,, ,,	,, between C and F.
Virtual	(i)	,, ,,	,, nearer than F.
	(ii)	Convex mirror:	object in any position.
Erect	(i)	Concave mirror:	object nearer than F.
	(ii)	Convex mirror:	object in any position.
Inverted	(i)	Concave mirror;	object beyond C.
	(ii)	,, ,,	,, between C and F.
Enlarged	(i)	,, ,,	,, ,, ,,
	(ii)	,, ,,	,, nearer than F.
Diminished	(i)	Concave mirror:	object beyond C.
	(ii)	Convex mirror:	object in any position.
Equal	(i)	Concave mirror:	object at C.

EXPT. 184.—**Focal length of a concave mirror.** (i) Allow a beam of parallel rays to fall upon the mirror, and adjust the position of a piece of white cardboard so that a well-defined image is formed on its surface. The distance of the cardboard from the pole of the mirror is the focal length of the mirror. A beam of sunlight is convenient for this experiment; or a distant chimney, or window frame, may be used.

(ii) Bore a small round hole in a piece of thin cardboard, and fix two thin threads, or wires, diametrically across the hole and at right angles to each other. Fix this vertically in front of a bright flame, and allow the rays passing through the hole to fall upon the mirror. Adjust the mirror so that its principal axis makes a small angle with the axis of the incident beam. Hold a piece of white cardboard vertically and normal to the reflected beam, and adjust its position so that a well-defined image of the cross-wires is seen on its surface. Measure the distance u of the cross-wires from the mirror, and the distance v of the image from the mirror; calculate the focal length by means of the formula

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

In this, and the following, method the surfaces of a bi-concave lens —that is, a lens having both faces concave, as in Fig. 198—may be used as mirrors, providing that the room is darkened partially.

(iii) Using the same source of light as in the previous method, adjust the position of the mirror so that a well-defined image of the cross-wires is seen upon the cardboard to which the cross-wires are attached and near to the round hole. Evidently the rays of light are reflected back along their own path, and the cross-wires must be situated at the centre of curvature of the mirror. Measure this distance; one-half of this distance is the focal length.

EXPT. 185.—**Focal length of a convex mirror.** Let O (Fig. 195) represent the position of the illuminated cross-wires, and let L be a bi-convex lens—that is, a lens having both faces convex, as in Fig. 197. The rays of light passing through L will converge to a point I, where a well-defined image will be formed upon a cardboard screen. Adjust the position of the screen until the best possible definition is obtained. Support the mirror M in a vertical position between L and I, and adjust its position until a well-defined image of the cross-wires is formed upon the cardboard at O. Since the rays are now reflected back along their own path they must fall normally upon the surface of M, and the distance IM between the screen at I and the mirror M must be the radius of curvature of the mirror. One half of this distance is the focal length.

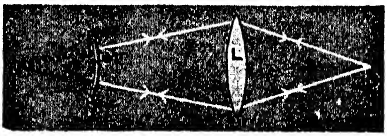


FIG. 195.—Experimental method of determining focal length of a convex mirror.

Magnification.—The ratio of the linear dimensions of the image and of the object is termed the **magnification**. Thus, in Figs. 192-194, the magnification obtained by means of the mirror is equal in each case to the ratio IM/OB.

It is convenient to express the magnification in terms of the quantities u and v. This relationship may be deduced in the following manner: In Fig. 192,

$$IM/OB = CI/CO,$$

or, expressed in words, the magnification is equal to the ratio of the distances of the image and of the object from the centre of curvature.

Since $\quad CI = (r - v), \text{ and } CO = (u - r),$

then
$$IM/OB = (r - v)/(u - r).$$

But
$$1/v + 1/u = 1/f = 2/r,$$

or
$$1/v - 1/r = 1/r - 1/u,$$

or
$$(r - v)/vr = (u - r)/ur,$$

or
$$(r - v)/(u - r) = vr/ur = v/u.$$

Hence,
$$IM/OB = v/u,$$

or, **the magnification is equal to the ratio of the distances of the image and of the object from the mirror.** The same result may be deduced by means of either Fig. 193 or Fig. 194.

NUMERICAL EXAMPLE.—A luminous object 3 inches high is situated 12 inches in front of a concave mirror, the focal length of which is 9 inches. Find the position and size of the image.

Using the formula $1/v + 1/u = 1/f$; since $u = +12$ in., and $f = +9$ in.,

then
$$\frac{1}{v} = \frac{1}{9} - \frac{1}{12} = \frac{1}{36},$$

or
$$v = +36 \text{ in.}$$

Hence, the image is 36 inches *in front of the mirror.*

Also,
$$\frac{\text{size of image}}{\text{size of object}} = \frac{36}{12},$$

or
$$\text{size of image} = 3 \times \frac{36}{12} = 9 \text{ inches.}$$

Solution of problems on mirrors by means of squared paper.—The solution, by graphical methods, of simple problems on spherical mirrors may be rendered more simple in many cases if squared paper be used for the purpose. It may suffice if suitable examples of the method are given.

EXAMPLE.—1. A luminous object, 10 cm. high, is placed in front of a concave mirror, the radius of curvature of which is 80 cm. Find the nature, size, and position of the image, when the object is (*a*) 20 cm., (*b*) 60 cm., (*c*) 110 cm. distant from the mirror.

In Fig. 196 one division of the squared paper represents 2 cm. OB is the object when situated 20 cm. from the mirror; and the position

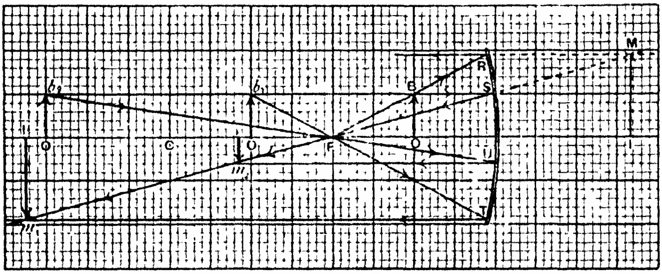


FIG. 196.—Graphical construction for image and object with a concave mirror.

of the image IM is determined by the paths of the two rays BR and BS. Similarly, the position of the image Im_1 of the object Ob_1 is determined by the rays b_1T and b_1S; and the image Im_2 of the object Ob_2 is determined by the rays b_2U and b_2S. The results obtained by measurement should be compared with those obtained by calculation from the general equation, and tabulated in the following manner:

Distance of Object.	Nature of Image.	Distance of Image		Size of Image.	
		By Geometry.	By Calculation	By Geometry.	By Calculation
i. 20 cm.	Virtual, and upright	33·5 cm.	40 cm.	19 cm.	20 cm.
ii. 60 cm.	Real, and inverted	114·6 cm.	120 cm.	19 cm.	20 cm.
iii. 110 cm.	Real, and inverted	62·5 cm.	62·8 cm.	6 cm.	5·71 cm.

The student should endeavour to explain why the results obtained by geometry and by calculation agree more closely when the object is removed further from the mirror.

EXERCISES ON CHAPTER XXI.

1. Explain what is meant by an image. What is the difference between a real and a virtual image? In the case of a concave mirror, find the positions of the object when the image is virtual.

2. A small luminous object is placed in front of a concave spherical mirror of 12 inches focal length at distances of 3 feet, 2 feet, and 1 foot. Draw figures showing the positions and relative sizes of the images, and explain your construction.

3. A small object is placed six feet in front of a convex mirror of 3 feet radius. Give a diagram showing the nature and position of the image, and find its size relative to that of the object.

4. An object 5 cm. long is placed at a distance of 40 cm. from a convex mirror of 24 cm. focal length. Find the size and position of the image.

5. The middle of a small object is placed on the axis of a concave spherical mirror (i) half-way between the centre and the principal focus, (ii) half-way between the principal focus and the mirror. Draw diagrams to determine the position and circumstance of the image in each case.

6. Find the size of an image of the sun formed by a concave mirror 6 feet in radius, assuming that the distance of the earth from the sun is 100 times the diameter of the sun.

7. Prove that the focal length of a concave mirror is half its radius of curvature. How would you determine experimentally the focal length of such a mirror?

8. An object is placed 20 inches in front of a convex mirror of 10 inches radius. Find the position of the image, and draw a diagram to scale.

9. Explain the terms *radius of curvature*, *focal length*. How could you find by experiment the radius of curvature and the focal length of a concave mirror?

10. A mirror is fastened to the ceiling of a room and forms images of objects in the room. How can you learn from these images whether the mirror is convex or concave, and what information can you gather as to the degree of its convexity or concavity?

11. What is meant by the focal length of a mirror?
Determine graphically the position of the image formed of an object placed 30 cms. from a reflecting surface of which the radius of curvature is 40 cms.

12. Explain carefully how it is possible to find by a graphical method the position and size of the image of an object formed by reflection in a concave mirror.

A pin, 4·5 cms. long, is placed 10 cms. from a concave mirror, of which the radius of curvature is 15 cms. Find the position and size of the image.

13. Similar objects are placed close up to a plane, a concave, and a convex, reflecting surface; how would you distinguish between the surfaces from the images formed in them? How will the images change as the objects are moved away from the surfaces?

14. A concave mirror has a radius of 30 cms. Where must the object be placed so that the magnification may be 3, when the image is (*a*) real, (*b*) virtual?

15. Find the relation between the radius of curvature of a mirror and the distances of an object and its image from it. How would you test the relation by experiment?

16. Distinguish between a real and a virtual image. How would you find experimentally the position of a virtual image?

17. In an experiment with a concave mirror the following measurements of u and v were obtained:

u	v	u	v
27 cm.	77 cm.	60 cm.	30 cm.
30 ,,	60 ,,	80 ,,	26·7 ,,
40 ,,	40 ,,	100 ,,	25 ,,
50 ,,	33 ,,	120 ,,	24 ,,

Plot the readings on squared paper, and find from the curve the positions of the image when (i) $u=35$ cm., (ii) $u=70$ cm., (iii) $u=90$ cm.

CHAPTER XXII.

REFRACTION AT SPHERICAL SURFACES.

In the following paragraphs the consideration of the refraction of light rays at spherical surfaces must be limited to the deviation of rays and the formation of images by dense transparent media, which are bounded by two surfaces, of which both may be spherical, or one may be plane and the other spherical. A medium of this form is termed a **lens**.

Refraction through a lens.—Most lenses are of glass with curved surfaces, which are portions of spheres. In some lenses, however, one surface is quite plane. All lenses can be divided into two classes—**convex** or **converging**, and **concave** or **diverging**. Converging lenses are thicker in the middle than at the edges, and have the power of forming real images of objects. Concave lenses are thinner in the middle than at the edges, and are unable to form real images of objects.

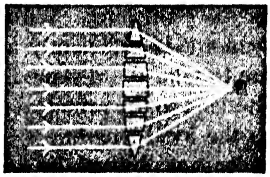


Fig. 197.—A converging lens.

To understand the action of the lenses upon the course of rays of light through them, it is simplest to regard them as being built up of parts of prisms in contact, as shown in Fig. 197, where a convex lens is built up in this way. A ray of light falling upon any one of these prisms is refracted towards its thicker part, and, as in a thin lens bounded by spherical surfaces each prism has a refracting angle proportional to its distance from the axis, the rays converge toward a point, which, if the incident rays are parallel, is known as the **principal focus** of the lens, as F in Fig. 197.

Fig. 198 represents the corresponding effect due to a concave lens. The section of such a lens may be regarded as being built up of parts of prisms of gradually increasing angle, and arranged with their bases outwards. Fig. 198 represents a pencil of parallel rays converted by a concave lens into a diverging beam of rays.

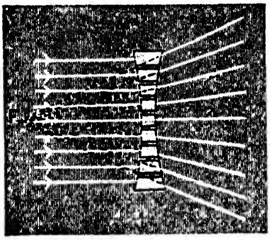


FIG. 198.—A diverging lens.

Principal axis, principal focus, and focal length.—The line connecting the centres of curvature of the two surfaces of a lens is termed the **principal axis** of the lens. The **principal focus** of a converging (or diverging) lens is the point towards which the rays converge (or from which they appear to diverge) when a pencil of rays parallel to the principal axis passes through the lens. Thus, in Figs. 197 and 198, the point F represents the principal focus. The distance from the principal focus to the centre of the lens is termed the **focal length** of the lens. With thin lenses it is sufficiently accurate to take the distance from the lens to the principal focus as the focal length.

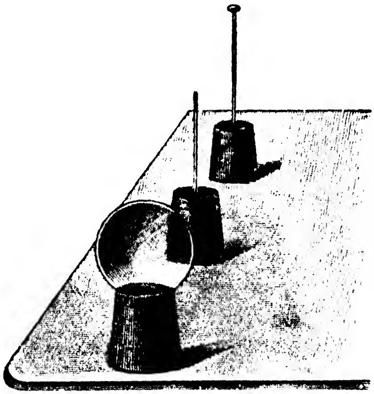


FIG. 199.—Apparatus for experiments with lenses.

Images formed by lenses.—When rays of light diverge from a luminous point on the principal axis of a lens, the rays which pass through the lens will either pass through, or appear to pass through some other point, which is termed the **image** of the luminous point. The image is **real** or **virtual** according as the rays actually pass through the point or only appear to pass through it.

The preceding paragraph does not apply in the case of a converging lens when the luminous point coincides with the

principal focus of the lens; for the emergent rays are parallel, and the image is situated at an infinitely great distance.

An image of an object seen through a lens may be either (i) *real* or *virtual*, (ii) *upright* or *inverted*, and (iii) *magnified* or *diminished*.

Much information concerning the different types of image may be derived by means of the simple appliance shown in Fig. 199. It consists of three corks which serve as supports for a lens, a needle, and a long pin: the pin should be sufficiently long for its head to be well above the top edge of the lens. If a lens of 2 inch diameter be used the pin should be $2\frac{1}{2}$ inches long.

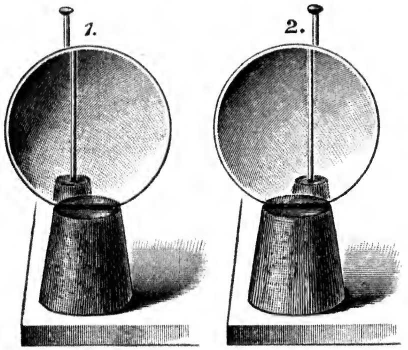


FIG. 200.—Expt. 186.

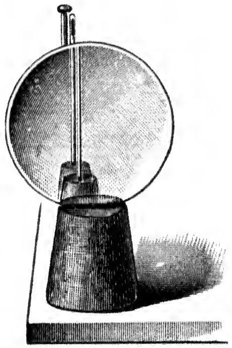


FIG. 201.—Expt. 187.

EXPT. 186.—**A concave lens.** Place the lens vertically at distances of 5, 10, 15, ... cm. in front of the pin, adjusting the height of the pin so that its head is seen well above the edge of the lens. View the image of the pin with the aid of one eye only and looking along the principal axis of the lens. Note (i) whether the image is erect or inverted; (ii) whether it is magnified or diminished; (iii) whether it is in front of, or behind, the lens; (iv) whether it is in front of, or behind, the object.

The Parallax Method (p. 252) may be applied in order to obtain information as to the position of the image, remembering that if two objects are placed in line with the eye, and the eye is then moved to the left (or right), the more distant object appears to move to the left (or right) relatively to the near object. Thus, when viewing the image of the pin through a concave lens, if the eye is moved to the left (or right), the image appears

to move towards the left (or right) edge of the lens (Fig 200); hence the image must be behind the lens. At the same time, compare the movement of the pin's head (seen above the lens) with that of the image; the former moves relatively to the latter in the same direction as that in which the eye moves; hence, the pin must be behind the image.

It is interesting to attempt to locate accurately the position of the image. The observation is not easy, but it is worth the attempt.

EXPT. 187.—**Position of image formed by a concave lens.** Place the needle vertically between the lens and the pin, and adjust its height so that its upper end is seen just above the lens. Move the eye to-and-fro and adjust the distance of the needle from the lens until *the eye of the needle* always appears to be a continuation of *the image of the pin*; the needle then occupies the position of the pin's image. Fig. 201 represents the appearance of the experiment when the eye has been moved to the left.

Enter your observations thus:

Distance of object from lens.	Is image erect or inverted?	Is it magnified or diminished?	Is it in front of, or behind, lens?	Is it in front of, or behind, object?	Distance of image from lens.
10 cm.	erect	diminished	behind	in front of	7·6 cm.

EXPT. 188.—**Convex lens.** Adjust the height of the needle so that an image of its eye can be seen through the lens. Place the lens at distances of 5, 10, 15, ... cm. in front of the needle; and, for each position, find the information required in order to fill in a Table similar to that recorded in Expt. 187.

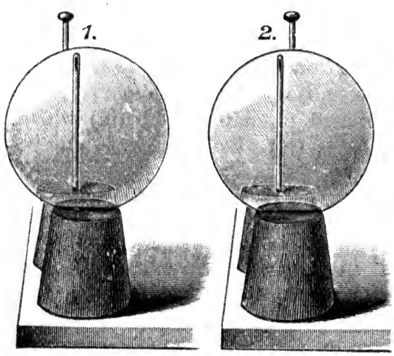


FIG. 202.—Expt. 188.

Find the position of the image of the needle by the Parallax Method, using either a long pin or a short pin according as to whether the image is behind or in front of the lens. Fig. 202, (i) and (ii), represent what would be seen when the eye is moved

to the left, and when a long pin is (i) behind, (ii) in front of the image. Notice that at one distance no distinct image can be seen, but if the distance of the object is increased slightly, an image is formed in front of the lens; in this case the image is located most readily by using a short, instead of a long, pin.

Summary.—It is evident from the preceding experiments that images due to a concave lens are always erect, diminished, and virtual (*i.e.* on the same side of the lens as the object). In the case of a convex lens, when the object is very near to the lens the image is erect, enlarged, and virtual; at one distance only, there is no image; and at greater distances the image is inverted, real, and either enlarged or diminished (depending upon the distance of the object).

The conditions under which the image assumes any of these characters may be summarised thus:—

Real	(i) Convex lens: object in any position except nearer the lens than F.
Virtual	(i) Convex lens: object nearer the lens than F. (ii) Concave lens: object in any position.
Erect	(i) Convex lens: object nearer the lens than F. (ii) Concave lens: object in any position.
Inverted	(i) Convex lens: object in any position except nearer the lens than F.
Enlarged	(i) Convex lens: object nearer the lens than F. (ii) Convex lens: object between F and 2F.
Diminished	(i) Convex lens: object at a greater distance than 2F. (ii) Concave lens: object in any position.
Equal	(i) Convex lens: object at distance 2F.

Position and nature of the images formed by lenses.—Just as in the case of images formed by reflection from a spherical mirror, it is possible to determine approximately the position of the image formed by a lens by making use of the fact that the paths traced out by certain rays are known, providing that the positions of the lens and its principal focus are known. Thus,

(i) Rays which pass through the centre of a lens do so without change of direction.

(ii) **Rays parallel to the principal axis of the lens are refracted so as to meet at the principal focus.**

(iii) **Conversely, rays from the principal focus of a lens assume a direction parallel to the principal axis after passing through the lens.**

In practice it is sufficient to make use of two only of these general principles, since the point of intersection of the paths after refraction of any pair of these rays suffices to determine the position of the image. The diagrams so obtained afford information as to (i) the position, (ii) the nature, and (iii) the size of the image.

In calculating the position of the image, by means of the general equation for lenses (p. 295), it is necessary to apply the following rule as to the signs *plus* and *minus*.

Rule of Signs.—Distances measured from the lens towards the source of light are positive; but when measured from the lens and away from the source of light they are to be considered negative.

By reference to Figs. 198 and 197, it is evident that the focal length of a diverging (or concave) lens is **positive**, and that of a converging (or convex) lens is **negative.**

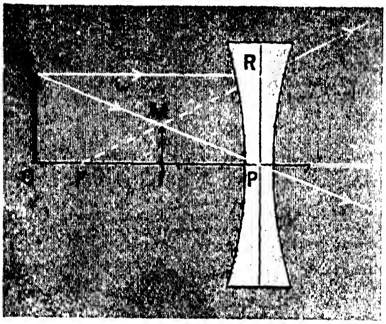


FIG. 203.—Virtual image, by means of a concave lens.

Image due to a concave lens.—In Fig. 203, let F be the principal focus of a diverging lens, of which PR is the median line. In determining by geometrical construction the paths of rays transmitted through the lens it is sufficiently approximate to assume that the lens is very thin, and that the divergence of the rays takes place abruptly at the median line.

Let OB be a luminous object. Consider the rays proceeding from the point B : one ray BR, which is parallel to the principal axis, is refracted in such a direction that it appears to originate from F. The ray BP passing through the centre of the lens undergoes approximately no deviation. The image can be

situated only at the point M, where the paths of these rays intersect. Similarly, the image of any other point of OB can be proved to be vertically under M. Hence, IM is the image of OB. It is evident that the image is **erect**, **diminished**, and **virtual**. The same result would be obtained if the object were situated nearer to the lens than the principal focus.

Relationship between the focal length and the positions of the object and image.—In Fig. 203, let $PO = u$, $PI = v$, and $PF = f$, then

$$OB/IM = PO/PI = u/v,$$

also $$PR/IM = PF/IF = f/(f-v).$$

But $$OB = PR.$$

Hence, $$u/v = f/(f-v),$$

or $$uf - uv = fv,$$

or $$uf - fv = uv,$$

which gives the **general equation for lenses**, viz.,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad \text{or} \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}.$$

This equation is applicable to all types of lenses, providing that the rule given already (p. 294), as to positive and negative values, is observed. The points at which object and image may change places without necessitating any change of construction except the reversal of the directions of the rays are known as **conjugate foci**.

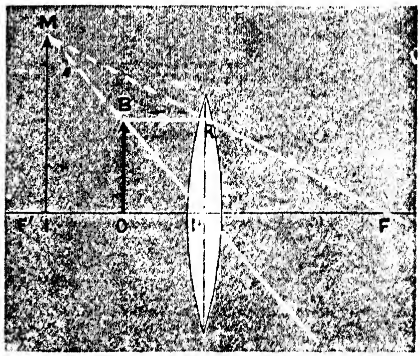


FIG. 204.—Virtual image, by means of a convex lens.

Image due to a convex lens.—The character of the image due to a convex lens depends upon the nearness of the object to the lens. In Figs. 204 and 205, let F be the principal focus of the lens. Mark off, to the left of the lens, a distance PF′ equal to the focal length PF.

CASE 1.—**Where the object is nearer to the lens than F.**

Consider the rays proceeding from a point B of a luminous object OB (Fig. 204). A ray BR, which is parallel to the principal axis, is refracted so as to pass through the principal focus F; and a ray BP, passing through the centre of the lens, is practically undeviated. The image of B must be at the point M where these paths intersect. Hence, IM represents the image of OB; and it is **erect, magnified** and **virtual.**

Fig. 204 explains the principle of the **pocket-lens** or **reading-glass** when used for slight magnification.

The general equation for lenses may be deduced from Fig. 204 in the following manner:

Since $OB/IM = PO/PI = u/v$,

and $PR/IM = FP/FI = -f/(-f+v)$;

then, since $OB = PR$, $u/v = -f/(-f+v)$.

Hence, $-uf + uv = -fv$

or $uf - fv = uv$,

or $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

CASE 2.—**Where the object is more distant than F.**

In Fig. 205 let OB be the object. By the same construction as before, the image of the point B is situated at M; and IM is the

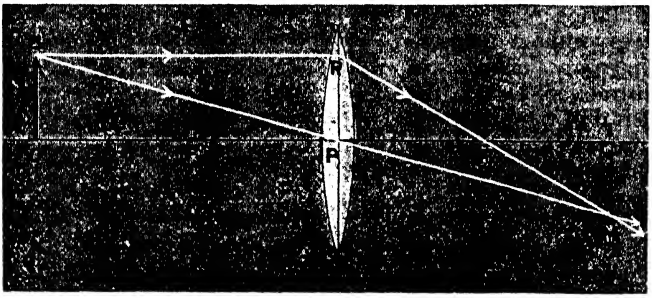


FIG. 205.—Real image, by means of a convex lens.

image of OB. Evidently the image is **real** and **inverted**; but the relative size of the image depends entirely upon the relative distances of the object and the image from the lens.

It will serve as a useful exercise for the student to deduce the general equation for lenses from Fig. 205.

Experimental determination of focal lengths.—The choice of methods of determining focal lengths of lenses depends entirely upon the apparatus available. Although a simple optical bench is desirable, yet fairly accurate results can be obtained without the aid of this appliance. The student is recommended to attempt the following methods.

CONVEX LENSES.

EXPT. 189.—**Parallel rays.** Support the lens in a vertical position and adjust its distance from a cardboard screen so as to form on the screen a real inverted image of some distant object (*e.g.* the window bars, or a distant chimney). The rays from the distant object are practically parallel, and the image is formed at the principal focus of the lens. Measure the distance of the screen from the lens.

EXPT. 190.—**Reflection method.** Bore a small circular hole through a white cardboard screen, and fix thin cross-wires across the hole with sealing-wax. Place a bright light behind and near the cross-wires. Support the lens so that the centre of the hole is on the principal axis of the lens. Place a plane mirror M (Fig. 206) close behind the lens, and adjust the distance of the lens L from the screen until an image of the cross-wires is formed on the screen P and close beside the circular hole. Evidently each ray of light is reflected back along its own path; and this can only be the case if the rays fall normally on the surface of M. Hence, the rays after passing for the first time through the lens are parallel to the principal axis, and they must therefore have originated from the principal focus. Therefore the distance f is the focal length of the lens.

FIG. 206.—Determination of the focal length of a convex lens.

EXPT. 191.—**Image and object.** In this method, use is made of the general equation $1/f = 1/v - 1/u$. The student may use the simple appliances (lens and two pins or needles) as described on p. 291, determining the position of the image by the Parallax Method. Or, he may use the illuminated cross-wires (p. 284) and a second cardboard

screen. In either case, several perfectly independent observations should be made, the distance u being different in each case. In calculating the value of f it must be remembered that, if the image is real, the distance v is negative.

Concave Lenses.

Expt. 192.—**Combination method.** Select a convex lens of known focal length, and of such converging power that, when fastened to the concave lens the combination is still converging. If necessary, determine the focal length (f_1) of the convex lens, preferably by the method of Expt. 190. Fasten the two lenses together and determine the focal length (F) of the combination. The focal length (f_2) of the concave lens can then be calculated by means of the equation $1/F = 1/f_1 + 1/f_2$. When substituting the values of f_1 and F it must be remembered that the focal length of a convex lens is always negative.

Expt. 193.—**Parallax method.** Determine the focal length of a concave lens by locating the position of the virtual image of a pin or needle by the Parallax Method (Expt. 188), and applying the equation $1/f = 1/v - 1/u$.

Magnification.—The ratio of the linear dimensions of the image and the object is termed the **magnification** (m). Thus, in Figs. 203-205, the magnification due to the lens in each case is equal to the ratio IM/OB.

It is evident from either of the diagrams that

$$m = \frac{\mathrm{IM}}{\mathrm{OB}} = \frac{v}{u}.$$

It is more convenient in some problems to express the magnifying power of a lens in terms of u and f. This can be deduced from the general equation, thus:

Since $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u},$$

then $$\frac{u}{f} = \frac{u}{v} - 1,$$

or $$\frac{u}{v} = \frac{u+f}{f}.$$

Hence, $$m = \frac{v}{u} = \frac{f}{u+f}.$$

Power of a lens.—The **power** of a lens is defined as the reciprocal of the focal length. A lens of focal length 100 cm. is recognised as having unit power; and this unit is termed the **dioptre.** It is also generally recognised that the power of a converging lens is *positive*, and that of a diverging lens is *negative.* Hence, the power of a converging lens of 50 cm. focal length is equal to $+1/0{\cdot}5=2$ dioptres; and that of a diverging lens of 25 cm. focal length is equal to $-1/0{\cdot}25=-4$ dioptres.

EXAMPLES.—1. The focal length of a converging lens is 50 cm. Find the nature, position, and size of the image of an object 5 cm. long which is placed vertically (i) 25 cm., (ii) 75 cm., (iii) 125 cm. from the lens

(i) Since $f=-50$ and $u=+25$, then

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f},$$

$$\frac{1}{v}=\frac{1}{u}+\frac{1}{f},$$

or
$$v=\frac{uf}{u+f}=\frac{25\times(-50)}{25-50}=\frac{-(50\times 25)}{-25}=+50 \text{ cm.};$$

$\therefore$ the image is virtual, and 50 cm. distant from the lens.

Also,
$$m=\frac{v}{u}=\frac{+50}{+25}=2.$$

Hence, the length of image $=2\times 5=10$ cm.

(ii) Since $u=+75$ cm., then

$$v=\frac{uf}{u+f}=\frac{75\times(-50)}{75-50}=\frac{-(50\times 75)}{25}=-150 \text{ cm.};$$

$\therefore$ the image is real, and 150 cm. distant from the lens.

Also,
$$m=\frac{v}{u}=\frac{150}{75}=2.$$

Hence, the length of the image is $(2\times 5)=10$ cm.

(iii) Since $u=+125$ cm., then

$$v=\frac{uf}{u+f}=\frac{125\times(-50)}{125-50}=\frac{-(50\times 125)}{75}=\frac{-250}{3}=-83{\cdot}3 \text{ cm.};$$

$\therefore$ the image is real, and $83{\cdot}3$ cm. distant from the lens.

Also.
$$m=\frac{v}{u}=\frac{83{\cdot}3}{125}=0{\cdot}664.$$

Hence, the length of the image is $0{\cdot}664\times 5=3{\cdot}32$ cm.

2. A piece of cardboard, in which a narrow slit is cut, is placed in front of a bright light. A screen is placed vertically at a distance of 49 inches from the slit. Where must a convex lens, of focal length 6 inches, be placed so as to give on the screen a well-defined image of the slit?

The data given are $u+v=49$ inches and $f=-6$ inches.

Since the image is real and on the opposite side of the lens to that of the slit, its distance from the lens must be regarded as *negative*. Hence, in the general equation $1/v-1/u=1/f$, we may substitute $v=-(49-u)$ and $f=-6$.

$$\therefore \quad -\frac{1}{49-u}-\frac{1}{u}=-\frac{1}{6},$$

or
$$\frac{-49}{u(49-u)}=-\frac{1}{6},$$

or
$$u^2-49u+294=0,$$

or
$$(u-42)(u-7)=0.$$

Hence, $u=+42$ inches, or $+7$ inches.

A distinct image will be obtained, therefore, if the lens is placed either 42 inches or 7 inches from the slit. In the former position the image is diminished, and in the latter position enlarged.

Solution of simple problems by means of squared paper—Squared paper may be used with advantage in solving simple problems on lenses, especially if the diagram so obtained is used as a verification of the result obtained by calculation. The following examples will indicate the method:

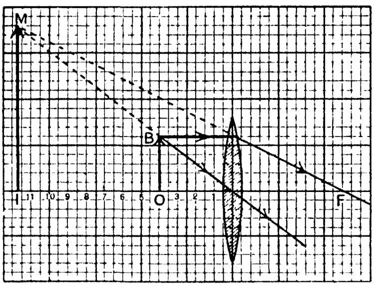


FIG. 207.—Graphical construction for image and object with convex lens.

EXAMPLES.—1. An object 3 cm. high is placed at a distance of 4 cm. from a convex lens of focal length 6 cm. Find the position and size of the image.

In Fig. 207, each division of the squared paper represents 0·5 cm. The position of the image IM is determined by the construction explained on p. 295. The diagram shows that the image is (i) virtual, (ii) approximately 11·8 cm. distant from the centre of the lens, and (iii) 11·8 cm. in height.

This result will be found to agree closely with that obtained by calculation from the general equation for lenses.

2. When an object is placed at a distance of 15 cm. from a converging lens the image formed on a screen is found to be twice the size of the object. Find the focal length of the lens.

Since the image is twice the size of the object, the distance of the former from the lens must be twice the distance of the latter from the lens. Hence, since the image is real, and therefore on the opposite side of the lens to that of the object, the distance of the screen from the object must be 45 cm.

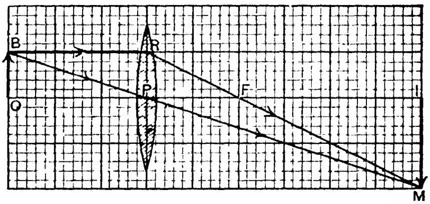


FIG. 208.—Focal length of a convex lens by graphical construction.

In Fig. 208, each division of the paper represents 1 cm.; mark off OP = 15 cm., and OI = 45 cm. Draw OB to any convenient scale, *e.g.* 5 cm., and make IM = 2OB. Draw the ray BR parallel to the principal axis, and join RM. The point F, where this ray intersects the principal axis, must be the principal focus; and PF is the focal length. By inspection PF = 9·8 cm. approximately.

EXERCISES ON CHAPTER XXII.

1. You are given a lens through which you can look, but which you are not allowed to handle. What tests would you apply in order to determine if it be concave or convex?

2. How does the image seen in a plane glass mirror differ from that seen on the ground glass of a camera, and how does each differ from the object in respect of size, and of position or inversion?

3. Define the principal foci of a thin convex lens. Draw a diagram showing the paths of the rays through such a lens from a luminous object situated on the axis of the lens at a distance of twice the focal length from its centre.

4. An object at a distance of 10 inches from a lens, when viewed through the lens appears to be at a distance of 30 inches from the lens. Find the focal length of the lens, and give a diagram showing the nature of the image.

5. A converging lens is employed to form an image of an object placed in front of it at a distance of 20 inches from the lens. If the image behind the lens is twice as large as the object, find, by a geometrical construction, the focal length of the lens.

6. A converging lens having the same focal length (5 cm.) as a concave mirror is placed with its axis coinciding with that of the

mirror, and with its centre at the centre of curvature of the mirror. Rays of light diverge from the principal focus of the lens remote from the mirror and fall on the lens. Draw a diagram to indicate their path through the lens to the mirror, and thence, after reflection, back through the lens. What difference would there be if the lens were placed with its centre at the principal focus of the mirror?

7. A pin, $\frac{1}{4}$ inch long, is placed at a distance of 3 inches from a convex lens of focal length 2 inches. Draw to scale a diagram to show the position and length of the image. Explain your construction.

8. A luminous point is situated 30 cm. in front of a lens, and an image is formed 10 cm. behind the lens. What kind of lens is used? and what is its focal length? Draw a diagram to scale, and verify the result of the calculation.

9. A pocket magnifying glass has a focal length of 5 cm. Find, by means of a diagram drawn to scale, the position and length of an object, 0·2 cm. long, placed 3 cm. from the lens.

10. The focal length of a camera lens is -20 cm. How far from the lens should the sensitized plate be in order to photograph an object 180 cm. in front of the lens? How large a surface at this distance could be photographed on a $\frac{1}{2}$-plate ($6\frac{1}{2}$ in. $\times\, 4\frac{3}{4}$ in.)?

11. A candle stands at a distance of 3 ft. from a wall. In what position must a convex lens of 8 in. focal length be placed between them so as to produce upon the wall a distinct image of the candle?

12. An object, 2 in. long, is placed 8 in. from a concave lens of 4-inch focal length. Find, by means of a diagram, the position and length of the image.

13. In order to find the focal length of a concave lens, it was blackened, with the exception of a circle 4 cm. in diameter at its centre. A beam of sunlight was allowed to pass through this, when it was found that an illuminated circle of 20 cm. diameter was formed on a screen held 64 cm. behind the lens and parallel to it. What was the focal length of the lens?

14. An object is placed 100 cm. from a concave lens. What must be the focal length of the lens so that a virtual image of the object can be formed at a distance of 25 cm. from the lens?

15. The focal length of a concave lens is 6 in., and a small object is placed 18 in. from the lens. Draw a diagram to scale, showing the path of the rays by which the image is formed, and determine its position.

16. A screen is fixed at a convenient distance from a lighted candle. It is found that a convex lens may be placed in two positions between the candle and screen so as to throw a distinct image of the flame on the screen. Explain this.

Being allowed to measure any distances you like, how would you determine the focal length of the lens?

17. A boy has a convex lens the focal length of which is 10 cm. How far from a screen must it be to get an image of the sun on the screen? How far from the screen must it be to get an image of a candle which is a metre from the lens? What is the power of the lens?

18. How would you find the focal length of a convex lens? An object is placed 20 cm. from a convex lens, and an inverted image is formed 4 times as large as the object. Find the focal length of the lens.

19. On a screen 1 foot behind a lens an image 6 inches long is formed of a man 5 feet 6 inches in height standing in front of the lens. Find the distance between the man and the lens, and the focal length of the latter.

20. Describe some optical (or other) method for measuring the radii of curvature of the surfaces of an ordinary lens. Upon what does the focal length of a lens depend besides the curvatures of its surfaces?

21. What procedure would you adopt to determine most correctly and easily the power of a concave lens?

22. A converging beam of light falls upon a diverging lens, the axis of the beam being coincident with the axis of the lens. Explain by means of a figure, or otherwise, the conditions for the emergent beam being convergent, parallel, or divergent.

23. A lantern slide is $3\frac{1}{4}$ inches square, and an enlarged image of it is to be formed by the aid of a lens of 6 inches focal length upon a screen 20 feet distant from the lens. What kind of lens should be used, at what distance from the slide must it be placed, and what will be the size of the image?

24. A converging lens has one of its faces plane, and the plane face is silvered. The lens is held in the path of light diverging from a small hole in a screen, the curved surface being towards the hole. When the lens is at a certain distance from the screen, a sharp image of the hole is thrown upon it. Explain, by means of a drawing, the path of the rays in their course from the hole to the image. What is the distance from the lens to the screen? Would the effect be the same if the experiment were repeated with the curved face of the lens silvered and the plane face turned towards the hole? Give reasons.

CHAPTER XXIII.

THE EYE AND OPTICAL INSTRUMENTS.

The eye.—Fig. 209 represents a horizontal cross-section through the centre of an eye. The outer coating consists of a hard thick membrane *s*, called the **sclerotic**. The front part *c* of the sclerotic is transparent, and is termed the **cornea**. A transparent lens-shaped body *o* of hard gelatinous consistency, termed the **crystalline lens**, is supported from the walls of the eye near to the cornea. The lens divides the eye into two chambers; the anterior chamber is filled with a watery liquid *a*, termed the **aqueous humour**, and the posterior is filled with a jelly-like substance *v*, termed the **vitreous humour**. Just in front of the lens is a contractile diaphragm *i*, the **iris**, with a circular orifice (the **pupil**) near to its centre. The **retina** (*r*), which is the portion of the eye's inner surface sensitive to light, is liberally supplied with nerve-fibres and blood-vessels. The nerve-fibres originate from the **optic nerve** (*n*), which enters the eye on the inside of the centre of the retina. Light falling upon these nerve-fibres appears to set up nervous stimuli which are transmitted to the brain, and these are interpreted as the phenomenon termed sight.

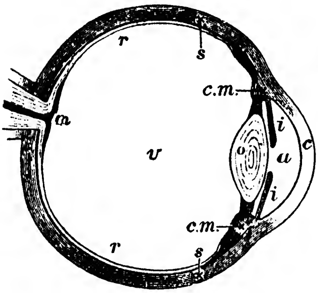


FIG. 209.—Horizontal cross-section through the centre of an eye.

Fig. 210 indicates how the cornea and crystalline lens give rise

to an inverted image of a distant object on the retina. Although the image is inverted, yet the mental interpretation of the effect upon the retina is just as though the images were erect.

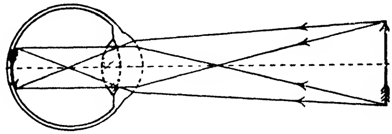


FIG. 210 —Formation of an inverted image on the retina.

It might be thought that a clear image of any object is only obtained when the object is at one given distance from the eye; but there is really a wide range of distance through which distinct vision is possible to a normal eye. This **power of accommodation**, as it is termed, is effected by an involuntary change in the curvature of the surfaces of the crystalline lens. The change in curvature is brought about by the unconscious action of the **ciliary muscle** (*c.m.*, Fig. 209) which surrounds the edge of the lens and is connected to the inner walls of the eye. When a near object is looked at, the ciliary muscle contracts and causes the lens to bulge more, thus increasing its diverging power. On the other hand, when looking at a distant object the ciliary muscle is relaxed, and the lens is thereby flattened. The power of accommodation is limited, for objects which are very near cannot be focussed clearly: nor can the details of very distant objects be seen clearly. The distance at which objects are seen with greatest distinctness by a normal eye varies from 25 to 30 cm.

Defective eyes.—The most common ocular defects are (i) **Short-sight** (or, Myopia), (ii) **Long-sight** (or, Hypermetropia), and (iii) **Loss of accommodative power** (or, Presbyopia).

(i) **Short-sight.**—A short-sighted eye cannot see distant objects distinctly. The eye-ball is usually too long; and parallel rays falling upon the cornea are brought to a focus F (Fig. 211, A) in front of the retina. If the object from which the rays proceed is brought nearer to the eye, the position of the image recedes; and, at a certain distance, vision becomes distinct. If brought still nearer, the power of accommodation enables the lens to thicken, and vision remains distinct. Since the power of accommodation is limited, vision becomes indistinct again if the object is brought too near.

Fig. 211, A, represents how parallel rays may be brought to a focus on the retina by placing a diverging lens L in front of the eye. The dotted lines indicate the path of the rays after passing through the lens. Thus, a short-sighted eye needs a diverging lens in order to see distant objects.

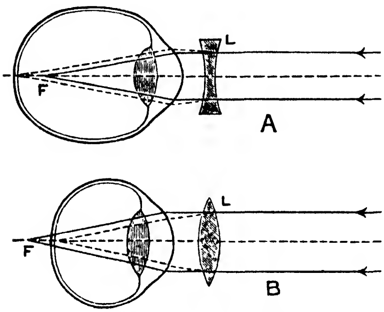


FIG. 211.—In A a double concave lens corrects the short-sighted eye, and in B the double convex lens corrects the long-sighted eye.

(ii) **Long-sight.**—A long-sighted eye cannot see near objects distinctly. When the eye is at rest, parallel rays are focussed behind the retina; but, as a rule, the accommodative power is sufficient to enable distant objects to be seen clearly, though this is not sufficient to focus near objects. The dotted lines in Fig. 211, B, indicate how, by using a converging lens L, the focus of parallel rays may be moved forwards on to the retina of a long-sighted eye.

(iii) **Loss of accommodative power.**—This defect is found usually in elderly people. It is noticed generally that, as the condition develops, the nearest point at which vision is distinct gradually recedes. For this reason, it is often the case that an elderly man finds that printed matter can be read only if held at arm's length.

An object (such as printed matter) held near the eye can only be seen clearly by an eye with this defect if seen through a convex lens, since by this means the image is more distant than the object (Fig. 204, p. 295). On the other hand, distant objects can only be seen with the aid of a concave lens, since the image is brought nearer than the object. Persons suffering from loss of accommodative power find it necessary, therefore, to use two different kinds of lenses, according to the distance of the object viewed.

Near point and far point.—The power of accommodation varies in different individuals, and in the same individual it

changes with progressive age. The normal eye of an adult can focus clearly an object which is not more than 10 or 12 inches distant from the cornea.

The nearest point to the eye at which a small object can be seen clearly is termed the **near point.** The point to which the eye is focussed when at rest is termed the **far point**; and, in the case of a normal eye, the far point is obviously at an infinite distance; but it is frequently found that the far point is within measurable distance of the eye.

The determination of the near point and the far point constitutes an instructive experiment. One method is based upon the fact that when an object is viewed through a convex lens of known focal length and placed quite near to the eye, a well-defined image is seen only when the image is situated between the near and far points. When the object is brought gradually nearer to the lens a point is reached when the image just ceases to be clearly defined; by using the formula on p. 296 the position of the image, and therefore of the near point, can be calculated. Similarly, when the object is withdrawn gradually, a position is reached when the image again ceases to be defined clearly, and the position of the image, obtained by calculation, gives the distance of the far point.

EXPT. 194.—**Determination of near point and far point.** Select a convex lens of known focal length (20 cm. is convenient). Draw a small square, in pencil, on a piece of cardboard, and support this vertically on a movable support behind the lens. Place one eye as near as possible to the lens, and move the cardboard slowly towards the lens until the image is just becoming indistinct; measure this distance. Repeat the measurement twice, and take the average d_1. The distance of the image, *i.e.* the distance of the *near point*, can be calculated by means of the formula $1/f = 1/v - 1/u$. Thus, in an actual experiment, $f = -20$ cm., and $d_1 = 10{\cdot}1$ cm.; hence

$$-\frac{1}{20} = \frac{1}{v} - \frac{1}{10{\cdot}1},$$

or

$$\frac{1}{v} = \frac{1}{10{\cdot}1} - \frac{1}{20},$$

or

$$v = \frac{202}{9{\cdot}9} = 20{\cdot}4 \text{ cm.}$$

Similarly, note the maximum distance, d_2, at which the image is just distinct, and calculate the distance of the far point. Enter the observations thus:

Eye.	d_1.	Near Point (calculated).	d_2.	Far Point (calculated).
Right	6·9	10·5	13·6	42·5
Left	10·1	20·4	14·7	55·5

The visual angle.—The apparent size of an object is proportional to the size of the image of the object upon the retina, and this depends upon the distance of the object from the eye—the nearer the object the larger the image. The apparent size of the object is measured usually by the angle which the object subtends at the eye; and this angle is called the **visual angle.** Thus, in Fig. 212, if E represents the eye and AB the object, the visual angle is AEB; if *ab* represents the object the visual angle is *a*E*b*.

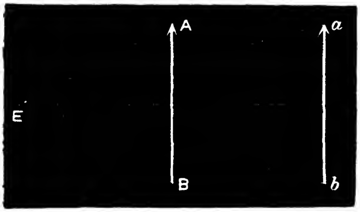


FIG. 212.—Visual angles AEB and *a*E*b*.

The simple magnifying glass.—The apparent size of an object, when viewed by the naked eye, is a maximum when the object is situated at the near point. The apparent size would be increased if the object were brought still nearer, but the image would be indistinct. When, however, the object is viewed through a convex lens held near to the eye, the object may be brought nearer to the eye than the 'near point,' and still the image may be well-defined, since the virtual image due to a convex lens is further from the lens than the object. Fig. 213 represents how a virtual image of an object AB is seen

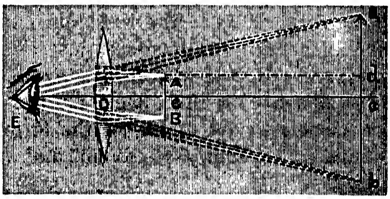


FIG. 213.—The simple magnifying glass.

at ab; and it is evident that the visual angles of the object and of its image are the same. Hence the lens serves the purpose of enabling the visual angle to be increased while maintaining at the same time distinct definition.

In determining the **magnifying power** of a lens it is usual to adjust the position of the object so that the image is situated at the near point; and the magnifying power may be defined as *the ratio of the visual angle of the image to that of the object if it were situated at the 'near point.'*

In Fig. 213 suppose that c is the 'near point,' then

$$\text{magnifying power } (m) = \frac{ac}{dc} = \frac{ac}{\text{AC}} = \frac{\text{O}c}{\text{OC}}.$$

If Oc be the distance d of the 'near point,' then, since the lens is converging and its focal length therefore negative,

$$-\frac{1}{f} = \frac{1}{d} - \frac{1}{u},$$

and
$$\frac{1}{u} = \frac{1}{d} + \frac{1}{f},$$

or
$$u = \frac{df}{d+f}.$$

Hence,
$$m = \frac{\text{O}c}{\text{OC}} = \frac{d}{u} = \frac{d+f}{f}.$$

EXAMPLES.— 1. A long sighted person cannot see objects clearly at a distance of less than 50 cm. Find the power, in dioptres, of the glasses required in order to read print held 15 cm. in front of the glasses.

The focal length must be such that when an object is 15 cm. in front of the glasses, the image is at least 50 cm. distance. Hence, $u = +15$ cm., and $v = +50$ cm.

$$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{50} - \frac{1}{15} = -\frac{35}{750},$$

or
$$f = -21{\cdot}4 \text{ cm.},$$

or
$$\text{power} = +\frac{1}{0{\cdot}214} = +4{\cdot}67 \text{ dioptres}.$$

2. A person with short-sight is able to read print only when held 15 cm. from the eye. What kind of glasses, and of what focal length, are necessary in order that print held at a distance of 25 cm. from the eye may be read clearly?

In this case, $u = +25$ cm., and $v = +15$ cm.;

$$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{15} - \frac{1}{25} = +\frac{2}{75},$$

or $$f = +37 \cdot 5 \text{ cm.}$$

Hence, concave glasses of focal length 37·5 cm. are required.

The photographic camera.—The photographic camera consists of a rectangular box with two opposite sides vertical and rigid.

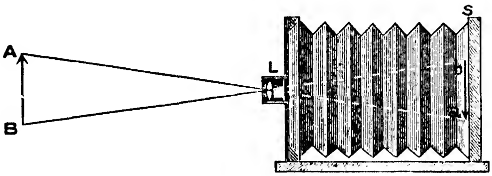


FIG. 214.—Principle of the photographic camera.

The distance between these sides can be adjusted by the remaining sides being made extensible, and consisting of folded leather or other opaque material. A circular opening in one of the rigid sides serves to carry a converging lens (L, Fig. 214); and the opposite side is fitted with a translucent screen S of matt glass, which can be replaced by a photographic plate.

When the camera is placed with its lens directed towards a distant object AB, a real inverted image *ab* is formed on the other side of the lens. The screen S is moved towards the lens until its position coincides with *ab*, when a well-defined image of the object can be seen on the screen. A photographic plate consists of a glass sheet covered on one side with a gelatine film loaded with chemical compounds sensitive to actinic (p. 233) light rays. When such a plate is substituted for the screen, and exposed for a definite period to the rays passing through the lens, subsequent development of the film reveals a permanent reproduction, in black and white, of the distant object. The plate, when developed, is termed a **photographic negative.**

If the upper part of the picture be more distant than the lower part from the lens, then in order that the whole image may be defined with equal clearness in all parts, the lower part of the screen must be placed nearer to the lens than the upper part. For this reason, the frame carrying the screen is frequently hinged along its lower side to the base of the apparatus, so that it may be tilted backwards or forwards according to the relative distances of the upper and lower parts of the picture.

The optical lantern.—The optical lantern consists essentially of a bright source of light and a system of converging lenses

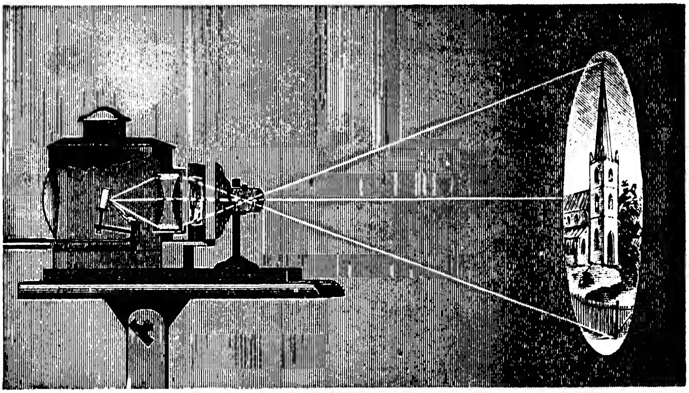


FIG. 215.—The optical lantern.

arranged so as to throw on a screen an enlarged inverted image of a transparent photograph or drawing, or of any object constructed in a manner suitable for projection on a screen. The rays of light proceeding from the source of light (Fig. 215) pass through the transparency (or 'lantern slide') which is supported in a suitable carrier; the rays which are not absorbed by the dark parts of the slide then pass through a **focussing lens,** which is adjusted in position so that the slide is slightly beyond its principal focus, and a real inverted and enlarged image of the slide is thrown on a distant screen. The **condensing lenses** which are usually a combination of two lenses and are nearest the source of light, serve two purposes: (i) to increase the total amount of light concentrated on the slide, and (ii) to deflect the rays of light

passing through the outer portions of the slide sufficiently for their paths to pass through the focussing lens. In the absence of a suitable condenser, the image on the screen would be much less bright, and it would represent the middle part only of the slide.

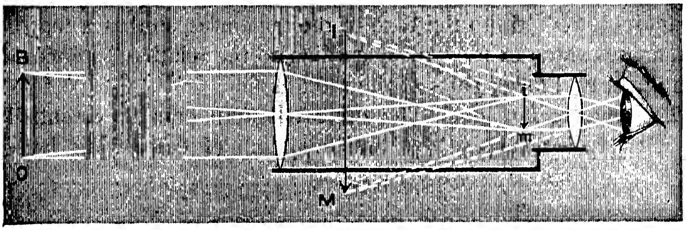


FIG. 216.—Principle of the astronomical telescope.

The astronomical telescope.—In order to obtain a magnified image of a distant object it is necessary to use more than one lens. Fig. 216 represents the principle of the astronomical telescope, which consists usually of a large lens, of considerable

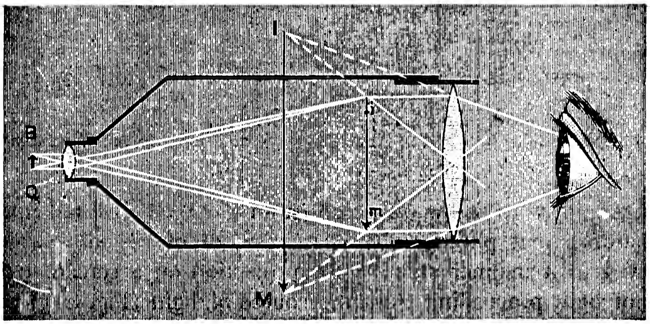


FIG. 217.—Principle of the compound microscope.

focal length, called the **object-glass**, and a smaller lens of short focal length called the **eye-piece**. The diagram represents several rays coming from a point of a distant object, and, since the distance is assumed to be great, these rays are practically parallel. A real, inverted, and diminished image *im* of the object is formed in a position which practically coincides with the focus of the object-glass. The eye-piece is adjusted so that *im* is just within

its principal focus. On looking through the eye-piece a virtual and enlarged image of *im* is seen at IM. The dotted lines in the diagram indicate how the position of this image is determined.

The compound microscope.—In the compound microscope (Fig. 217) a small object-glass of short focal length is used instead of the large object-glass of long focal length employed in the telescope. If the object OB under observation be placed just beyond the principal focus of the object-glass, a magnified, real, and inverted image is obtained at *im*. When this image is viewed through the eye-piece, a virtual image IM of a much larger size is seen.

EXERCISES ON CHAPTER XXIII.

1. A man who can see distinctly at a distance of 1 foot, finds that a certain lens when held close to his eye magnifies small objects 6 times. Determine the focal length of the lens.

2. A man who can see most distinctly at a distance of 5 in. from his eye, wishes to read a notice at a distance of 15 ft. off. What sort of spectacles must he use, and what must be their focal length?

3. A long-sighted person can only see distinctly objects which are at a distance of 48 cm. or more. By how much will he increase his range of distinct vision if he uses convex spectacles of 32 cm. focal length?

4. A long-sighted person uses convex glasses of 40 cm. focal length, and finds that he cannot read print through them comfortably when it is held nearer than 30 cm. What is his nearest point of distinct vision?

5. Describe the simple magnifying glass and explain its use. Draw a figure showing the position of object and image and the course of light when a magnifying glass (focal length = 3 cm.) is used by a person whose distance of most distinct vision is 15 cm.

6. A person, whose distance of most distinct vision is 10 cm., uses a simple magnifying glass to view a certain object. The focal length of the magnifying glass is 2·5 cm. Draw a figure showing the relations between the linear dimensions of object and image, and trace the course of a small pencil of light from a point of the object to the eye.

7. Describe, explaining how the image is formed, either a simple microscope or a simple astronomical telescope.

8. A patient cannot see objects clearly at a distance of less than 36 inches; find the focal length of the glasses required to enable him to read at 10 inches.

9. A short-sighted person cannot see objects clearly at a distance greater than 6 inches. What spectacles would be required to enable him to see distant objects clearly? If his least distance of distinct vision without glasses is 3 inches, what would it be with the above spectacles?

10. Two converging lenses, A and B, are fixed vertically with their axes coinciding and with their centres 70 cm. apart. The focal lengths of A and B are +35 cm. and +20 cm. respectively. A capital letter 10 cm. high, cut from a poster, is fixed vertically 100 cm. from the centre of A and on the distant side from B. The letter is viewed through both lenses by an eye placed near to B. Draw a diagram to scale, showing the position and size of the image of the letter.

11. A reading-glass of 8 cm. focal length is used by a person whose 'near point' is at a distance of 24 cm. What is the magnifying power of the glass?

12. How would you arrange two convex lenses to form a telescope? What is the purpose of each lens?

CHAPTER XXIV.

COMPOSITION OF LIGHT.

Dispersion.—The multicoloured rainbow, varying from violet at the inner edge to red at the outer, is a familiar phenomenon; and the colour effects seen when the edge of a window frame is viewed through a prism, held with its refracting edge parallel to the object, are also familiar appearances. In this simple experiment we are repeating that historical experiment with which Newton proved that ordinary white light is a complex mixture of coloured lights. Fig. 218 represents the nature of this experiment. A beam of sunlight was caused to fall upon a glass prism held so as to refract the light downwards upon the opposite wall of a dark room. The light was found to be drawn out into a long coloured band, violet at the lower end and red at the upper. This coloured band Newton termed the **spectrum.** The colours violet, indigo, blue, green, yellow, orange, and red are clearly distinguishable, although each colour changes, by insensible

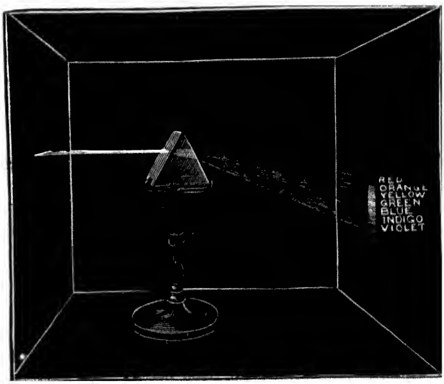


FIG. 218.—Dispersion of light by a prism. (Newton's experiment.)

gradations, into the next. It is evident that, when passed through a prism, violet rays are bent more than yellow rays, and yellow rays more than red rays; or, expressing the same fact in other words, violet rays are more **refrangible**, that is, deflected more from their original path than yellow rays, and yellow more refrangible than red rays. The separation of the different colours owing to this difference in refrangibility, is termed **dispersion.**

In order to understand why we can obtain a spectrum by this simple means, it is necessary to realize that the condition which gives rise to the sensation of light really consists in the rapid transmission of a kind of wave-motion through the luminiferous ether (p. 233)—a medium which fills all space, even a perfect vacuum, and occupies the intermolecular spaces in all forms of matter. There is reason to believe that when light is being transmitted through the medium, there exists within it a vibratory

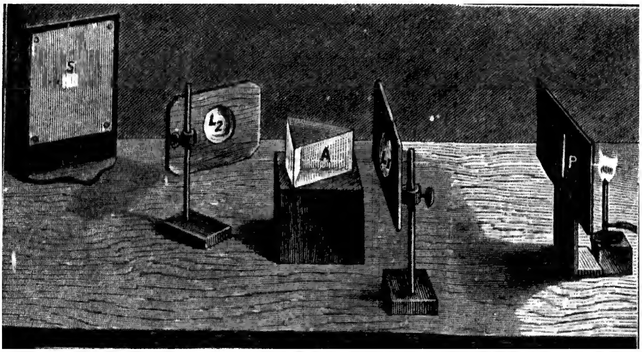


FIG. 219.—Experiment to show the dispersion of light by a prism.

condition, the vibrations taking place at right angles to the direction in which the light is travelling; in this sense light waves may be compared to the ripples on a liquid surface. In just the same way that we can set up either long or short ripples in water, so the waves of light may be of different length; and it is this difference in wave length which determines the colour of the light. The shortest waves to which the eye is sensitive give rise to the sensation of violet, and the longest to that of red. By special means it is possible to detect waves shorter than violet, or longer than red; but they cannot be observed by means of the unaided eye.

EXPT. 195.—**Dispersion by a prism.** In a piece of card cut a slit (P) about 2 cm. long and 1 mm. wide. Place the card, with the slit vertical, in front of a fish-tail gas flame (Fig. 219); and adjust the position of a converging lens (L_1) so that it is approximately at a distance equal to its focal length from the slit. Place a second lens (L_2) a few inches in front of L_1 and so that their axes coincide. Adjust the screen (S) so that a well-defined image of the slit is seen on its surface. Measure the distance between L_2 and S. Arrange a prism (A) on a stand, so that it is of the same height as the slit, and has its refracting edge vertical; and adjust its position so that the rays emerging from L_1 are incident at a suitable angle upon one of its faces. Catch the light emerging from the prism by a second lens (L_2). Move the position of the screen (S) until the coloured band of light is best seen. The distance of the screen from L_2 should be the same as before. Observe that the light is refracted towards the base of the prism, and that it is decomposed into constituent colours, which are differently bent by the prism. The violet light is refracted most and the red light least. Colours between these limits are bent by intermediate amounts. Name the colours you can see.

Analysis of light by a prism.—When a beam of sunlight is made to pass through two prisms similarly arranged instead of one, the coloured band or spectrum produced is longer, that is, the dispersion is greater. The amount of dispersion also depends upon the material of which a prism is made. Glass produces a much greater amount of dispersion than water; flint glass possesses twice the dispersive power of crown glass; carbon bisulphide, again, has even more dispersive power than flint glass.

Although a continuous band of colour is observed when sunlight, or limelight, or a gas- or candle-flame is seen through a prism, this **continuous spectrum** is not always produced. For when substances such as sodium, strontium, and lithium, or their compounds, are burnt in a non-luminous flame, and the light from the coloured flame observed through a prism, a spectrum is seen consisting of bright lines, which are different for different substances. A prism may thus be used, and is used, to analyse light. The light of incandescent sodium vapour, produced by burning common salt in a flame, when observed through a prism is characterised by a yellow line, and the light emitted by other substances when burning are each distinguished by rays of a

1. By interposing a second prism of the same material and angle as the first, with its angle reversed. The dispersion of the first prism is neutralised, and the beam of light leaves the second prism in a direction parallel to the beam incident upon the first prism.

2. By the colour disc.

EXPT. 196.—**Recomposition of light by means of a second prism.**—Form a spectrum as described in Expt. 195. Place a second prism (B, Fig. 221), of the same material and having the same refracting

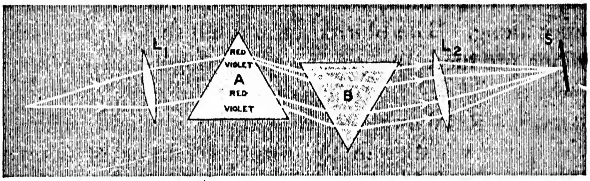


FIG. 221.—Recomposition of light.

angle as prism A, in the position shown in the diagram, adjusting the second prism so that the rays fall upon one of its faces, and so that its faces are parallel to those of the first prism. Place the lens (L_2) and the screen (S) in the same relative position as before, and observe on the screen the *white* image of the source of light. Evidently the different colours of the spectrum have been recombined to form white light.

It is an important fact that, by the simple laws of refraction, the path of each of the rays through B must be parallel to its path through A; also, since the rays emerging from B must be parallel to the rays incident upon the prism A, the former rays must constitute a parallel beam, and they will be brought to a focus at the principal focus of the lens L_2, and a white image therefore is formed.

EXPT. 197.—**Recomposition by colour disc.** Upon a round piece of card paint sectors of the different colours contained in the spectrum, arranging the areas of the coloured sectors as nearly as possible in the proportion in which they occur in the spectrum.

Place the card upon a whirling table or upon a top, and rotate it rapidly, when it will be found that light from the card gives rise to the sensation of an impure white or grey.

The colour disc.—The explanation of the recombination of the separate colours of the spectrum by means of a rapidly revolving disc, as in Expt. 197, is very simple. It is due to

what is called the **persistence of images** on the retina of the eye. Each impression the retina receives lasts for a certain length of time—about one-tenth of a second. It is not an instantaneous impression only. Think of the common trick of whirling round a stick with a spark on the end which gives rise to the impression of a continuous circle of light. This is because the second impression of the spark is received by the eye before the first impression has died away. Similarly, the impression of one sector, say, a red one, has not disappeared before the next is received, and while these compounded impressions linger a third one comes along. The blurred total of all these rapidly occurring impressions produces the greyish white tinge seen when a colour disc is whirled.

Rainbows.—Rainbows are caused by sunlight falling upon drops of water whether in the form of rain or of spray. The observer must have his back to the sun; and the centre of the bow is the point in the sky directly opposite to that occupied by the sun at the time of observation. In the **primary rainbow** the red colour is on the outer edge and the violet on the inner edge, the other colours of the spectrum being between them: in the **secondary rainbow** sometimes seen above the primary one this order of colours is reversed, the red being on the inner edge and the violet on the outer. Suppose the sun to be on the horizon when an observer sees these rainbows. The centre of the bow would be on the opposite point of the horizon, the red top of the primary rainbow would be at an angle of about 42° above the horizon, and the violet edge would be about 2° below the red: the angular elevation of the secondary rainbow would be about 52°. The radius of the primary rainbow as a whole is always about 41° and that of the secondary bow about 52°. When, therefore, the sun is on the horizon the bows seen are the largest possible. As the position of the sun above the horizon increases, the centre of the bows gets more and more below the horizon, the arcs visible become smaller and smaller until, when the sun has an altitude of about 41°, the primary bow disappears; while the secondary bow also becomes invisible when the sun's altitude exceeds 52°. This explains why rainbows are never seen in the British Isles in the middle of the day in summer.

The optical cause of the rainbow is a little difficult to explain exactly in an elementary book, but Fig. 222 may enable the student to comprehend the general principle involved. Suppose the sun's rays to be falling in the direction indicated upon the raindrops A_1, A_2, A_3, A_4. The light falling upon A_1 passes into the raindrop, is internally reflected along the path *abc* and emerges dispersed into the spectrum colours from violet *v* to red *r*. Light entering the second drop A_2 is similarly affected. From the lower drop violet light reaches the observer's eye; and red light from the upper drop, while the various drops which may be imagined between these two contribute the intervening colours of the spectrum according to their positions. In the case of the secondary rainbow, the sun's rays undergo double internal reflection in the raindrops, as shown in A_3 and A_4. Light enters A_3, and after traversing the path *mopq* emerges, broken up into the spectrum colours from red to violet in the order indicated in the figure. Red light thus reaches the observer's eye from A_3 and violet light from A_4; and the intermediate colours are formed in similar manner.

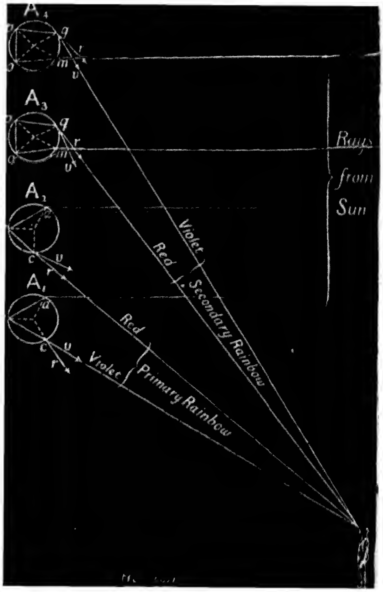


Fig. 222.—The formation of a rainbow. Adapted from the *U.S. Monthly Weather Review*.

Colour.

Colour of transparent bodies.—The colour of transparent bodies is due to the constituents of white light transmitted by them. A blue solution through which the light from a lantern is passed is blue because, of all the colours of the spectrum, it is able to transmit easily only the blue rays; the others—green, yellow, orange, red, etc.—are absorbed by the solution. Consequently, if this trans-

mitted blue light falls upon a sheet of red glass it is, in its turn, absorbed; red glass only transmits red light, that is why it is red. So that a combination of the blue solution and a piece of red glass is quite opaque to light—none of the colours of the spectrum can pass. Similarly, pieces of red and blue glass together are, if thick enough, quite opaque. When a strip of coloured glass, or a solution in a narrow test tube, is held between a spectrum and a screen, it appears as a black shadow upon the screen in all parts of the spectrum except in the colour which it is able to transmit. Transparent bodies like glass, water, and so on, transmit all the colours of the spectrum with nearly equal facility, and therefore appear colourless when only thin layers are used. As, however, no substance transmits light of all wave-lengths equally, there is no perfectly transparent body.

EXPT. 198.—**Absorbed and transmitted rays.** (i) Make an Oxford blue liquid by adding ammonia solution to copper sulphate dissolved in water until the precipitate formed is re-dissolved. Place the blue liquid thus made in a glass cell. Focus the round hole of a lantern cap on the screen. Interpose the filled cell. Notice the pure blue colour. Now interpose red glass either before or behind the cell. Notice that no light can pass now.

(ii) Use a hollow prism containing carbon bi-sulphide to produce a long spectrum of a slit on the lantern cap. Pass the filled glass cell from the last experiment through the spectrum, and notice that it is only able to transmit blue light. Repeat the experiment with as many coloured transparent solutions as possible, *e.g.* a solution of bichromate of potash and of permanganate of potash. Notice in each case that a particular liquid is only able to transmit light of its own colour.

Colour of opaque bodies.—The colour of opaque bodies is due to the constituents of white light which they reflect. If the light from a lantern in an otherwise dark room be made to fall upon sheets of cardboard which have been painted with various brilliant colours, and the light reflected from the coloured sheets be caught on a white surface, it is at once seen that the colour of the light reflected is the same as that of the card from which it comes.

Coloured opaque bodies when passed through a spectrum only appear coloured when in that part of the spectrum which is the colour they appear to have in white light. A red substance like sealing-wax is red only when there are red rays falling upon it

which it can reflect. The sealing-wax absorbs all the other constituents of white light; and hence when it is held in blue light, or light of any other colour than red, since all the light rays of this colour are absorbed, no light is reflected and it appears black. A white opaque substance, like a sheet of paper, appears white because it reflects all the constituents of white light equally well. Similarly, when a card painted violet is passed through a spectrum it only appears violet when in the violet rays, and in all other colours it seems black, because it cannot reflect these colours.

EXPT. 199.—**Absorbed and reflected rays.** (i) Paint sheets of cardboard with various brilliant colours. Send the light from a lantern in an otherwise dark room upon them, and catch the reflected light on white sheets of cardboard. Notice that the colour of the light reflected is the same as that of the card from which it is reflected.

(ii) Pass through the same spectrum various coloured opaque bodies, *e.g.* a rod of sealing-wax. Notice in this case it is only coloured when passing through the red rays. It appears a dull grey colour in most parts of the spectrum. Observe that green leaves are only coloured when passing through the green part of the spectrum.

Selective absorption and transmission.—In every case the colour of a body depends on selective absorption or selective transmission. Of the coloured rays of white light one portion is absorbed at the surface of the body. The body is **coloured and transparent** if the unabsorbed portion traverses it; if, on the contrary, the unabsorbed light is reflected the body is **coloured and opaque.** In both cases the colour depends upon the constituents of white light which are left to reach the eye after the other constituents have been absorbed. Bodies which reflect or transmit all colours in the proportion in which they exist in the spectrum are white; those which reflect or transmit none are black. Between these extreme limits infinite tints exist depending on the smaller or greater extent to which bodies reflect or transmit some colours and absorb others.

Bodies have no colour of their own; the colour of a body changes with the light which falls upon it. It is interesting to remember that this absorption of certain constituents of light necessitates a using up of energy. But since energy cannot be destroyed it is in these cases converted into heat. Theoretically,

a blue glass would get hotter than a red one, because the former absorbs all the red rays, and these have a greater heating effect than blue rays.

EXERCISES ON CHAPTER XXIV.

1. Describe and explain the effects observed when cards coloured bright red, green, and blue respectively, are passed from the red to the blue end of the spectrum.

2. Some glass houses in which ferns are grown are constructed of green glass. Describe the appearance, to an observer in such a house, of a lady in a red costume carrying a book with a bright blue cover. Give reasons for your answer.

3. How would you explain to a class of children the effect of a stained glass window upon sunlight? What simple experiments would you perform to convince them of the truth of your statements?

4. A ray of white light is passed through a glass prism; make a sketch showing how the direction of the ray is changed by its passage through the prism and the order of the colours seen when the light falls on a screen.

How would you show that when these colours are re-combined white light is produced?

5. Describe an arrangement by means of which a spectrum may be formed upon a screen.

If the light is made to fall upon a piece of red glass before reaching the screen, how and why will the spectrum be affected? What would the effect have been if blue glass had been used?

6. How can it be proved that:

(*a*) White light is a mixture of many colours? (*b*) Different colours have different degrees of refrangibility?

7. What is meant by the dispersion of light? On what fact does it depend?

8. Explain the term refrangibility as applied to a ray of light. Are rays of all colours equally refrangible?

9. It is sometimes said that "red glass colours the sunlight red," and that "blue glass colours the sunlight blue." Mention facts or experiments which show that this is not accurate. Put the statement in a more accurate form.

10. Bright sunlight falls obliquely upon the surface of the water contained in a white china basin; a penny is held near the surface of the water, and in such a position that its shadow falls upon the bottom of the basin. Parts of the shadow are found to be edged with colour. What colours may be observed? On what part of the shadow is each to be seen? How do you account for the colours?

11. What is a prism ? Give a diagram showing the course of a ray of white light through a prism made of glass. Which colour is refracted most, and which least ?

12. Describe the essential parts of either (*a*) a spectroscope, or (*b*) an astronomical telescope.

13. Explain the coloured fringes seen when objects are viewed through a glass lustre or prism.

14. Why does a field poppy appear red? What experiment could you arrange to make it appear black?

15. Several coloured cards (*e.g.* red, blue, green) are lying on a table in a lighted room. State how far the colours could be identified by a person seeing them through a piece of cobalt (blue) glass. Give reasons for your answer.

16. Light from a slit is allowed to fall on a prism. State and explain what may be observed when the slit is illuminated with (i) sodium flame, (ii) white light.

17. Describe a spectroscope. What will be observed in the instrument when the light passing through the slit comes from (i) a spirit lamp with a salted wick, (ii) an electric light, (iii) an electric light in a red glass globe?

18. How would you test, in a given instance, whether the light transmitted by a blue glass is homogeneous or not?

PART V.

SOUND.

CHAPTER XXV.

VIBRATORY MOTION.

Wave motion.—The circular waves set up when a stone is dropped into still water is a phenomenon familiar to all. Although the waves appear to travel outwards, yet the water itself is not travelling so; for, if a row of corks be floating on the water the corks move *up and down* only, and their distance from the centre of the waves is not increased: in fact, it is the 'disturbance' alone which travels outwards. This is an example of true wave motion, which can be defined in the following terms: **Wave motion consists in the repeated motion of a series of particles, the motion being handed on from each particle to its neighbour.**

When the stone touches the water, it causes a depression; and, like all elastic bodies which, when deformed, tend to resume their original shape, the water tends to recover its original level. Consequently, water flows into the depression; but, in so doing, the property of **inertia** (p. 103) causes the inflowing water to 'overshoot the mark,' and the depression is followed by an elevation. This is again followed by a depression, and so on. But the disturbance is not restricted to this one point; for, through the agency of the property termed cohesion (p. 57), the disturbance is handed on to neighbouring particles. In fact, the condition resembles what might be observed with a row of pendulums, the bobs of which are joined by spiral springs. A transverse motion to and fro of the **first**

pendulum will influence the next pendulum, imparting to it a similar motion. This will be handed on to each pendulum in turn, resulting in a kind of wave motion along the row. In this case the persistence of the motion is due to the inertia of each pendulum, and its transference along the row is due to the elasticity of the springs connecting the bobs. Subsequent paragraphs will show how a wave motion, of one type or another, may be propagated through any medium possessing these properties of inertia and elasticity.

Simple harmonic motion.—Suppose a simple pendulum to be set swinging so that its bob describes a circular path: it will traverse the path with uniform velocity. If looked at downwards in a sloping direction, the path will appear to be an ellipse (Fig. 223); but if the eye be placed at the same horizontal level it will appear to move along the straight line A*bcd*.... Suppose the circular path to be divided into 16 equal portions, AB, BC, etc.; then when viewed horizontally, the distances A*b*, *bc*, *cd*, etc., will be the apparent distances traversed in equal time intervals. At both A and I the bob will appear to be momentarily at rest: and its velocity will appear to be a maximum at *e*. This apparent motion along the path A*bcd* is termed **Simple Harmonic Motion** (S.H.M.).

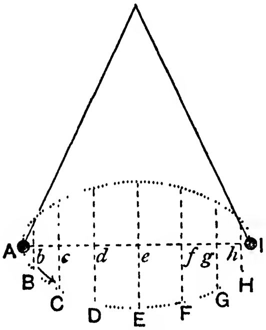


Fig. 223.—A simple pendulum, describing a circular path.

The total time-interval occupied in passing from its position of rest, at *c*, until it again passes the same point in the same direction is termed the **period** of the vibrating body. The **phase** of the vibrating body at any instant is the fraction of a total period which has elapsed since passing the position of rest in a given direction, *e.g.* from left to right. Thus, the phase when passing *f* from right to left will be 7/16. The **amplitude** is the extreme distance to which the body moves from its position of rest; thus, in Fig. 223, the amplitude is *e*A.

Transverse wave motion.—In Fig. 224 (i), let A, B, C, ... represent the path of a particle moving up and down with S.H.M. The positions A, B, C, etc., occupied after equal intervals of time, are located by means of the generating circle,

the circumference of which is divided in this case into 12 equal parts.

Suppose a similar motion to be set up in a number of particles, 1, 2, 3, 4, etc., arranged along a straight line at equal distances apart; and suppose the motion to be handed on from one particle to the next. Let the amplitude of each particle be equal to AD, and let consecutive particles differ in phase by $\frac{1}{12}$ of a complete period.

If particle 1 be moving downwards through its position of rest, particle 2 will then be moving downwards at *a*; particle

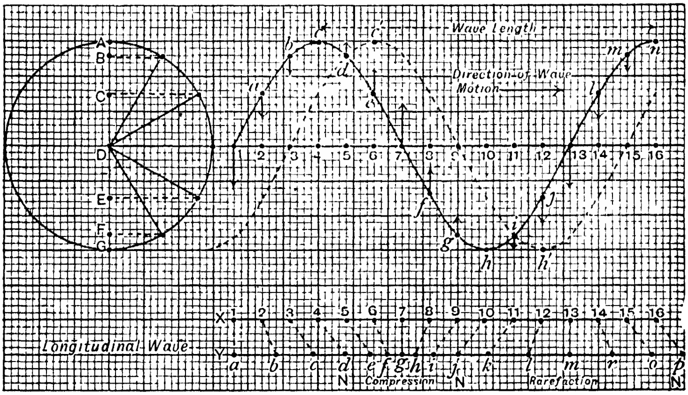


FIG. 224.—Wave motion.

4 will be momentarily at rest at *c*; particles 5 and 6 will be moving upwards at *d* and *e* respectively; 7 will be passing its position of rest in an upward direction, and so on. If these instantaneous positions are joined by a continuous line, a wave outline is obtained which closely resembles the outline of a water wave.

The dotted outline indicates the instantaneous positions occupied by the particles at a later moment, viz., $\frac{1}{6}$ of a period later. The elevation at *c* has now moved forward to the position *c′*; and, after one complete period has elapsed, the elevation at *c* will have travelled to the point *n*. Thus, the wave motion is propagated onwards; and, since the particles vibrate

transversely to the direction in which the waves are moving, the effect is termed a **transverse wave motion.**

The **wave length** is defined as the least distance between any two particles which are in the same phase; thus, the distances *cn*, *bm*, or *al* are equal in each case to the wave length.

The **velocity** of the wave motion is the distance through which the disturbance is propagated in unit time. The **frequency** is the number of complete waves which pass a given fixed point in unit time.

Suppose that n wave crests pass any fixed point in unit time, and that the wave length is l. Then, if V be the velocity of the wave motion, the first crest will have travelled in unit time to a distance V; hence

$$V = n \times l,$$

or velocity = frequency × wave length.

Transverse waves cannot be transmitted through gases, for the elasticity of a gas is brought into operation only when the gas is compressed or rarefied. Suppose, then, that the consecutive particles of a gas are set in vibration to and fro along, instead of transversely to, the line of disturbance. The distances apart of consecutive particles will vary—at one instant the particles being closer together than normally, and further apart at another instant. Such waves are termed **longitudinal waves**; and it may be proved that the transmission of sound through gases involves this type of wave only. Sound is transmitted also through solids and liquids by means of the same type.

Longitudinal wave motion. In Fig. 224 (ii) a row of equidistant particles 1, 2, 3, ... are arranged along the straight line X. Suppose that the particles are set in S.H.M. horizontally, and that the phase difference between consecutive particles is equal to $\frac{1}{6}$ of a complete period. The wave curve *abcd*... of Fig. 224 (i) may be regarded as a *displacement curve* for determining the instantaneous positions of the particles: an upward displacement in Fig. 224 (i) corresponding to a displacement forwards in Fig. 224 (ii), and a displacement downwards to a displacement backwards. Thus, the instantaneous positions of the particles will be represented by the points *abcd*... along the line Y. It is evident that, at the instant represented, the

particles 5 to 9 are closer together, and the particles 11 to 15 are further apart, than normal; or, in other words, these are regions of compression and of rarefaction respectively. Also, the pressures in the neighbourhoods of particles 4 and 10 must be normal. A longitudinal wave may be considered, therefore, to consist of a sequence of compressions and rarefactions separated by regions of normal pressure; and this condition is transmitted onwards with definite velocity.

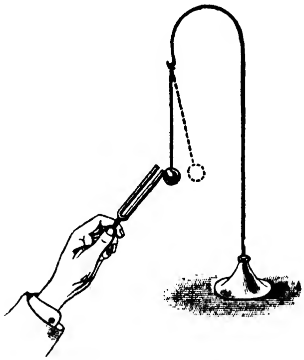


FIG. 225.—Expt. 200.

The production of sound. The origin of all sound is motion. Thus, when sound is derived from a stretched string, its indistinct outline shows that it is vibrating rapidly to and fro.

EXPT. 200.—**Vibration and sound.** Strike the end of one prong of a tuning-fork on the knee, or on a hard cushion. Hold the fork so that the outer side of one of the prongs touches either (i) the lip, or (ii) a pith ball suspended by thread (Fig. 225) or (iii) the surface of water contained in a beaker. The effect, in either case, demonstrates that the prongs are vibrating to and fro.

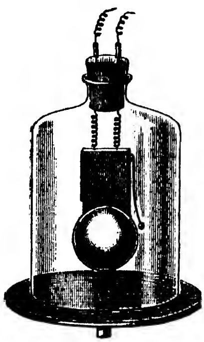


FIG. 226.—Experiment to show that sound is not transmitted through a vacuum.

The transmission of sound.—The fact that some definite medium is necessary for the transmission of sound may be demonstrated by means of the apparatus shown in Fig. 226. An electric bell is suspended by wire springs inside the bell jar of an air-pump. The bell is connected electrically to a voltaic cell by means of wires passing through the rubber stopper of the bell jar. When the jar is full of air the ringing of the bell can be heard distinctly; but, as the air is exhausted, the sound becomes almost inaudible.

Solid media, also, may serve for the transmission of sound: thus, if a watch be placed on one end of a table (or, in contact with one end of a long rod) the ticking is heard distinctly by an ear placed in contact with the other end of the table or rod.

The generation of sound waves by a vibrating body.—In Fig. 227 (i), the letters *a*, *b*, and *c* represent the successive positions of one prong of a vibrating tuning fork. The movement of the prong resembles closely that of a simple pendulum: when in position *a* the prong is momentarily at rest and is commencing to move towards the right; its velocity increases until it is a maximum at *b*, and then diminishes until it reaches *c*, when it is again momentarily at rest. Moreover, the movements are **isochronous**, that is, they are performed in the same time, whether they are small or large.

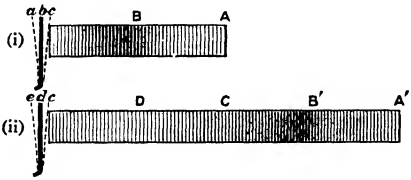


FIG. 227.—Compression wave caused by a vibrating prong of a tuning-fork.

Consider the effect of this movement upon a column of air situated to the right of the prong. The first movement from position *a* causes in the air a slight compression which is propagated outwards with the velocity of sound; subsequent movement of the prong through position *b* to position *c* causes similar compression pulses, but the compression will be most pronounced at the moment when the prong has maximum velocity, *i.e.* when it is passing position *b*. When the prong reaches position *c*, the condition of the air column will resemble that shown in Fig. 227 (i); the first compression will have travelled to a position A, and the pulse of maximum compression will be situated at B. Fig. 227 (ii) represents the subsequent condition of the air column when the prong has moved back again through position *d*, to its original position *e*, thus completing one vibration. The movement of the prong from right to left tends to leave a partial vacuum behind it, and the air is rarefied partially; this rarefaction is a maximum when the prong is moving with maximum velocity through the position *d*.

At the moment when the prong has completed one vibration, the air column will be in the condition represented. The first compression A will have travelled outwards to A′; the region of maximum condensation B will have moved to B′. The region at C will be momentarily in its normal condition; and the region of maximum rarefaction will be at D. The condition thus set up is one complete **sound wave.** After the second vibration of the prong, the first disturbance will have travelled away to a distance twice as far away as A′, and the space between A′ and the fork will be again in the same condition of condensation and rarefaction as shown in Fig. 227 (ii).

It must be borne in mind that the sound waves are not restricted to narrow columns of air, as shown in Fig. 227; in fact, the surrounding air in almost all directions is affected in a similar manner. This would be realised more fully if a point source of sound waves could be considered; and, in such a case, we could regard the point as being surrounded with spherical envelopes of compression and rarefaction spreading rapidly outwards with the velocity of sound.

The **loudness** or **intensity** of the sound depends simply upon the energy contained in that portion of the waves which strike the drum of the ear. The energy contained in any single wave remains practically constant; and, since each wave is distributed over a spherical surface which is enlarging rapidly, the energy passing through each unit area of a wave surface depends upon the distance of that surface from the source.

Since the area of a sphere varies directly as the square of the radius, the energy of the pulse when at a distance of 2 metres from the source will be distributed over an area four times as great as when the pulse was at a distance of 1 metre; and an ear placed at the former distance will experience the effect of only one-fourth the energy which it would experience when placed at the latter distance. Hence, **the loudness of the sound varies inversely as the square of the distance from the source.**

Reflection of sound.—Sound waves are reflected according to the same laws which govern the reflection of waves of light by plane or spherical mirrors; but the conditions under which these two phenomena may be observed differ on account of the wide

difference between the length of light waves and the length of sound waves and also because light waves are transmitted by the ether, whereas sound waves require a material medium for their transmission. The wave length of the lowest audible note is about 36 ft., and that of the highest audible note is about half an inch: these lengths are very great in comparison with the wave length of light. In order that the phenomenon of reflection may be observed, the surface must be large in comparison with the wave length of the vibrations falling upon it. Hence, for the reflection of sound, a surface of considerable area is required. On the other hand, long waves do not require the surface to be so smooth. Consequently, a comparatively rough surface, such as a sheet of cardboard, a wooden board, or a brick wall, serve to produce the phenomenon of reflection of sound.

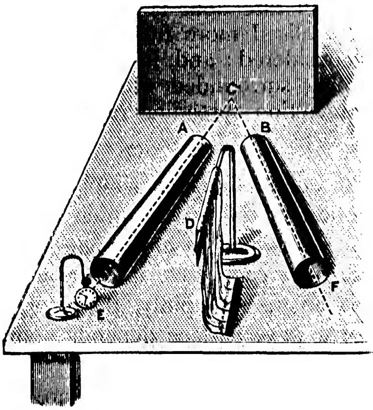


Fig. 228.—Experiment on the reflection of sound.

Expt. 201.—**Reflection of sound.** A and B (Fig. 228) are two tin-plate tubes, about 1 yard long and 3 inches diameter. Support them horizontally in the positions shown. Suspend a watch at E near to the end of one tube. Place the ear near to the other end of the tube at F. If the sound can be heard, hang up a screen (such as a wet towel) at D so as to prevent the sound waves from passing directly to the ear. Now place a flat reflector in a vertical position at C, and slowly rotate it round a vertical axis. Notice the position of the reflector when the sound can be heard; and observe also the effect of tilting the reflector forwards or backwards.

Echoes are familiar examples of the reflection of sound, and they can be observed frequently in the neighbourhood of houses, high walls, cliffs, or hillsides. If the sound be generated near the observer, it is necessary that the reflecting surface be not less than a certain distance away, otherwise the echo is confused

with the original sound, owing to the fact that the impression on the ear of a sound wave persists for at least $\frac{1}{10}$ of a second. The reflecting surface must be therefore at such a distance that the sound wave will require at least this interval of time to travel to the surface and back again.

A peculiar phenomenon of echo can be observed occasionally when a single wave-pulse of sound is reflected from a series of receding steps, as in a staircase, or from a series of separate flat wooden posts along a roadside. The greater distance of consecutive reflecting surfaces causes consecutive echoes to be retarded slightly in reaching the ear. When the interval between the echoes is regular, the sequence will give rise to a note of which the pitch corresponds to the frequency of the echoes.

Masses of water vapour, such as clouds, or of any gas denser than the air, will serve as surfaces for the reflection of sound. Thus, the irregular roll of thunder which often follows a flash of lightning consists in the numerous and overlapping echoes due to direct reflection from cloud surfaces at different distances and to multiple reflection of the waves between two or more cloud surfaces, or between the clouds and the earth. The original noise is extremely brief, and its duration corresponds with that of the lightning flash.

When a source of sound, such as a watch, is placed at the focus of a concave reflecting surface, the sound waves are reflected along parallel paths, and they can be detected at a greater distance than if the 'sound-mirror' is not used. In the **speaking-tube** the waves are reflected repeatedly from side to side of the tube; and, instead of the energy of the sound waves being distributed through a rapidly increasing space, it remains more or less concentrated within the limits of the tube, and sufficiently so for the sounds to be detected by an ear placed at the distant end.

EXERCISES ON CHAPTER XXV.

1. A regular series of waves is transmitted along a flexible cord. What do you understand by the wave-length and the amplitude of the waves? Would you call the waves longitudinal or transverse? Explain the meaning of these terms.

2. An observer on the sea-shore notes that the waves are breaking at the rate of 10 per minute, and that they take two minutes to reach the shore from a rock 100 yards out at sea. Find the mean wave-length and the velocity of propagation in feet per second.

3. What is meant by saying that the vibration of a pendulum or tuning-fork is isochronous? What would be the effect observed after striking a tuning-fork if the frequency of the vibration increased in the same proportion as the amplitude diminished?

4. Explain the formation of echoes. How is it that the sharp crack accompanying a flash of lightning produces the long roll of the thunder?

5. Two observers are stationed at a distance of 1 mile and $\frac{1}{2}$ mile respectively from a point where a bugle note is sounded. If there are no reflections of the sound, how much louder will the sound appear to the second observer than to the first?

6. Describe an experiment showing that air or some other medium is necessary for the transmission of sound. What practical difficulty arises in such an experiment?

7. Describe an experiment to show that sound is not propagated through a vacuum.

How does the air move when sound waves travel through it?

8. Explain the formation of echoes. A person standing 92 yards from the foot of a cliff hears the echo half a second after clapping his hands. Deduce the velocity of sound in air.

9. Show what must be the direction of sound vibrations, relative to the direction of propagation, in order that sound may be transmitted by a gas. What characteristics of the vibrations determine the character of the sound heard?

10. Describe the action of a vibrating tuning-fork.

11. Describe two experiments which show that sound waves can be reflected. A sharp tap is sounded in front of a long flight of stairs. What impression would you get if you were standing in front of the stairs?

CHAPTER XXVI.

ELASTICITY. VELOCITY OF SOUND.

Elasticity.—The influence of elasticity upon the transmission of wave motion has been referred to previously (p. 328). All forms of matter, when subjected to external forces, undergo changes of volume or of shape; and when the forces are removed, the matter tends to recover more or less completely its original volume or shape. This power of recovery is termed **elasticity.** Thus, a bent clock-spring or a stretched steel wire are examples of bodies which possess elasticity to a high degree. Liquids and gases offer resistance to change of volume only, and not to change of form; and we may say that such substances possess considerable *volume-elasticity.*

The change of volume, or of form, which a substance undergoes, is termed the **strain**; and the force causing it is termed the **stress.** The ratio **stress/strain** is termed the **coefficient of elasticity.**

Elasticity of gases.—When a given volume of a gas, confined under an observed pressure, is subjected to an increased pressure, the volume is diminished by a definite amount, in accordance with Boyle's Law (p. 83). Thus, suppose v c.c. of a gas be measured at a pressure p dynes/cm.^{2}; and let the volume be reduced to $(v - dv)$ c.c. when the pressure is increased to $(p + dp)$ dynes/cm.^{2}, where dv and dp represent small changes in volume and pressure respectively. Then, since the strain is measured by the change in volume of each unit volume of gas, it is represented by the ratio dv/v; and the stress producing this strain is dp dynes. Hence,

$$\textbf{coefficient of volume-elasticity} = \frac{dp}{dv/v} = \frac{v \cdot dp}{dv}.$$

It can be shown that, in the case of a gas, *this coefficient is numerically equal to the original pressure.* For, by Boyle's Law,

$$pv = (p + dp)(v - dv)$$
$$= pv - p \,.\, dv + v \,.\, dp - dp \,.\, dv.$$

But, since both dp and dv are very small, their product may be neglected;

$$\therefore \quad p \,.\, dv = v \,.\, dp$$

$$\text{or} \quad \frac{v \,.\, dp}{dv} = p.$$

Velocity of sound in gases.—Steam is seen issuing from the whistle of a distant locomotive sooner than the sound is heard: the flash of a gun, or the striking of a cricket ball with the bat, is seen before the sound reaches the observer: so also the lightning flash precedes the thunder. Such observations demonstrate that time is required for the transmission of sound from one point to another.

It was proved theoretically by Sir Isaac Newton that the velocity (V) of sound in a gas varies directly as the square root of the volume-elasticity (E) of the gas, and inversely as the square root of the density (D). Or, expressed as an equation,

$$V = \sqrt{\frac{E}{D}}.$$

For example, the density of air, at 0° C. and at a pressure of 76 cm. of mercury, is 0·001293 gm./c.c.; and, by the previous paragraph, E is measured by the pressure acting upon it. The pressure on each sq. cm., due to a column of mercury 76 cm. high, is $(76 \times 13{\cdot}6 \times 981)$ dynes. Therefore

$$V = \sqrt{(76 \times 13{\cdot}6 \times 981)/0{\cdot}001293} = 28000 \text{ cm./sec.}$$

This value is not in agreement with that obtained by actual experiment, viz. 33180 cm./sec.; and the cause of the discrepancy was not explained until a much later date, when it was proved that the numerator of the above equation should be increased by the product 1·41. In this case,

$$V = \sqrt{(1{\cdot}41 \times E)/D} = 33170 \text{ cm./sec.}$$

Experimental determination of velocity of sound in air.—The first attempt to measure the velocity of sound in air was made in 1738 under the auspices of the French Academy of Sciences. Two groups of observers, with cannon, were stationed on hills

about 17 miles apart; one of the cannon was discharged, and the observers on the distant hill measured the time interval between the flash of the cannon and the sound of the discharge. The observation was made in both directions so as to eliminate the effect of wind. From the observations it was calculated that the velocity at 0° C. is about 332 metres per second. Taking these and more recent experiments into consideration, the mean value deduced for the velocity at 0° C. is 331·7 metres (or 1088 ft.) per sec.

Effect of various conditions on the velocity of sound.—Since, by Boyle's Law, the volume occupied by a given mass of gas is inversely proportional to the pressure, it follows that the *density of a gas must be directly proportional to the pressure.* The elasticity of a gas is also directly proportional to the pressure. A change of pressure thus affects equally both the density and the pressure of a gas. The relation of numerator to denominator in the ratio *elasticity/density* is therefore unaltered by any change in the pressure. Hence, the velocity of sound is the same at any atmospheric pressure. This conclusion has been verified by experimental determinations of the velocity at high altitudes.

An increase in temperature reduces the density of a gas. Hence, the velocity of sound increases with rise of temperature, and diminishes with fall of temperature. Based upon the known rate at which a gas expands (p. 164), and upon the fact that the density of a gas is inversely proportional to the volume occupied by a given mass of the gas, it can be proved readily that $V_t = V_0\sqrt{(1 + \alpha t)}$, where V_t and V_0 are the velocity of sound at $t°$ C. and 0° C., and α is the coefficient of expansion of a gas (viz. 0·00368). From this equation the velocity of sound at any temperature $t°$ C. is

$$(33170 + 61t) \text{ cm. per sec.,}$$

or

$$(1088 + 2t) \text{ ft. per sec.}$$

Damp air is a mixture of ordinary dry air and water vapour. Since the density of water vapour, at ordinary temperatures, is less than that of dry air in the ratio 0·62 : 1, the density of damp air is less than that of dry air at the same temperature and pressure. Hence the velocity of sound in damp air must be greater than in dry air.

As the velocity varies inversely as the square root of the density, other conditions being the same, its relative value in any two gases can be obtained from the densities of the gases. For example, air has a density of 14·3 compared with hydrogen, hence

$$\frac{\text{velocity of sound in hydrogen}}{\text{velocity of sound in air}} = \sqrt{\frac{14 \cdot 3}{1}}$$
$$= 3 \cdot 8.$$

Taking the velocity of sound in air to be 1088 feet per second, that of hydrogen is therefore $1088 \times 3 \cdot 8$ or 4134 feet per second. Similarly, as oxygen is 16 times denser than hydrogen, and $\sqrt{16} = 4$, the velocity of sound in oxygen is $\frac{1}{4}$ of 4134, or 1033 feet per second.

Velocity in solids and liquids.—By striking with a hammer one end of a long series of connected iron pipes and determining the interval between the sound transmitted by the pipes and that conveyed by the air, the relative velocity of sound in iron and air was determined by two French investigators. The total length of the pipes was 951 metres and the temperature of the air was 11° C. Calculations showed that at this temperature the sound waves in air would take 2·8 seconds to travel 951 metres. The sound was heard through the iron 2·5 seconds before that transmitted by the air, so that it took only 0·3 second to travel along the pipes. The sound was therefore transmitted by the iron about nine times (2·8/0·3) faster than by the air.

The velocity of sound in water was determined by two observers on the Lake of Geneva in 1826. Two boats were moored about eight miles apart. Suspended from one was a large bell immersed in the water and from the other a tube with a trumpet-shaped receiver to catch the sound. When the bell was struck some gunpowder was ignited, and the interval between seeing this light and hearing the sound of the bell gave the velocity of sound in the water. The value found was about 1430 metres per second. These direct methods of determining the velocity of sound are chiefly of historical interest; indirect methods are now employed in most cases. Thus, the formula $V = \sqrt{E/D}$ can be applied to solids or liquids, provided that the elastic constant E is known for the direction in which the waves travel, and is expressed in suitable units. Methods of determining the velocity in bodies which can be obtained in the form of rods are described in Chapter XXVIII.

EXERCISES ON CHAPTER XXVI

1. Explain how the velocity of sound in air may be measured by two observers stationed some distance apart. How could the measurement be made independent of the velocity of the wind between the stations?

2. A man stationed between two parallel cliffs fires a gun. He hears the first echo after two seconds, and the next after five seconds. What is his position between the cliffs, and when will he hear the third echo?

3. A rifle-bullet is fired against a target one mile distant with an average velocity of 1200 ft. per second. Does the bullet, or the sound of the firing, reach the target first? If the temperature of the air is 61° F., what is the interval of time between the two arrivals?

4. Describe a method of determining the velocity of sound in air. Will the result obtained be the same in summer and in winter? Give reasons for your answer.

5. Two sources of sound A and B are situated at distances of 100 metres and 300 metres respectively from an observer, who estimates the intensity of the sound from A to be four times as great as that from B. Compare the amplitudes of vibration of the two sound waves (i) near the observer, (ii) at equal but small distances from the respective sources.

6. Describe a method of determining the velocity of sound in the open air. How will the result be affected by wind, and how can the effect of wind be allowed for?

7. How is sound propagated? Is the velocity of sound in air constant?

8. If the velocity of sound in a gas varies directly as the square root of the pressure and inversely as the square root of the density of the gas, show what effect change of temperature has on the velocity.

9. How is the velocity of sound in air affected by changes of temperature? Is it affected by changes of pressure?

CHAPTER XXVII.

MUSICAL SOUNDS.

Distinction between a musical note and a noise.—In considering continuous sounds, as distinct from short sharp sounds such as an explosion, it is necessary to realise the physical difference between a **musical note** and a **noise.**

In the case of a musical note, the vibrations are comparatively simple and they fall upon the ear with uniform frequency ; and, in the case of a noise, the vibrations are complex and of irregular frequency. The sound obtained from an organ pipe and the screech of a parrot are examples of these two classes of sound.

Loudness and pitch of musical notes.—The loudness of a note depends simply upon the amplitude of vibration of the source of the sound : the greater the amplitude the louder the sound.

Observation of a vibrating tuning-fork or of a stretched string will show that as the amplitude of vibration diminishes the loudness of the note diminishes ; but the pitch, which depends solely upon the frequency of vibration, remains absolutely the same.

EXPT. 202.—**Smoked-glass record or vibration.** Make a thin metal style, about 1 cm. long, from thin sheet brass, or from brass wire beaten out flat with a hammer, and fix it with wax to one prong of a tuning-fork. Blacken the surface of a glass plate by holding it over the flame of burning camphor or over a yellow gas flame. Lay the glass on a table, strike the tuning-fork, and rapidly draw it across the plate

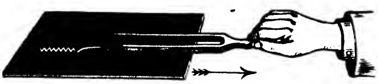


FIG. 229.—Trace of a vibrating tuning-fork.

so that the smoked surface is just touched by the style. Notice how the amplitude of the wave-trace is greater at the beginning than at the end.

Pitch of a note.—The influence of frequency of vibration on the pitch of a note may be observed by means of **Savart's toothed wheel** (Fig. 230), which consists of a toothed wheel (A) capable of being rotated rapidly while a piece of thin board is held in a position so that the teeth strike against the edge of the board. Although the sound emitted is harsh and unmusical, yet a definite pitch is unmistakable; and, by varying the speed of rotation, it can be made evident that the pitch is raised when the frequency with which the teeth strike the board is increased.

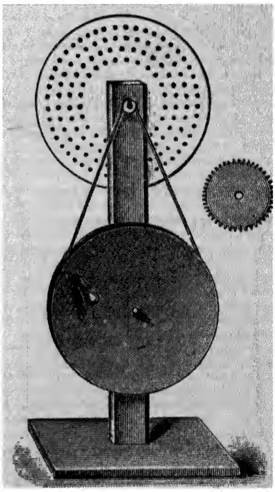


Fig. 230.—Disc siren. The toothed-wheel can be fixed upon the spindle instead of the disc.

The **disc siren** affords more complete information on the relationship of pitch to frequency of vibration. It consists of a cardboard disc, pierced with several concentric rows of holes, which can be rotated rapidly on a whirling-table.

The rows, commencing with the innermost, should have 24, 30, 36, and 48 holes respectively. When a stream of air, blown through a narrow jet such as can be made from glass-tubing, is made to impinge on either row of holes of the rotating disc a note of definite pitch is obtained. The sound arises from the fact that when a hole passes in front of the jet a momentary wave of compression is formed immediately behind the disc; and, during the interval between the passage of two consecutive holes, the inertia of the air causes a partial rarefaction to be set up behind the disc. This is followed by a compression when the next hole passes in front of the jet. Thus, we obtain a regular sequence of pulses of compression and rarefaction, which travel outwards with the velocity of sound.

If the speed be increased the same row of holes will give rise to a note of higher pitch.

If the speed remains constant, and the jet be brought successively in front of each row of holes, commencing with the inner row, the series of notes obtained will be recognised by those acquainted with musical intervals as the **third**, **fifth**, and **octave** above the lowest note.

Relation of pitch to rapidity of vibration.—If while the disc siren, described in the previous paragraph, is rotating, the jet be held near to the inner row and then to the outer row, the frequency of the impulses will be in the ratio 24 : 48, or 1 : 2, and the note of higher pitch is an octave above the lower. When the speed of rotation is increased, both notes become sharper, *but the higher note is still an octave above the lower.* When any two musical notes having this striking relationship are sounded, we can always state that the pitch, or rate of vibration, of one note is double that of the other.

Both Savart's wheel and some forms of siren often have an arrangement by which the number of teeth or holes passing a point in one second is recorded. The number of vibrations which produces a note of a particular pitch can thus be determined. Similarly, to find the vibration number of a tuning-fork, or any other body sounding a single note, a Savart's wheel or a siren having a counter connected with it is brought in unison with the note and the number of vibrations per second indicated by the dial is then observed.

A fact of everyday experience will help the student to understand how the pitch of a note depends upon the number of impulses which reach the ear per second. If the whistle of a passing express train be listened to it will be noticed that the pitch of the note while the train is approaching is distinctly higher than when it is receding. The whistle itself is sending out sound waves at exactly equal intervals of time; but, in the interval between the sending of any two consecutive waves, the train will have moved slightly forward, and the distance between any two consecutive condensations (or rarefactions) will be *less* than it would be if the train were stationary. The waves, therefore, are of shorter length, and the pitch of the note correspondingly higher. Similarly, when the train is receding, the wave-length will be correspondingly lengthened, and the pitch lowered. The explanation of this phenomenon is known as **Doppler's principle.**

Musical intervals, and the major diatonic scale.—The ratio between the rates of vibration of two notes is termed the **interval** between the notes. Thus, when the disc siren is rotating at constant speed, the interval between the notes derived from the innermost row of holes and the second row is 30/24, or 5/4. This interval is termed a **major third**. The interval between the notes given by the 2nd and 3rd rows is 36/30, or 6/5: this interval is termed a **minor third**. The interval between the notes of the 1st and 3rd rows is 3/2, and is termed a **major fifth**. Since $3/2 = 5/4 \times 6/5$, it is evident that when two intervals are added together the resultant interval is expressed by their numerical product. If the three notes given by the inner rows of holes in the disc siren, or the three notes C, E, and G, of a piano, are sounded together they form a pleasing combination: it is now known as the **major chord**. The relative frequencies of these notes may be expressed by the numbers 24, 30, and 36; and these numbers have the ratio 4 : 5 : 6.

The major diatonic scale, such as is represented by the sequence of white notes of a piano, commencing with middle C, is built up in the following manner: A second major chord is obtained by starting from C′, the octave of C, and descending in the ratio 6 : 5 : 4. This gives frequencies of 48, 40, and 32; and these correspond to the notes C′, A, and F. This set of three notes is known as the **sub-dominant chord**. Finally, a third major chord is obtained by starting from G, and ascending in the ratio 4 : 5 : 6. This gives frequencies of 36, 45, and 54; and these correspond to the notes G, B, and D′. This is known as the **dominant chord**. The note D′ is above the octave of C, and its lower octave D, having a frequency 27, falls between C and E. Thus we obtain the following sequence of notes into which the octave may be divided:

Notes	C	D	E	F	G	A	B	C′
Vibrations per second	256	288	320	341·3	384	426·6	480	512
Relative frequency	24	27	30	32	36	40	45	48
Interval (compared with C)	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

The sonometer.—The vibrations of wires or strings can be studied conveniently by means of the **sonometer** or **monochord**,

one form* of which is shown in Fig. 231. The essential parts are a sounding-box with wires stretched along it, upon one of which weights can be hung. Near the ends are fixed metal edges called 'bridges'; and similar bridges, which are movable, are required for the purpose of shortening the length of the vibrating string.

FIG. 231.—A sonometer.

When a string is vibrating the point which has greatest amplitude is termed an **antinode** or **loop**, and the points which are stationary are termed **nodes**. This simplest mode of vibration of a string occurs when there is one antinode, in the centre, and a node at each end. When vibrating in this manner the wire gives out its **fundamental note**.

It will be explained in a subsequent paragraph how, by fixing points of the wire other than the extreme ends, the wire may be made to vibrate in 2, 3, 4, or more separate portions (Fig. 233); in such cases the number of nodes and antinodes is greater than when the wire is giving its fundamental note.

Laws of vibrating strings.—The rate of vibration of a stretched wire or string depends upon the length, the stretching force, and the mass of unit length of the wire. The relationship between the rate and these several conditions is expressed in the equation

$$n = \frac{1}{2l}\sqrt{\frac{F}{m}},$$

where n is the rate of vibration of a string of length l cm., and of mass m gm. per unit length, when stretched by a force F dynes. The above equation suggests that the rate of vibration is

(i) directly proportional to the square root of the stretching force, and

* Experiments are more satisfactory if the sonometer is suspended against a wall in a vertical position, and with its lower end projecting slightly forwards so as to ensure good contact between the wire to which weights are attached and the lower bridge.

(ii) inversely proportional to the length and to the square root of the mass of unit length.

EXPT. 203 (i).—**Length of the wire.** Tune the two wires until they are in unison. Shorten by means of a movable bridge one of the wires A until it gives the octave above its original note as compared with the unaltered wire B. Its rate of vibration is twice its previous rate. Measure its length and note whether this is equal to one-half its previous length.

(ii) By means of another movable bridge tune the previously unaltered wire B until in unison with the shortened wire A. Move the bridge under A until its shorter position gives a note one octave above B, and therefore two octaves above its fundamental note. Its rate of vibration is now four times as great as its initial rate. Measure its length and note whether this is equal to one-fourth of its initial length.

(iii) If two tuning-forks of known rate of vibration are available, measure the lengths of one of the wires required to give notes in unison with the two forks respectively, keeping the tension the same. Note whether the ratio of the lengths is equal to the inverse ratio of the rates of the two forks.

EXPT. 204.—**The stretching force.** Stretch a thin wire on the sonometer with a weight of one kilogram, and tune the other wire to unison. Increase the stretching force to four kilograms, and find by comparison with the other wire whether the note now given is one octave above the previous note.

Try to verify the law when the stretching forces are weights of two and of three kilograms.

EXPT. 205.—**Diameter and nature of the wire.** Select two wires (A and B) of different material, *e.g.* brass and steel, or two wires of the same material but of different diameter. Stretch one of them (A) with a known weight, and determine the length l_1 of the fixed wire C which is in unison with it. Make file marks on the wire A where it touches the bridges, unhang the weight, and cut the wire at the file marks by means of wire cutters. Weigh this length of wire, and determine the mass m_1 of unit length. Stretch wire B with the same weight as before, and determine the length l_2 of the wire C which is in unison with it. Proceed, as before, to find the mass m_2 of unit length of wire B.

If n_1 and n_2 are the frequency of vibration of the wires A and B, then $n_1/n_2 = l_2/l_1$. Also, by the above equation, $n_1/n_2 = \sqrt{m_2/m_1}$. Cal-

culate the values of the ratios l_2/l_1 and $\sqrt{m_2/m_1}$, and observe whether they are equal.

Beats.—When two notes very nearly in unison, and of the same quality, are sounded together, the ear is unable to hear either of them separately, as would be the case when their pitch differs considerably; but the ear can detect a throbbing effect, as though the note were being sounded alternately loudly and softly. This is due to the sequence of waves which originate from the two sources of sound alternately reinforcing and interfering with each other.

Fig. 232 represents two such sets of waves, one represented by a continuous line and the other by a dotted line: the wave-length of the former being slightly longer than that of the latter, but the amplitude is the same in both sets. At A the condensations of

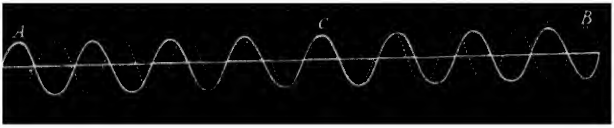


FIG. 232.—Formation of beats.

one set coincide with rarefactions of the other, and the resultant effect is silence. At C the condensations coincide, and the sound will be a maximum. At a further point B the waves again mutually interfere. If the ear be placed at B the momentary silence will be followed by maximum sound when the waves at C reach it, and this will be followed by silence when the waves at A reach it. These pulses of loudness are called **beats.**

It necessarily follows that if two forks vibrate 256 and 257 times respectively in one second their mutual interference will produce one beat in each second. **The number of beats in each second is equal to the difference in the number of vibrations per second made by the two vibrating bodies.** The difference in the number of vibrations must be small for beats to be heard. When more than 16 beats per second are formed, the ear cannot resolve them and a resultant note is heard.

EXPT. 206.—**Beats caused by vibrating wires.** Tune the two wires of a sonometer to apparent unison. Try to detect the presence of

beats: it is easier to detect them when the ear is placed in contact with the end of a wooden rod, the other end of which is pressed against the board of the sonometer. If they cannot be detected, alter *very slightly* the length of one of the wires by means of a movable bridge. Notice how the frequency of the beats increases as the previous unison is more and more disturbed.

EXPT. 207.—**Beats between vibrating wire and tuning-fork.** Adjust the length of a sonometer wire so as to be in perfect unison with a given tuning-fork, and verify the absence of beats by bringing the stem of the vibrating fork in contact with the board of the sonometer. Now load one prong of the fork with a small pellet of wax: this will have the effect of lowering the rate of vibration of the fork, and it should be possible to detect beats when the fork is sounded with the sonometer wire. Attach a larger pellet of wax and observe how the beats are still more frequent.

Harmonics, or overtones.—In previous paragraphs a string has been considered to vibrate with nodes at each end only (Fig. 233 (i)), in which case its fundamental note is produced. But, by preventing the movement of the wire at any intermediate point, a node is set up at that point and the wire vibrates in two or more short segments. Thus, when the wire is touched at its centre (Fig. 233 (ii)) and bowed or plucked at a place half-way between this point and either end, the wire vibrates in two segments. The wire now has two antinodes, A_1 and A_2, or points of maximum movement. When the wire is touched at a point one-third of its length from one end, and then bowed at the middle of the shorter portion, it will divide into three segments (Fig. 233 (iii)). Fig. 233 (iv) shows how the string divides into four segments when touched at a point one-fourth of its length from one end. The positions of these nodes

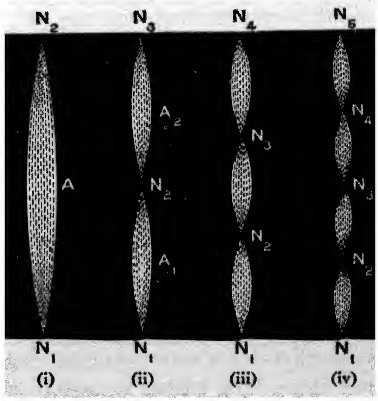


FIG. 233.—Nodes and antinodes of a vibrating string.

and antinodes can be verified by placing upon the wire short narrow strips of paper, usually called riders; those at the nodes remain in their places when the wire is vibrating, but the riders at the antinodes are thrown off.

The pitch of the note emitted depends upon the length of each vibrating segment only; and since, when vibrating in the modes shown in Fig. 233 (i)–(iv), the lengths of the segments have the ratios $1 : \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$, the pitches of the notes emitted have the ratios $1 : 2 : 3 : 4$. The lowest note is the **fundamental**; and those produced by the vibration of aliquot parts of the sounding body are termed **harmonics** or **overtones.** In the case of a string vibrating as shown in Fig. 233 (i), the fundamental note only is sounded and is said to be pure. This simple condition, however, is rare; for the vibration of the string may be a combination of the motions shown in both (i) and (ii). When this is the case, the note is not pure, but it is richer on account of the presence of the first harmonic, which is an octave above the fundamental note.

The accompanying diagram (Fig. 234) represents the positions on the musical stave of the sequence of harmonics, which may be

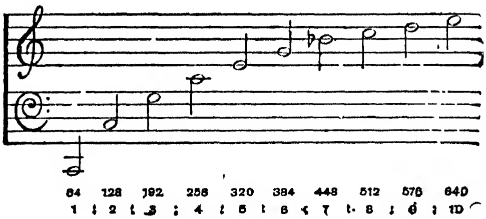


FIG. 234.—Sequence of harmonics.

obtained from a wire tuned in unison with the C note below the bass clef. The **quality** or **timbre** of a note is determined by the presence of these harmonics. Thus, the same note may be sung by the human voice, or played upon a violin, cornet, organ, or other instrument; but though the pitch and intensity may be alike, the timbre differs because of the different number or relative strengths of the harmonics produced when the note is sounded.

EXERCISES ON CHAPTER XXVII.

1. A stretched string 4 feet long is in unison with a tuning-fork which vibrates 256 times a second. What will be the rate of vibration of the string when it has been shortened 6 inches?

2. A sonometer string is stretched with a force of 16 lb. weight. What load must be attached so that the note may be an octave lower?

3. At what point must the G string of a violin be pressed by the finger of the player in order to give the note C′?

4. A fork A has a frequency of 256. When this fork and a second fork B are sounded close together, 3 beats per second can be heard. A pellet of wax is attached to one of the prongs of B, and the frequency of the beats is found to be reduced to 2 per second. What is the frequency of the fork B when unloaded?

5. Determine the vibration number for each tone of a scale the key-note of which has a vibration number 260.

6. A copper wire (density 8·8 gm. per c.c.) 100 cm. long and 1·8 mm. in diameter is stretched by a weight of 20 kilograms. Calculate the number of vibrations which it makes per second when sounding its fundamental note.

7. A given note is sounded first on a piano and then on a violin. How is it that the notes can be distinguished easily though we say the same note has been sounded?

8. How does the frequency of vibration of a stretched string depend upon the length of the string, the stretching force, and any other physical property of the string? How are these laws applied in the piano?

9. If the frequency of a tuning-fork be 128, and the number of vibrations per hour of a second fork exceeds that of the first by 300, how many beats will there be in a minute if the two are sounded together?

10. Describe experiments to show that the impression of a musical interval as judged by the ear depends solely upon the ratio of the frequencies of vibration of the two notes concerned and not upon the difference of their frequencies. The frequencies of vibration of two notes being 400 and 900, what is the frequency of a note that would appear to the ear to lie midway between them?

11. What is the pitch of a tuning-fork? What may be heard when two forks of nearly the same pitch are sounded together? How would you determine which of the two was vibrating the faster?

12. Describe an experiment for showing that, when a musical note is produced, the greater the frequency of the vibrations the higher is the note.

13. Describe some form of siren, and explain how you would use it to determine the frequency of a given tuning-fork.

14. How does the frequency of the note sounded by a string vibrating transversely depend on (i) the length, (ii) the tension of the string?

15. Describe a monochord and explain how to use it to compare the frequencies of two tuning-forks.

If the frequency of the middle C on a pianoforte be 256 vibrations per second, what will be the frequency of the next higher E?

16. A man standing by a railway notices that the pitch of the note due to the whistle of an engine diminishes as the engine passes him. Explain this result.

If the frequency of the whistle is 256 vibrations per second and the velocity of the engine is $\frac{1}{20}$ of that of sound, what will be the frequencies of the notes heard by the man before and after the engine passes him?

17. Explain the formation of the beats heard when two tuning-forks, which are not quite in unison, are sounded together.

A standard fork A has a frequency of 256 complete vibrations per second and, when a fork B is sounded with A, there are 4 beats per second. What further observation is required for determining the frequency of B?

18. A note on a harmonium and a string of a violin have been tuned to be in unison with a given tuning-fork. How do you account for the difference in the quality of the sounds produced by the two instruments?

19. Draw diagrams to show how a stretched wire may vibrate. Upon what does the note emitted by a stretched wire depend?

20. The vibration frequency of a tuning-fork A is 256 complete vibrations per second, and the frequency of another fork X differs from that of A by 4 complete vibrations per second. Describe what will be heard when the two forks are sounded simultaneously, and explain how to determine whether the frequency of X is greater or less than that of A.

21. Explain the meaning of the 'pitch' and the 'intensity' of a musical sound.

How do they respectively depend on the nature of the sound wave which produces the note?

22. A heavy goods train was approaching a railway station. To an observer at the station the puffs of steam from the funnel appeared at a certain instant to coincide with the sound of the blasts. Presently, however, the sounds seem to precede the puffs, the difference between them continually increasing until a second coincidence was established and the process was repeated. Explain this.

CHAPTER XXVIII.

INDUCED VIBRATIONS.

Natural and impressed periods of vibration.—In order to understand clearly the phenomenon of **resonance** it is necessary to distinguish **free** vibrations from **forced** vibrations.

Every simple pendulum has a natural period of vibration when swinging freely—the period depending upon the length of the pendulum. But, by taking hold of the bob with the hand it is possible to impart to it any rate of vibration we please; in this case a forced vibration is produced.

The sound of a tuning-fork can be heard when the fork is held close to the ear. But if the handle of the vibrating fork be brought into contact with a board or table the sound can be heard at a considerable distance. The reason for this is that the vibrations are transmitted through the handle to the board, which is thus forced to vibrate at the same rate. The waves set up in the air by the vibrating board are added to those originating from the fork, and the sound seems much louder. The board is thus caused to vibrate at a rate which is not necessarily its natural rate; or, in other words, it is in a state of **forced vibration.** The tone of a violin is due, to a considerable extent, to the forced vibrations set up in the wooden case of the instrument; similarly, the tone of a piano is due to the forced vibrations set up in the sounding-board across which the wires are stretched.

Induced vibrations.—Free vibrations may be set up in a heavy pendulum, or in any other suspended body, by applying a sequence of small repeated blows, providing that the interval between the blows corresponds to the natural period of free

vibration of the suspended body. If, however, the blows come irregularly or at an incorrect frequency, they may have little or no effect in setting up vibration. A regiment of soldiers crossing a suspension bridge may set up dangerous oscillations if the frequency of their step corresponds with the natural period of vibration of the bridge: for this reason the soldiers are often ordered to fall out of step when crossing such a bridge. The same effect may be observed when walking along a plank bridge; for considerable oscillation may be set up if the steps are rightly timed, but the oscillations cease when the rate of step is increased or diminished.

EXPT. 208.—**Sympathetic vibrations.** (i) Select two tuning-forks of the same pitch, one of them being fixed upon a sounding-box or resonator as in Fig. 235. Strike the other fork and hold its stem upon the sounding-box for a moment; then remove it and stop its vibration. Notice that the fork upon the sounding-box has taken up the vibrations, and gives out the same note.

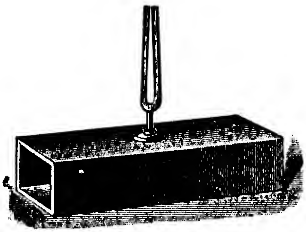


FIG. 235.—Tuning-fork on a resonator.

(ii) Stretch a wire upon the sonometer until it is in unison with a tuning-fork. Set the fork in vibration and hold the stem for a moment upon the sonometer. Notice that the wire has taken up the vibrations and sounds the same note.

(iii) Sing any note loudly near a piano and stop suddenly. The note in unison with it will be sounded by the piano.

It is easy to realise the process by which such waves of sound set up vibrations in a wire in unison with the note sounded. For, suppose a condensation to strike the wire; this will thrust the wire slightly aside in the direction in which the waves are travelling. During the passage of the succeeding rarefaction, the wire has time to swing back past its position of rest, when it is thrust forward again by the next condensation. Thus, a series of slight, but well-timed, impulses are given to the wire, which soon acquires a considerable amplitude of vibration. By such reasoning it is clear that a wire not in unison with the note will not be affected so readily by the waves of sound.

The familiar phenomenon of the sound obtained by blowing across the open end of a key shows that vibrations may be set up

in an air column; and an air column of definite length has a definite natural period of vibration. When a vibrating tuning-fork is held over a tall glass cylinder, into which water is poured gradually so as to vary the length of the air column, a length can be obtained which will resound loudly to the note of the tuning-fork.

The term **Resonance** is applied to all such effects of vibratory motion produced in one body by the influence of another.

Vibrations of air columns.—The circumstances in which a vibratory condition may be set up in an air column which is closed at one end may be compared to those which determine a vibratory condition in a spiral spring of which one end is fixed. Thus, suppose a weight to be suspended from a spiral spring (Fig. 236 (ii)); by imparting a succession of small taps, directed upwards, to the weight, a considerable vibration may be set up in the spring, *providing that the frequency of the taps coincides with the natural period of vibration of the spring.* In the same manner, when a succession of small taps is imparted to the open end of an air column (Fig. 236 (i)) by means of a vibrating tuning-fork, considerable vibratory motion is set up in the air column if the rate of vibration of the fork coincides with the natural period of vibration of the air column; and the sound waves originating from the fork are augmented considerably by those from the vibrating air column. It is important to notice, in this analogy, that the fixed end of the spring and the closed end of the air column are stationary, and that the opposite ends in each case are regions of maximum motion.

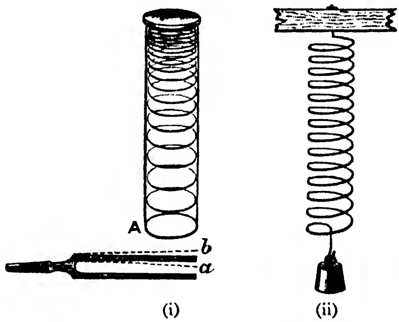


FIG. 236.—Vibrations of a spring and a closed air column.

Tube closed at one end.—A **stationary** vibration is the only type which can be set up within a column of air contained in

a cylinder closed at one end. The relation between the length of the air column and the wave-length of the note given out when the air column is vibrating in its simplest manner may be derived by comparing it with a stretched string in a state of stationary vibration. In the case of the string there are points of maximum motion called antinodes, mid-way between the fixed points called nodes. In the case of the air column, the air near the closed end must be analogous to the node of the vibrating string, since the closed end prevents motion; but, at that end, rapid alternations of condensation and rarefaction are possible. The conditions at the open end are just the reverse, for the air in that position may be in a state of rapid movement; but, since it is exposed freely to the outer air, changes in density are impossible. In other words, *the closed end will be a region of maximum changes of density, and the open end will be a region of maximum motion.* The analogy between a vibrating string and a vibrating air column is shown in Fig. 237. In the former the distance between a node N and an antinode A is equal to *one-fourth* of a wave-length; similarly, the length of the air column B is equal to *one-fourth* of the wave-length of the note which it will emit. This can be verified by the following experiment.

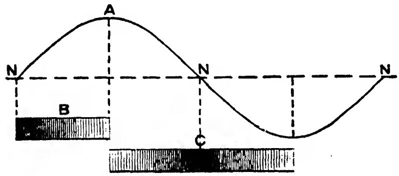


FIG. 237.—Relation between wave-length and vibrating columns of air.

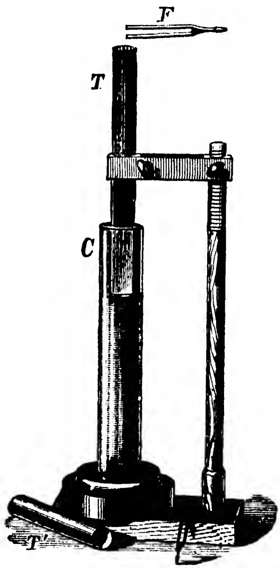


FIG. 238.—Determination of the velocity of sound in air by means of a vibrating air column.

EXPT. 209.—**Resonance of air column.** Support a glass tube T (Fig. 238), about 20 cm. by 3 cm., open at both ends, in a vertical

position with its lower end dipping into water contained in a wider cylinder C. Hold over the upper end of tube T a vibrating tuning-fork F, of which the rate of vibration is known; and adjust the position of T so that the greatest reinforcement of the sound is obtained. Measure the distance from the top of T to the water level. Put T out of adjustment, and repeat the observation at least four times, and take the mean of these results.*

If the length of the air column, as measured above, be l, the wave length of the note emitted is $4l$; and if n be the frequency of vibration of the fork, the velocity v of sound in air, at the temperature of the room, is given by the equation $v = n \times 4l$. Calculate the value of v.

Note the temperature of the room, and calculate the theoretical value from the formula $v = (33200 + 60t)$ cm.; where t is the temperature measured on the Centigrade scale.

By similar experiments the velocity of sound in any gas may be determined. When the column of gas in a closed tube responds to a certain note, the wave-length of the note is four times the length of the column. Suppose the vibration-frequency of the note to be known by comparison with the note of a tuning-fork or other standard, then the velocity of sound in the gas is this number multiplied by four times the length of the vibrating column. Similarly, to find the vibration number of a tuning-fork, a column of air is adjusted as in Expt. 209 until it is in sympathetic vibration with the fork, and its length is measured. Let this be 0·22 metre. Then taking the velocity of sound to be 340 metres per second at the temperature of the air column, we have

$$V = nl.$$

But $$l = 4 \times 0{\cdot}22 = 0{\cdot}88.$$

Hence $$n = 340/0{\cdot}88$$
$$= 386.$$

Tube open at both ends.—In the case of an air column enclosed in a tube open at both ends, the ends of the tube are always antinodes; and, when the fundamental note is sounded there is a node at the middle of the tube. By analogy with stationary vibrations in a string, as shown in Fig. 237, C, the wave-length of the fundamental note is equal to twice the length of the tube. This can be verified by holding the same fork which was used in Expt. 209 in front of an open tube, the effective length

* Strictly speaking, the position of the antinode is slightly *outside* the end of the tube, and this distance depends on the diameter of the tube A more exact measure of the quarter wave-length is obtained by adding 0 6 of the radius of the tube to the length measured from the water surface to the top of the tube.

of which can be adjusted by means of a paper cylinder made to slide over the outside of the tube. The length which gives maximum reinforcement will be found to be twice as long as the air column closed at one end.

Organ pipes.—The conditions of vibration of air columns in closed or open tubes apply to organ pipes. In the case of reed pipes, a small tongue is set in vibration at the mouthpiece by air being forced past it; but in ordinary organ pipes, the air blasts strike against a lip, as with a boy's whistle-pipe, and vibrations are thus set up which are reinforced by sympathetic vibrations of the air column. Vibrations of many wave lengths are produced at the mouthpiece, but only those with which the air column is in unison are taken up and strengthened to form the musical note. As described for tubes, the wave-length of the fundamental note given by an organ pipe closed at one end is four times the length of the pipe, but if the pipe be open at both ends, the wave-length of the fundamental note is twice the length of the pipe. The wave-length of the note of an organ pipe can thus be determined from the length of the pipe. Neither the material of which the pipe is made, nor the diameter of the pipe, provided it is small in comparison with the length, need here be considered to affect the pitch of the note produced. The pitch of the note varies directly, however, with the velocity of sound in the vibrating column, and therefore rises as the temperature increases or when a lighter gas than air fills the pipe. As the pitch of a note depends upon the frequency or number of vibrations per second, the equation $V = nl$ (p. 330) can be used to determine the velocity of sound in a gas when the vibration frequency of the note is known and the length of the pipe emitting it. The following examples illustrate the use of this relation.

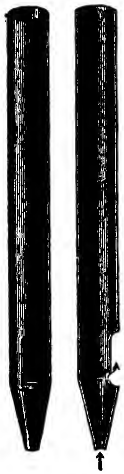


FIG. 239.—Organ pipes.

EXAMPLE I. An open organ pipe 0·65 metre long gives the note middle C, the vibration number of which is 256 per second. Find the velocity of sound in the air at the temperature of the tube.

The wave-length (l) of the note is twice 0·65 metre, that is, 1·30 metres. So that

$$V = 256 \times 1{\cdot}30$$
$$= 332{\cdot}8 \text{ metres per second.}$$

EXAMPLE 2. The velocity of sound in hydrogen is 1269·5 metres per second. What would be the length of a closed organ pipe which gives a note having a vibration frequency of 512 per second when blown with hydrogen?

$$1269{\cdot}5 = 512 \times l.$$

Therefore $$l = 1269{\cdot}5/512$$
$$= 2{\cdot}48.$$

The length of a closed organ pipe is $l/4$; hence the answer required is $2{\cdot}48/4 = 0{\cdot}62$ metre.

Longitudinal vibrations of rods.—When a rod is clamped at one end and made to vibrate longitudinally, the wave of compression set up travels along the length of the rod in much the same way as it does in a column of air. The relative velocities of sound in the rod and in air can therefore be determined by measuring the length of a closed air column which resounds to the same note as that emitted by a rod clamped at one end and set in vibration. Similarly, the velocity in rods of different material can be found by cutting the rods until they give the same note, for then the relative lengths give the relative velocities.

With a rod clamped at one end and set in longitudinal vibration the point of maximum motion (antinode) is at the free end and that of maximum compression (node) is at the clamped end. The vibration of such a rod may be compared with that of the air in a closed tube or organ pipe The wave-length of the fundamental note is, therefore, four times the length of the rod. Similarly, the longitudinal vibrations in a rod clamped at the middle are like those of the air in an open tube or organ pipe, for both ends are antinodes. The wave-length of the fundamental note of a rod clamped in this way is twice the length of the rod.

EXPT. 210.—**Longitudinal vibration.** Clamp a rod of brass loosely at its middle. Near one end hang a pellet of sealing-wax. Rub the other end with a resined leather. The rod gives out a note and the pellet is knocked away.

EXPT. 211.—**Pitch and length.** Obtain two long rods of the same kind of wood, one twice the length of the other, and clamp them

separately at the centre. Set the rods in longitudinal vibration by rubbing them with a resined leather. The longer rod will be found to give a note an octave lower than the shorter one, because a wave of compression has to travel twice as far, and consequently appears half as often.

EXPT. 212.—**Relative velocities in deal and oak.** Obtain rods of equal length in deal and oak and set them in longitudinal vibration. The deal rod gives out the higher note because the waves are propagated quicker by it than by the oak. Cut down the length of the oak rod until the two rods give the same note. It will be found that a deal rod 72 in. long emits the same note as an oak rod 49 in. long. The relative velocities of sound in deal and oak are therefore as 72 is to 49.

EXPT. 213.—**Determination of velocities.** Adjust a monochord string until it is in unison with a tuning-fork of known pitch. Measure the length of the string. Set a rod of mahogany clamped at the middle in longitudinal vibration, and adjust the monochord string in unison with it. Measure the length of this vibrating string. Knowing the pitch of the fork, the velocity of sound in mahogany can be found as follows:

Standard fork $C' = 543$ vibrations per second.
Length of string in unison with $C' = 60$
" " " rod $= 24$
Length of rod $= 6$ feet.

Then $$\frac{24}{60} = \frac{543}{n}$$

Therefore the frequency, $n = 1357 \cdot 5$.
Using the formula $V = nl$,
we have $n = 1357 \cdot 5$
and $l =$ twice the length of the rod.
Hence velocity of sound in mahogany

$$= 1357 \cdot 5 \times 6 \times 2$$
$$= 16290 \text{ feet per second.}$$

Find in the same way the velocity of sound in glass, oak, and brass.

EXERCISES ON CHAPTER XXVIII.

1. A tuning-fork produces strong resonance when held over a jar 22·35 cm. long and 2 cm. radius. Find the wave-length of the note emitted. If the temperature is 15° C., calculate the rate of vibration of the fork.

2. State how the air moves in different parts of a tube 1 ft. long, open at both ends, when sounding its fundamental note. Neglecting

the correction for the width of the tube, and assuming the velocity of sound in air to be 1116 ft. per second, calculate the frequency of the note emitted.

3. Find the number of vibrations per second of a fork which produces resonance in a pipe which is 10 inches long and 2 inches in diameter, and closed at one end. The temperature of the air at the time of the experiment is 50° F.

4. Explain the meaning of the term 'Resonance' by reference to an experiment in which a vibrating tuning-fork is held over a column of air. Show how this experiment can be used to determine the vibration frequency of a fork, the velocity of sound in air being assumed as 1100 ft. per second.

5. If there are 32 holes in the disc of a siren, which makes 1050 revolutions per minute, what is the frequency of the note emitted? What would be the length of an open organ pipe which, sounding its fundamental, emitted the same note? (Velocity of sound in air = 1120 ft. per sec.)

6. The end of one of the prongs of a tuning-fork is held over the mouth of a tube which can be raised or lowered in water. When the mouth of the tube is at a given height above the water the sound of the fork appears to swell out loudly. Carefully explain this. Would the height be different, if (i) the temperature of the air were higher, (ii) if the air in the tube were replaced by carbonic acid gas? Give reasons for your answer.

7. A glass tube one foot long and one inch in diameter is closed at one end. Describe the motion of the air within the tube when vibrating in the simplest possible manner. How, if at all, would the pitch of the note emitted by such a column be changed by a rise of temperature, by an increase of atmospheric pressure, and by the substitution of a denser gas for the air in the tube?

8. How would you determine the number of vibrations per second executed by the prong of a tuning-fork?

9. Explain the effect of temperature upon the frequency of the note emitted by an open organ pipe. Does the effect depend upon the material of which the pipe is made, or upon the nature of the gas in which the pipe is sounded? Give reasons.

10. In building an organ for use in a warm climate it is necessary, in order to produce notes of a given pitch, to make the pipes longer than if they were to be used in England. Explain why this is so.

11. How would you show that the velocity of sound is not the same in air as in carbon dioxide gas at the same temperature?

12. What is meant by resonance?
Give two illustrations of its use in acoustic experiments.

13. Explain what is meant by resonance. The length of the column of air in a tube closed at one end which gives the greatest

resonance with a tuning-fork is observed to be 32·5 cm.; find the wave-length of the note emitted by the fork.

14. Describe the method of determining the velocity of sound in air by means of a resonance tube and a tuning-fork of known pitch.

How may the correction due to the 'open end' of the tube be determined experimentally?

15. You are given a tuning-fork of known frequency, a deep gas jar, and a metre scale. How would you determine the velocity of sound in air?

On what acoustic principle does this experiment depend?

16 State and explain what may be noticed when a person sings a note in front of the strings of a piano.

17. How would you determine the vibration number of a tuning-fork?

18. How would you show that the velocity of sound in air is different from its velocity in carbon dioxide?

Explain how it is possible for a man to calculate roughly his distance from a cliff when the velocity of sound in air is known.

19. Why, when the handle of a vibrating tuning-fork is put in contact with a wooden board, is the amount of sound produced greatly increased? Is the time during which the fork goes on vibrating affected?

20. If you were provided with a fork of unknown pitch and the necessary apparatus, how would you determine the velocity of sound in air?

21. State how you could determine the velocity of sound in a solid which could be obtained in the form of a rod.

PART VI.

MAGNETISM.

CHAPTER XXIX.

NATURAL AND ARTIFICIAL MAGNETS.

Lodestone.—A **magnet** is a solid body possessing the property of attracting iron ; it has the same power of attracting a few other metals, but to a much less marked extent than in the case of iron.

Stones possessing the property of attracting iron are found abundantly near Magnesia (in Asia Minor), from the name of which place the word magnet originated. This stone is now termed **magnetite** ; it is an oxide of iron, and contains about 72 per cent. of iron ; it is distinctly heavy, and is dark-gray to black in colour. Only some specimens of magnetite possess the properties of a magnet, but all are capable of being attracted by a magnet.

A piece of magnetite, selected as showing the properties of a magnet, will, when suspended by a thread so as to swing freely, come to rest in a definite position, and point approximately **north** and **south.** This property possessed by magnetite was known to the people of other nations at a very early date ; for example, there is every reason to believe that the Chinese were aware of it in the year 2400 B.C. On account of this property of setting itself in a north and south direction like a compass needle, a piece of magnetite which behaves like a magnet is called a **lodestone** or leading-stone.

The points at which iron filings chiefly accumulate upon a lode-stone or any other magnet which has been dipped into them

are called the **poles**, and the imaginary line joining these points is termed the **axis**, of the magnet.

EXPT. 214.—**Attractive property.** Dip a lodestone into a small heap of iron filings: observe how the filings cling to it chiefly at two points.

EXPT. 215.—**Directive property.** Suspend a lodestone by means of silk *cord* (not *twisted* sewing-silk), in such a manner that the line joining the poles may move freely in a horizontal plane. Observe that the lodestone always comes to rest in one particular position. When at rest, the axis of the lodestone lies in a magnetic north and south direction. Mark the end which points north with a spot of sealing wax or red paint.

Magnetisation.—The properties of a lodestone can be imparted to iron or steel. When an ordinary steel needle is dipped into

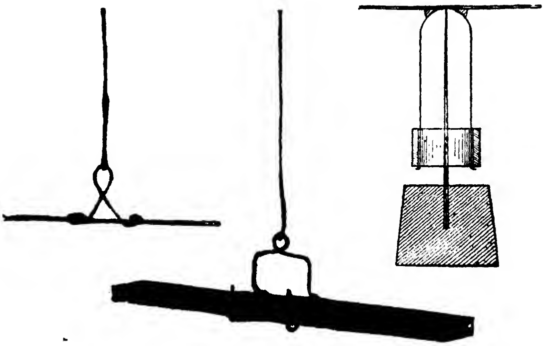


FIG. 240.—Methods of supporting magnets.

iron filings it has no more effect upon them than a copper wire or a splinter of wood. Also, such a needle, like a strip of copper or wood, does not exhibit the directive properties of a lodestone, even when it is suspended so as to be free to come to rest in any direction. When, however, a needle is stroked with either pole of a lodestone, it acquires the properties of a magnet, that is, it will attract iron filings and will set itself in a north and south direction if suitably suspended. The presence or absence of magnetisation in a needle or any other iron or steel object may thus be tested by observing whether the object has the magnetic properties of a lodestone.

EXPT. 216.—**Magnetisation of iron by a lodestone.** Support an ordinary needle horizontally in a silk fibre suspension (Fig. 240). The needle may swing to and fro, but it does not indicate any tendency to come to rest pointing in any one definite direction. Dip the needle into iron filings; it does not appear to have the power of attracting the filings. Stroke the needle several times in one direction with one end of a lodestone. Notice that the needle will now attract iron filings and will set itself in a magnetic north and south direction when suspended. (An alternative method of suspending needles is shown in Fig. 240. A short test-tube, 2 inches long, rests inverted on the point of a darning-needle fixed vertically in a wide cork, and a small lump of soft wax enables the needle to be attached to the closed end of the tube. In order to ensure stable equilibrium, a strip of sheet lead is cut to a length necessary to form a collar which can be slipped over the outside of the test-tube and resting on its expanded rim.)

Like and unlike magnetic poles.—That end of a suspended lodestone, or of any other magnet, which points toward the north is called the **north-seeking pole**, and the end which points toward the south is termed the **south-seeking pole.** The north-seeking end of a suspended lodestone is repelled by the north-seeking end of another lodestone brought near it. Similarly, the south-seeking end of one lodestone will repel the south-seeking end of another lodestone When, however, the north-seeking end is brought near the south-seeking end, attraction takes place. These results may be expressed briefly by the words **like poles repel, unlike poles attract.**

A piece of magnetite or of iron or steel which is not a magnet, and therefore does not set itself in a magnetic north and south direction when suspended, is always attracted by a magnet brought near it and is never repelled. Repulsion only takes place when both the bodies under examination are magnets. This fact enables a magnetised piece of iron or steel to be distinguished easily from unmagnetised iron or steel.

It has been seen that a lodestone can impart its properties to a steel needle, that is, it can convert an unmagnetised piece of steel into a magnet having north-seeking and south-seeking poles. To magnetise a needle in this way, one end of the lodestone is used to stroke the needle several times in one direction. It is

found that the magnetic polarity generated in the end of the needle which is last touched by the lodestone is of opposite kind to that of the pole used for the process.

EXPT. 217.—**Directive property of magnetised iron.** Place a needle on the table, and holding it firmly by pressing a finger on the eye of the needle, rub the marked pole of the lodestone along the needle from eye to point; lift the lodestone some distance away from the table, and bring it down again on to the eye of the needle, and repeat this operation several times. Replace the needle in its support, and observe how different is its behaviour from that observed before it was magnetised.

It comes to rest with its eye pointing in the same direction as does the marked end of the lodestone.

EXPT. 218.—**Attraction and repulsion.** Bring the marked end of the lodestone near to the point of the needle: attraction takes place. Bring the same end of lodestone near to the eye of the needle: repulsion is observed. Repeat the observations with the other end of the lodestone; the eye of the needle is attracted now, while the point is repelled.

EXPT. 219.—**Like and unlike poles.** Magnetise a second needle in exactly the same manner as described in Expt. 217, but stroke the needle with the unmarked end of the lodestone (instead of the marked end). Suspend the needle, and observe how it comes to rest with its eye pointing in the opposite direction to that obtained in Expt. 217 (when the marked end of the lodestone was used).

Magnetic substances.—The terms **natural magnet** and **artificial magnet** are used frequently to distinguish the lodestone from a piece of iron or steel which has acquired the same properties by artificial means. In the experiments performed, while the lodestone is a "natural magnet," the needles which have been magnetised by mechanical processes are termed "artificial magnets." Things like iron and steel which are attracted by a magnet are termed **magnetic substances.** Nickel and cobalt are also magnetic substances, but zinc, copper, paper, wood, glass, and air are examples of non-magnetic substances. The influence of a magnet can pass through any non-magnetic substance just as readily as it does through air.

EXPT. 220.—**Attraction of nickel and cobalt.** Bring a bar-magnet (or lodestone) in contact with some fragments of nickel and of cobalt; the fragments are attracted.

EXPT. 221.—**Non-magnetic substances.** Suspend a magnetised needle and bring the pole of a magnet near it. Hold successively in front of the pole a sheet of copper foil, of zinc foil, of paper, glass or wood. In no case is the deflection of the needle affected.

Magnetisation by means of an artificial magnet.—Using a lodestone it is only possible to magnetise comparatively small pieces of steel, and even then the magnetisation is not so marked as is the case when magnets stronger than the lodestone are used. It is more satisfactory therefore to dispense with the lodestone, and to use instead some form of artificial magnet, *e.g.* the long bars of magnetised steel known as **bar-magnets** (Fig. 241).

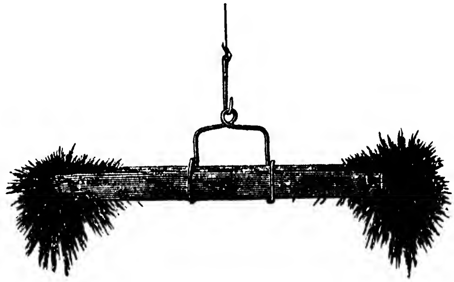


FIG. 241.—A bar-magnet which has been dipped into iron filings.

Another common form of artificial magnet is the **horse-shoe magnet,** in which the steel has been bent into the form of a horse-shoe previous to magnetisation (Fig. 252). The poles of the magnet are at the ends of the horse-shoe, and are thus situated close together.

EXPT. 222.—**Magnetisation of steel.** Break off a piece of clock-spring about 5 or 6 cm. long; hold it firmly on the table by a finger placed at one end (or, better still, fix it to the table by soft wax at the ends), and draw one pole of a magnet along the whole length of the spring, and proceed as in Expt. 217 (Fig. 242). Test the magnetisation (*a*) by means of iron filings, and (*b*) by suspending in a stirrup.

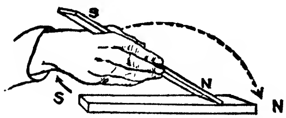


FIG. 242.—Method of magnetising steel.

Magnetisation by an electric current.—The strongest magnets are made by means of the electric current. If a close spiral of cotton-covered copper wire be wound round a rod of steel (Fig. 243) and a current of electricity sent through the spiral, the steel

becomes a magnet and retains its magnetic properties after the current has ceased. Soft iron also is magnetised strongly when an electric current is passed round it, but it soon loses its magnetism when the current ceases. A solid rod of soft iron, which is only a magnet so long as a current of electricity continues to flow around it, is termed an **electro-magnet**.

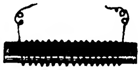


FIG. 243.—Magnetisation of an iron rod by an electric current.

EXPT. 223.—**Electro-magnetisation.** Wrap a spiral of cotton-covered copper wire round a piece of thin-walled glass tubing (about 10 cm. long and 0·5 cm. bore) (Fig. 244); place inside the tube a needle or piece of clock-spring, and pass a strong current through the wire for a few seconds; after stopping the current, remove the needle, and test it for magnetisation.

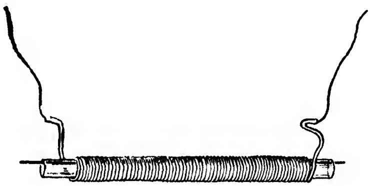


FIG. 244.—Method of magnetising a needle by an electric current.

The more common form of electro-magnet is the horse-shoe, which consists of a thick core of soft iron, bent either into the form of a horse-shoe with straight limbs, or into a form resembling three sides of a rectangle. Round each limb is wound a bobbin of several layers of thick cotton-covered copper wire, the direction in which the wire is wound on the limbs being *opposite* (Fig. 245). While an electric current is passing round the bobbins, a bar of steel may be magnetised by drawing it completely over one of the poles of the horse-shoe. (The polarity of the electro-magnet may be determined by means of a compass-needle.)

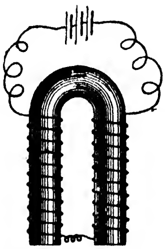


FIG. 245.—An electro-magnet.

Consequent poles.—Sometimes a magnet may be found which has similar poles at the ends; this may arise through faulty magnetisation, and it may be imitated artificially in a simple way. A magnet showing this peculiarity will always be found to have the poles of opposite kind somewhere along its length, and these may be located by dipping

the magnet completely into iron filings, or by means of a compass needle placed in a series of positions along its length.

EXPT. 224.—**Production of consequent poles.** Magnetise a long knitting-needle in four separate parts by the method of Expt. 222, and so that a north-seeking pole is found at each end; another north-seeking pole is found also at the centre, and south-seeking poles at one-quarter of the whole length from each end.

Destruction of polarity.—When a magnet is subjected to rough treatment it loses a considerable portion of its magnetism; for example, if it be dropped on the floor, or struck with a hammer several times, its strength is reduced to a marked extent.

Magnets also lose their magnetic properties when strongly heated. After a magnetised needle has been heated to bright red heat in a Bunsen (or blow-pipe) flame, and allowed to cool, it behaves like an ordinary unmagnetised piece of steel.

EXPT. 225.—**Effect of striking.** Magnetise a French wire-nail about 7 cm. long, and test its magnetisation by bringing it near to a compass-needle. Strike it several times with a hammer, and test again; it will be found to have lost a considerable portion of its magnetism.

EXPT. 226.—**Effect of heating.** Hold a magnetised needle in a Bunsen flame by means of metal tongs, or by wrapping the ends of a short length of copper wire round the needle; when red hot remove it, and allow to cool; test its magnetisation by means of a compass-needle.

EXERCISES ON CHAPTER XXIX.

1. Two steel needles are supplied to you, only one of which is magnetised. (i) How would you determine, by means of a cork floating on water and a lodestone, which of the needles is magnetised? (ii) How could you distinguish the needles without the aid of a lodestone?

2. Two sewing-needles are magnetised so that the eye of each is a north-seeking pole. The needles are stuck by their points into separate bits of cork, so that when thrown into water they float upright with the eyes downwards. How will they behave towards each other when floating in this way?

3. You are doubtful whether a steel rod is neutral or is slightly magnetised. How could you find out by trying its action on a compass-needle? If it is found to be magnetised, how would you determine its polarity?

4. A needle is to be magnetised so that its eye acquires north-seeking polarity. State fully how you would proceed to do this.

5. How would you determine experimentally whether a magnet has consequent poles or no ?

6. What conclusion would you come to if a magnetised piece of steel, when suspended, does not tend to come to rest pointing in a north and south direction ? If the steel be now broken into two parts, would you expect them to behave, when suspended separately, in the same manner as the unbroken piece of steel ? (Give diagrams to explain your answer.)

7. Describe the steps you would take to magnetise a piece of clock-spring as strongly as possible.

8. An unmarked magnet, with means for its suspension, is given you. How could you determine which is the north-seeking end ?

9. How may a bar magnet be demagnetised ?

How may it then be remagnetised so as to restore the original polarity (*a*) by means of other magnets, (*b*) by using an electric current ? (Bristol Univ., S.C.)

10. Given three rods A, B, C of similar appearance, and being told that one was a magnet, the other of magnetic material, and the third non-magnetic, explain how you could distinguish between them, no iron filings or other magnetic material of any kind being available. (Lond. Univ., Gen. Sch.)

CHAPTER XXX.

MAGNETIC INDUCTION.

Magnetic induction.—When one magnetic pole of a lodestone is brought near either end of an unmagnetised needle freely suspended, the attraction observed may, at first sight, suggest that it is simply a case of 'Unlike Poles attracting,' and that repulsion will be found when the other end of the needle is tested. But on completing the experiment in this manner we again find 'attraction.' It would seem either that an entirely new phenomenon is being brought into play, or that 'latent' magnetisation in the needle appears when the magnet is brought near one end, and reappears in a reversed direction, when the other end is tested. To decide this point, it is necessary to test the polarity of the distant end of the needle while the magnet still remains in its first position.

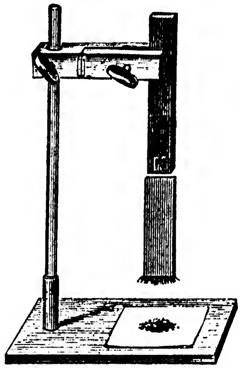


FIG. 246.—Magnetisation of a strip of iron by induction.

EXPT. 227.—**Induced magnetisation.** Cut strips of the thin galvanised iron which is used in making biscuit-tins and tobacco-tins. Strips about 10 cm. long by 1 cm. wide are convenient. Hold a strip of galvanised iron in line with the magnet's axis, the end not quite touching the north-seeking pole; bring the distant end of the iron in contact with iron filings; some filings cling to the end (Fig. 246). Reverse the strip of iron and again test.

EXPT. 228.—**Poles of an induced magnet.** Attach a strip of the galvanised iron to a 'test-tube' support (Fig. 240), and arrange a

bar magnet horizontally on blocks of wood so that it is at the same level as the strip, and so that its N.-seeking pole *almost* touches the end of the strip. We may anticipate that *the near end of the strip has S.-seeking polarity*: test this by bringing near to it the S.-seeking pole of another magnet, and observe any indication of repulsion. To make the effect more evident, make the latter magnet approach and recede from the strip, at a rate coinciding with the time of swing of the strip. The series of *small* impulses will set up a considerable amplitude of swing in the strip.

Reverse the magnet in the hand, and prove in the above manner that *the distant end of the strip has N.-seeking polarity.*

EXPT. 229.—**The induced polarity is temporary.** Remove the bar-magnet, and repeat the tests for magnetisation; we find that the strip now behaves like an unmagnetised piece of iron.

It is evident that a strip of iron actually becomes a magnet when it is near a bar-magnet, but ceases to be one as soon as the magnet is removed. We say that magnetism has been temporarily induced in it, and that its behaviour is due to **magnetic induction** from the bar-magnet.

When a piece of iron or steel is magnetised by induction, the end farthest away from the inducing pole acquires polarity of the same kind, the nearer end acquires polarity of the opposite kind, to that of the nearest pole of the permanent magnet. If the piece of iron be really a magnet, then it also should be capable of inducing magnetism in a second piece of iron held near to it.

EXPT. 230 —**Secondary induction.** Support a bar-magnet horizontally on a block of wood; and arrange two pieces of soft iron, each on a 'test-tube' support, in line with the magnet's axis; test by the method described in Expt. 228 the polarity of the induced magnetism in the more distant piece of iron.

We can understand now the cause of all the phenomena of attraction which a magnet displays towards magnetic substances. The experiments prove that **magnetic induction always precedes attraction,** and that these phenomena are all in accordance with the simple law that 'Unlike Poles attract.'

EXPT. 231.—**Magnetic chain.** Clamp a large bar-magnet in a vertical position, note the polarity of the lower end, and hang from it a strip of galvanised iron. Bring into contact with the iron a number of small wire-nails, and notice how long a chain of nails can be supported.

The iron and each nail are temporarily magnetised. Test the polarity of the extreme end of the chain by means of a compass-needle (Fig. 247).

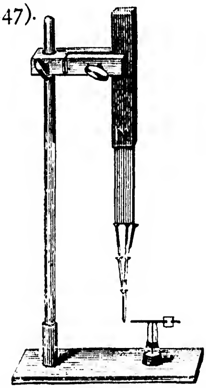


FIG. 247.—Test of polarity of the end of a magnetic chain.

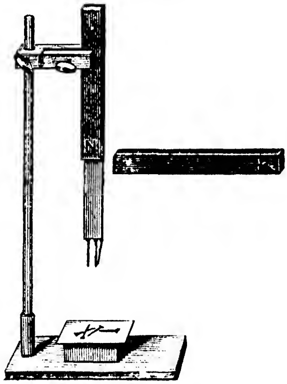


FIG. 248.—Effect of bringing unlike poles near one another.

EXPT. 232.—**Resultant induction.** Bring near to the N.-seeking end of the first magnet the south-seeking pole of a second bar-magnet. This second magnet will act inductively on the iron and nails also, but the induced polarity will be opposite to that already present. The

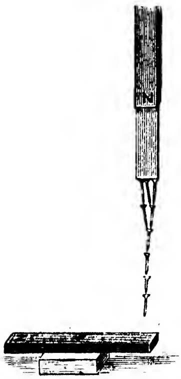


FIG. 249.—Increased effect due to induction.

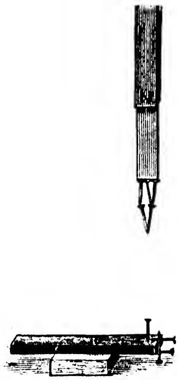


FIG. 250.—Decreased effect due to induction.

magnetisation of the iron and nails is consequently weakened, and most of the nails fall off (Fig. 248).

Replace everything as in Expt. 231. Now place the south-seeking pole of a second bar-magnet just below the end of the chain of

nails. We can add two or three more nails now; the induction due to the south-seeking pole tends to strengthen that originally present, and consequently the induced magnetism is increased (Fig. 249). Remove the south-seeking pole, several nails will fall off; if now the north-seeking pole of the second magnet be placed close to the end of the chain, more nails will fall (Fig. 250).

EXPT. 233.—**Repulsion between like induced poles.** Suspend from the pole of a vertically-clamped magnet a bunch of sewing-needles, or three or four strips of galvanised iron. Notice that the lower ends of all the needles have similar polarity, and mutually repel each other—the needles consequently bunch outwards (Fig. 251).

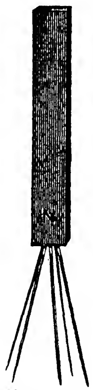


FIG. 251.—Expt. 233.

Induction in a magnet.—It has been shown that induced magnetism can be produced in a piece of iron which is already in a state of induced magnetisation. Magnetic induction can also be caused in a piece of iron which is a permanent magnet.

A long knitting-needle, for instance, which has been magnetised feebly may have its polarity entirely reversed by bringing a strong magnet near it. When the magnet is at some distance from the needle the induced magnetisation is weak, and its effect is hidden by that of the permanent magnetisation of the needle; but when the magnet is brought near the needle the induced magnetisation is not only sufficient to neutralise the permanent magnetisation, but completely overpowers it.

EXPT. 234.—**Effect of distance on degree of induced polarity.** Feebly magnetise a long knitting-needle, and suspend it in a stirrup. Hold the pole of a strong bar-magnet some distance away, and observe the repulsion between similar poles. Rapidly bring the magnet to within an inch of the repelled end of the needle, when the original repulsion is converted into a strong attraction.

This phenomenon, unless it is guarded against, often gives rise to incorrect conclusions in an experiment. It is most important in all such experiments gradually to bring the magnet from a distance near to the compass-needle, and to watch carefully for the effect. If the two ends, the polarities of which are to

be compared, have unlike polarity, then the induced magnetisation aids the true attraction which should be observed. It is only when the polarities are alike that the true repulsion may be masked by the attraction due to magnetic induction.

Retentivity.—Simple experiments on magnetic induction both with soft iron and with hard steel, show that the behaviour of these two materials differs considerably. This difference is not so easy to observe so long as the substance remains in actual contact with the magnet, but becomes very evident when it is removed from the neighbourhood of the magnet. The soft iron, when removed, immediately indicates a more or less complete loss of magnetisation. We may say that soft iron soon forgets the treatment to which it has been subjected. But hard steel, after removal, continues to exhibit the properties of a magnet; hence, we may say that hard steel does not forget its previous treatment. To remove the induced magnetisation from steel we must subject it to rough treatment, *e.g.* strike it several times with a hammer, or drop it on the floor, or in some other way disturb the arrangement of its minute particles.

The power of retaining magnetisation after the magnetising force is withdrawn is termed Retentivity. Some specimens of steel will retain as much as 90 per cent. of the original magnetisation; so also will soft iron, if not subjected to the slightest mechanical disturbance.

EXPT. 235.—**Retentivity of iron and steel.** Clamp a bar-magnet vertically, and suspend from the pole a piece of soft iron, from the lower end of which several wire nails are hanging. Carefully remove the soft iron, and observe the nails soon drop off, showing that the magnetisation induced in the soft iron is lost rapidly.

Now attach a short piece of hard steel to the pole of the magnet, and hang from the steel as many nails as possible. Carefully remove the steel, and notice that nearly all, or perhaps all, the nails continue to hang from the steel. The magnetisation in the steel remains, even though withdrawn from the influence of the bar-magnet.

If a piece of hard steel of suitable size is not to hand, the effect may be shown very well with ordinary steel pen-nibs.

Keepers or armatures.—The use of the keeper or armature is a practical application of the phenomenon of Magnetic Induction.

When a horse-shoe magnet is allowed to remain for a considerable time with its poles unprotected, its degree of magnetisation slowly diminishes; but when a short length of soft iron is placed so as to connect the poles, and in contact with the entire length of the pole-faces, this liability to loss of magnetisation is prevented. Any piece of soft iron serving this purpose is called a **keeper**. So long as it is in contact with the poles of the magnet the soft iron is also a magnet by induction. The stronger the induced magnetisation in the keeper the better its purpose is fulfilled.

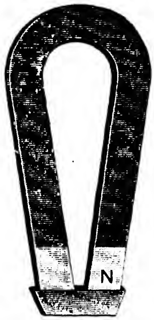


FIG. 252.—Horse-shoe magnet and keeper.

In Fig. 252, N induces south-seeking polarity at the near end of the keeper, and north-seeking polarity at the distant end. The pole S has the same effect, consequently N and S *help each other*, and produce a much greater degree of induced magnetisation than if they were acting alone.

The two poles of a bar-magnet cannot be connected in this simple manner, but the difficulty is overcome by keeping the bar-magnets in pairs, and placing them parallel to one another with opposite poles together. A piece of soft iron is placed at each end of the pair.

EXERCISES ON CHAPTER XXX.

1. Two similar rods of soft iron have each of them a long thread fastened to one end, by which they hang vertically side by side. On bringing near to the iron rods, from below, one pole of a strong bar-magnet, the rods separate from each other. Explain this.

2. If a compass-needle be deflected when a steel bar is brought near it, how can you find out whether the deflection is due to magnetism already possessed by the bar, or to the bar becoming magnetised by the compass-needle at the time of the experiment?

3. You have given to you two rods, one of soft iron, the other of hard steel; also a compass-needle and a bar-magnet. Describe experiments with the things provided whereby you could find out which was the iron and which was the steel rod.

If the rods are of the same size, describe how you could dispense with the bar-magnet and still distinguish the iron from the steel.

4. A bar-magnet is laid on a table with its N end projecting over the edge. A soft iron ball clings to the under side of the projecting

end. State and explain what happens when the S pole of a second magnet is brought (i) above and near to the N pole of the first, (ii) below and near to the iron ball. What will happen if the N pole of the second magnet is brought below and near to the iron ball?

5. A compass-needle and a straight strip of soft iron of the same length as the compass-needle are fastened together so as to be in contact with each other at both ends. Will the force which tends to make the combination point north and south be the same as that which would act on the compass-needle alone? Give reasons for your answer.

6. A bar-magnet is laid upon the table, and a soft iron bar of about the same length as the magnet is hung horizontally just above it by a flexible string. What will be the effect on the soft iron bar if a second bar-magnet be laid on the table and gradually brought near the first at right angles to it, and with its north-seeking pole pointing to the middle of the first magnet?

7. One pole of a magnet made of soft iron and only feebly magnetised is found to be repelled by the north pole of a strong magnet when the latter is some distance away, but to be attracted when the magnets are brought close together. Explain this.

8. What experiments would you perform to show the difference between the magnetic properties (*a*) of the two ends of a bar-magnet, (*b*) of soft iron and steel? (Lond Gen Sch.)

CHAPTER XXXI.

MAGNETIC FORCE, AND MAGNETIC FIELDS.

Application of the principle of moments to a magnetic experiment.—When a compass-needle is acted upon by two external magnets, as in Fig. 253, it comes to rest in a position such that the moments of the two forces F and F′ are equal and opposite.

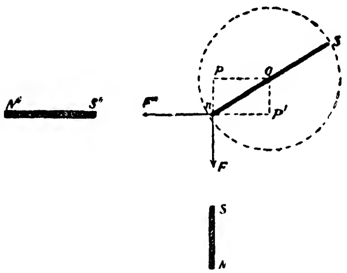


FIG. 253—Deflection of compass needle by two magnets.

The moment of

$$F = F \times OP = F \times nP'.$$

The moment of

$$F' = F' \times OP'.$$

Hence

$$(F \times nP') = (F' \times OP');$$

or

$$F' = F \times \frac{nP'}{OP'}.$$

In an actual experiment it is difficult to measure nP' and OP' separately, but the angle nOP' can be readily measured if the needle is pivoted at the centre of a wide circular scale, divided into degrees; and to the needle is attached a long light pointer with its ends moving just above the scale. The ratio nP'/OP' is the *tangent* of the angle nOP'; and when, in an experiment, the angle has been read, the value of the tangent of the angle can be obtained from Mathematical Tables.

If F′ is called the *deflecting-force,* the above result may be stated thus:—**The deflecting force is proportional to the tangent of the angle of deflection caused by it.**

The law of inverse squares.—The strength of the magnetic force which a bar-magnet exerts upon a compass-needle depends upon their distance apart. This suggests a resemblance to the law of inverse squares, which holds good with regard to gravitational forces (p. 103). To test whether this is the case, measurements are made of the deflection caused when a bar-magnet is placed at different distances from a compass made free to move in a horizontal plane. The earth's magnetic influence may be regarded as a constant force tending to pull the needle into a north and south direction; and a variable resultant force is created by placing the magnet at different distances from the compass-needle on which the forces due to the bar-magnet and the earth's magnetism act. As all magnets have two poles, it is evidently necessary to use a very long magnet, so that one of the poles is too distant to exert an appreciable disturbing effect on the needle. The apparatus with which the experiment is carried out is termed a **deflection magnetometer** (Fig. 254). This consists of a short

FIG. 254.—A magnetometer.

magnetised needle, pivoted at the centre of a circular scale; and a long pointer made of thin metal wire or foil is attached to the needle. The base-board is prolonged on opposite sides of the scale, and a wooden centimetre-scale is fixed on each of these arms and adjusted so as to measure distances from the centre of the needle.

Suppose that the pole of a long bar-magnet is placed d_1 cm. to the east or west of the centre of the needle, and that the deflection is $\theta_1°$; the force f_1 due to the pole is proportional to $tan\ \theta_1$. When the distance is increased to d_2 cm., the deflection is reduced to $\theta_2°$; and the force f_2 is proportional to $tan\ \theta_2$. If the force varies inversely as the square of the distance, then

$$\tan \theta_1/\tan \theta_2 = d_2^2/d_1^2,$$

or

$$\tan \theta_1 \times d_1^2 = \tan \theta_2 \times d_2^2.$$

Therefore if, in a series of such measurements, the product ($tan\ \theta \times d^2$) is approximately a constant quantity, it is evident that the magnetic force due to a distant pole varies inversely as the square of the distance.

For the experiment, it is necessary to use a very long thin magnet, 45-50 cm. long, so that the more distant pole does not exert an appreciable disturbing effect on the needle, and also so that the pole of the magnet may be regarded as being situated at the extreme end of the steel rod.

EXPT. 236.—**The law of inverse squares.** Adjust the magnetometer so that the graduated arms are in an East-West direction. Lay the long magnet on one of the arms, with its axis pointing towards the centre of the needle, and with its near pole 15 cm. from the needle. Gently tap the cover over the needle, so as to assist the needle to assume its true position of rest, and note the scale-reading of *each* end of the pointer. Transfer the magnet to the other arm of the magnetometer, and place it with the same pole at the same distance from the needle; tap the cover, and again note the two readings of the pointer. Calculate the mean of these *four* readings, and note it down as the *mean deflection* (θ). [These four readings are taken in order to eliminate possible errors due to several causes; *e.g.*, (i) the pointer may not be exactly at right angles to the needle, and (ii) the needle may not be pivoted exactly at the centre of the circular scale.]

Repeat all these readings when the magnet-pole is at greater distances from the needle. Tabulate the results thus:

Distance (d).	Mean deflection (θ).	tan θ.	d^2.	tan $\theta \times d^2$.
15 cm.	$33^\circ\cdot5$	0·665	225	149
20 cm.	$20^\circ\cdot6$	0·3775	400	151
25 cm.	$13^\circ\cdot6$	0·2425	625	151
30 cm.	$9^\circ\cdot7$	0·17	900	153

The poles of a magnet.—In the previous paragraph it was assumed that the magnetic forces originated from the extreme ends of the magnet. This assumption was sufficiently correct, owing to the fact that the magnet was extremely long as compared with its width. If such a magnet be dipped into iron filings, the filings only adhere to the ends in a small compact bunch.

In the case of a comparatively short thick magnet, filings will adhere chiefly to the ends, but some adhere even at a considerable distance from the ends. The pole of the magnet would therefore appear not to be a well-defined point, but a superficial area of considerable extent, each portion of which

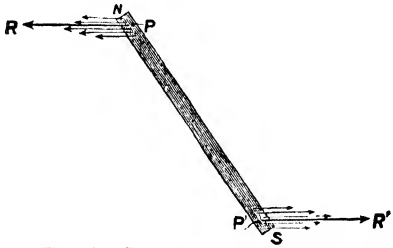


FIG. 255.—The points P and P′ are the poles of the magnet NS.

exerts magnetic force on a neighbouring magnet. The polarity seems to be more pronounced at the ends, and to diminish gradually towards the middle of the magnet.

In Fig. 255 let NS represent a bar-magnet which is suspended in a uniform magnetic field (pp. 384-5). In such a field the forces acting on the several small portions of the magnet which exhibit free polarity may be regarded as parallel to each other. Just as in mechanics a system of parallel forces (p. 125) may be replaced by a single force at a definite point, so in this case the system of parallel magnetic forces acting on the N-seeking pole may be replaced by a single force PR acting at a point P. Similarly, the forces acting on the S-seeking pole may be replaced by a single force P′R′ acting at a point P′. The points P and P′ are termed the **poles** of the magnet; and they may be defined as **the points of application of the resultant magnetic forces acting on a magnet which is situated in a uniform magnetic field.**

In short thick magnets (about 10 cm. long) the poles are situated about 1 cm. from each end. When the magnet is long, and only 1 or 2 millimetres wide, the poles approximately coincide with the ends.

In *Robison magnets*, which consist of long round steel rods of small cross-section, and fitted with a steel ball at each end, the pole may be regarded as being exactly at the centre of the steel ball.

A method of locating the poles of a bar-magnet is shown in Fig. 256. A sheet of *curve-paper* is fastened to a table and

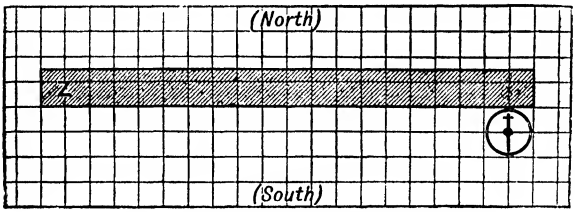


FIG. 256.—A method of locating the poles of a magnet.

adjusted so that one set of its lines have the same direction as that indicated by a compass-needle placed upon it. The bar-magnet is laid on the paper so that its long edge is at right angles to the same set of lines. A sensitive compass-needle (with glass top and bottom) is moved slowly along one side of the magnet until it comes to a position where it points exactly in the same direction as the lines of the curve-paper. The needle then points directly towards the pole of the magnet.

Magnetic forces due to both poles of a magnet.—Consider a single north-seeking pole situated at *n* near a bar-magnet NS (Fig. 257); it will be repelled by N in the direction *np*, and attracted by S in the direction *nq*. Since these forces are inversely proportional to the square of the distance, the force represented by *np* will be greater than that represented by *nq*, and in the ratio of $(nS)^2$ to $(nS)^2$. The resultant of these two forces is *nr*, and it is in the direction of this force that *n* will tend to move. The same method will determine the direction of the resultant force at other points, marked *n′*, in the magnetic field.

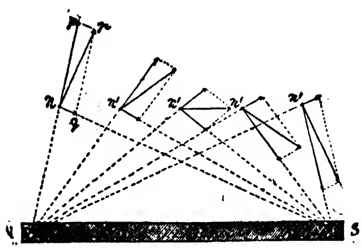


FIG. 257.—Graphical method of determining the magnetic forces due to a bar-magnet.

It is an instructive exercise to draw this diagram to scale on a sheet of paper, representing the magnet to be about 15 cm. long. The resultant forces obtained will show not only their relative directions, but the lengths of the lines will indicate their relative magnitude also.

If *n* be replaced by a south-seeking pole of equal strength to *n*, the forces due to NS will be equal in magnitude to those acting on *n*, but they will be opposite in direction. Consequently, if a short compass-needle be placed at the point *n*, it will not tend to travel bodily in any direction, since the forces acting upon it are equal and opposite, but it will simply come to rest pointing in the direction *nr*; at *n'* it will point in the direction of the resultant force at that point.

Magnetic Fields.

A field of magnetic force.—When a suspended magnetised needle is allowed to swing to and fro round its point of suspension, the manner in which it swings suggests that there are invisible forces acting on the needle, tending to bring it to rest with its magnetic axis pointing in a definite direction. Whenever these invisible magnetic forces appear to be influencing a suspended magnetised needle, it is said to be in a field of magnetic force. By observing their effects, the presence of these forces is detected, and, in addition to this, their direction also is determined by observing the direction in which the needle points when it comes to rest.

Since the needle behaves in this manner even when no other magnet is near to it, the only possible conclusion is that **the earth has a magnetic field of its own**; and if so, this must be due to a region of south-seeking magnetism situated in the direction of the north pole of the earth, and of north-seeking magnetism in the direction of the south pole.

If a bar-magnet be held near the swinging needle, a magnetic disturbance ensues which causes the needle to swing to and fro in the same characteristic manner, perhaps more rapidly perhaps more slowly; and, in nearly every possible position of the magnet relatively to the needle, the needle acquires a different position of rest. Evidently the bar-magnet has a field of magnetic force of its own, the effects of which have

been superposed upon those due to the earth's field. The needle will come to rest in a position indicating the direction of the **resultant magnetic force**, which is due partly to the bar-magnet and partly to the earth.

Again, it will have been observed that the needle swings sometimes rapidly, sometimes slowly. If it swings more rapidly, then the magnetic forces acting on it must be stronger ; if it swings more slowly, the magnetic forces must be weaker. In fact, by observing the rate of vibration, it is possible to compare the strengths of the magnetic forces at two different points.

Dr. Gilbert, who was physician to Queen Elizabeth, observed these effects in 1600, and he described a lodestone or magnet as being surrounded by an "orb of virtue." About the middle of last century Faraday substituted the term **magnetic field**.

The earth's magnetic field.—In order to investigate the character of any magnetic field, it is necessary to determine two factors at all parts of the field—(i) the direction of the magnetic force, and (ii) the strength of the force. Any diagram which represents the force-directions is termed a map of the magnetic field included within the area of the diagram.

The 'map' will consist of lines—perhaps straight, perhaps curved—the direction of which, at any point, indicates the direction in which a compass-needle will come to rest when placed at that point. Faraday, in 1837, gave to such lines the term **lines of magnetic force**.

The method of obtaining the map of a magnetic field by means of a compass-needle is indicated in the following experiment.

EXPT. 237.—**Map of the earth's magnetic field.** Fasten on a table a sheet of white paper (½-imperial). Mark off one of the edges into spaces, about 5 cm. wide, by pencil dots, as shown in Fig. 258. *Remove all magnets to a distance.* Place a sensitive compass-needle (with glass top and bottom) so that one of its poles is vertically over one of the dots, such as A, and indicate by means of another dot the direction in which the other pole is pointing. Move the needle until its first pole is exactly over the second dot, and mark a third dot in front of the other pole. Con-

tinue this process until a series of dots have been obtained completely across the paper. Join up these dots by a continuous pencil line. Plot out other lines in a similar manner. In Fig. 258 the line commencing with dot C is in process of being traced out. Indicate, by means of arrowheads, the direction in which the N-seeking pole of the needle tends to move: this is called *the positive direction of the magnetic field.*

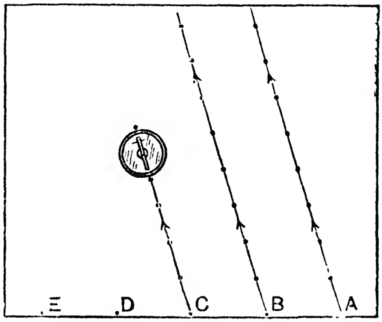


FIG. 258.—Method of obtaining a map of the earth's magnetic field.

The map thus obtained is a *horizontal* map of the earth's magnetic field, so far as the limits of the paper will allow: it indicates that the lines of force due to the earth are parallel. The same result would be obtained in any other part of the room; and it may be assumed that the earth's magnetic field is *uniform* over an area much larger than is required for the manipulation of ordinary experiments in magnetism.

Resultant magnetic fields.—In the neighbourhood of a bar-magnet the map of the field is much more complicated since, at any point, the magnetic force is the *resultant* of two forces—one due to the earth, and the other due to the magnet. At points near to the magnet, the force due to the magnet is predominant; but at more distant points, the force due to the earth is modified only slightly by that due to the magnet.

Fig. 259 (i) represents the resultant field due to the earth and to a bar-magnet placed with its N-seeking pole pointing direct South. In Fig. 259 (ii) the bar-magnet is pointing in the opposite direction. These maps can be obtained by the method previously described in Expt. 237; but a larger sheet of paper will be required.

Notice how, in Fig. 259 (i), the lines of force due to the earth appear to be pushed outwards on each side of the magnet; while, in Fig. 259 (ii), they appear to be drawn into the magnet to emerge at the opposite pole, to spread outwards, and gradually resume their parallel paths.

In both cases, two *neutral* regions are found in which the magnet's influence upon the needle is just neutralised by that of the earth, and where therefore the needle does not tend to point in any definite direction : these regions, marked X in the diagrams, are termed **neutral points**.

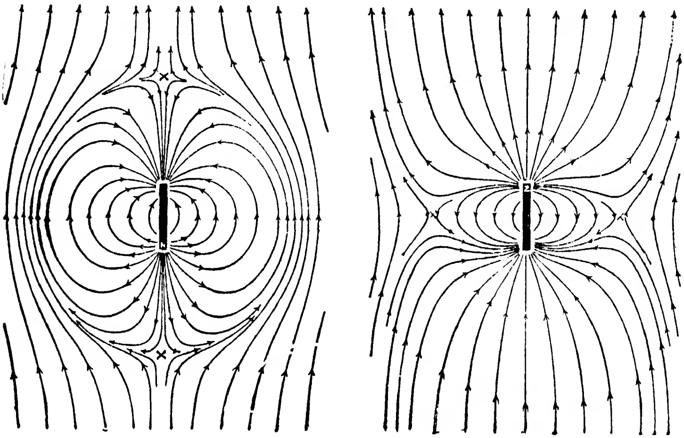


FIG. 259.—Map of the resultant field due to the earth and a bar-magnet. (i) N pole pointing south ; (ii) N pole pointing north.

It is easy to understand the reason for these neutral points ; for, in Fig. 259 (i), a single N-seeking pole, situated at the lower X, will be repelled towards the South by the magnet, while the earth attracts it towards the North. At a certain distance from the magnet these two forces will be equal in magnitude, and the single pole will not tend to move in either direction.

Fig. 260A represents the resultant magnetic field due to the Earth and a bar-magnet with its axis in an East-West direction, and with its N-seeking pole pointing towards the West. A rough sketch of the field due to the magnet alone will explain the reason for the neutral points being in the positions shown.

Fig. 260B is the map of the resultant magnetic field due to the Earth and a long cylindrical magnet clamped vertically, and with its N-seeking pole touching the surface on which

the map is drawn. In this arrangement there will be only one neutral point, and situated to the South of the magnet-pole. The distance of this point will depend of course upon the pole-strength of the magnet and upon the strength of the earth's field (see p 407)

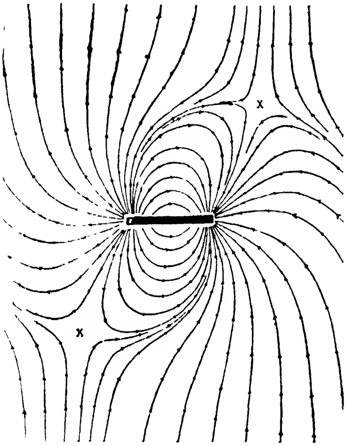


FIG. 260A.—Resultant field due to the Earth and to a magnet with its axis in an East-West direction

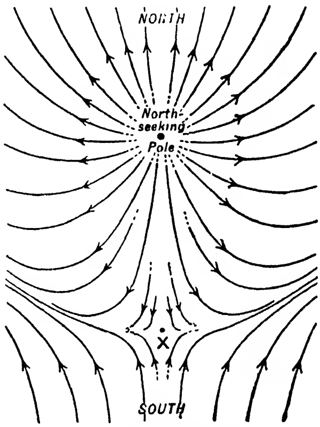


FIG 260B—Resultant field due to the Earth and to a single North-seeking pole

An instructive analogy, due to Faraday, compares the properties of the magnetic lines to the forces which would be exerted by stretched elastic threads, coinciding in direction with the lines of force, which tend to shorten themselves from end to end and to repel one another from side to side.

It is generally accepted that the **positive direction of a line of force is that direction in which a single north-seeking pole will tend to travel if placed at any point on that line of force.** The opposite direction is termed the **negative** direction of the line of force. Hence, a map of the magnetic field of a magnet will indicate the lines of force as emerging from the north-seeking pole and re-entering at the south-seeking pole.

It can be shown by experiment that a north-seeking pole actually does tend to travel along a line of force in the positive direction.

EXPT. 238.—**Movement along a line of force.** Support a bar-magnet, 20 cm. long, near and parallel to the edge of a large photographic dish filled with water. Magnetise a short fragment of sewing-needle, and fix it through a small piece of cork so that the needle can float freely in a vertical position. Let the north-seeking pole of the needle be uppermost. If floated near the north-seeking pole of the magnet the repulsion of the similar pole of the needle will be stronger than the attraction of the opposite pole of the needle, since the latter is more distant. The needle will travel slowly over the surface of the water, tracing out a curved path connecting the north- and south-seeking poles of the magnet.

Iron-filing maps of magnetic fields.—The compass-needle method of obtaining a map of the magnetic field of the earth or of a magnet has the advantage of accuracy, and it is also capable of affording information in parts of a magnetic field which would be too weak to be mapped out by the methods now to be described. The latter will give accurate maps of the field near a magnet, but will not do so for the more distant parts where the earth's magnetic field is predominant. The compass-needle method, however, cannot well be adopted in the lecture-room owing to the time required to obtain even one complete map.

The more rapid methods depend upon the principle of magnetic induction, whereby a piece of soft iron, when placed in a magnetic field, becomes magnetised by induction. Soft iron filings may be used for the purpose; each fragment becomes a temporary magnet, and, if free to move, behaves in the same manner as a compass-needle. The effect is approximately the same as would be obtained if an extremely large number of compass-needles were used, and, moreover, the general contour of the whole field is visible simultaneously.

The chief disadvantage is that the filing has far less freedom of movement than a compass-needle: this difficulty is partly overcome by supporting the filings on a *smooth* surface (*e.g.*, glass or smooth paper), and afterwards tapping the surface gently, say, with a pencil-point, so as to give to the filings some freedom of movement. This limitation prevents the filings from being influenced by a weak magnetic field, and the

method therefore is only suitable for mapping fields comparatively near to magnets.

When permanent maps are required, sheets of *paraffined paper* are used : these are prepared by passing sheets of thin white paper, such as thin typewriting paper, through melted paraffin-wax contained in a tin. The filings distributed on the surface of this paper are fixed in position by passing the flame of an inverted Bunsen burner over the surface, thus melting the paraffin-wax.

EXPT. 239.—**Iron-filing maps.** Lay a sheet of paper over a magnet, or group of magnets, and support it horizontally on slabs of wood placed round the magnet. By means of a metal pepper-box, or a muslin bag of coarse mesh, sprinkle filings uniformly over the paper, and gently tap it in several places with a pencil point. Sketch, in your note-book, the general contour of the lines of force, *selecting only a few typical lines*, and indicate by arrow-heads the ' positive ' direction of the force along each line. Do not attempt to imitate, in your sketch, the appearance of the iron-filing map.

Obtain maps of the fields due to the following arrangement of magnets :

(i) One bar-magnet (Fig. 261).

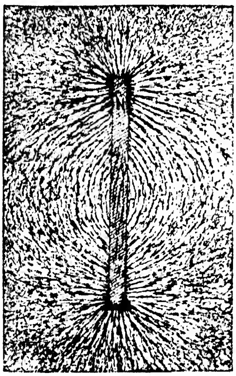


FIG. 261.

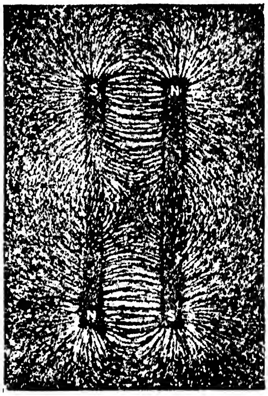


FIG. 262.

(ii) Two bar-magnets side by side, with unlike poles together (Fig. 262).

(iii) Two bar-magnets side by side, with like poles together (Fig. 263).

(iv) Two bar-magnets, with their axes in line, and unlike poles together.

(v) Two bar-magnets, with their axes in line, and like poles together.

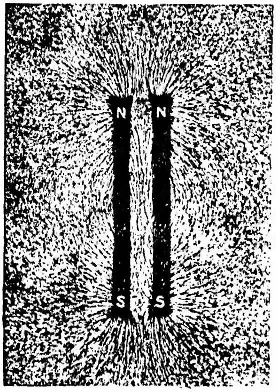


Fig. 263.

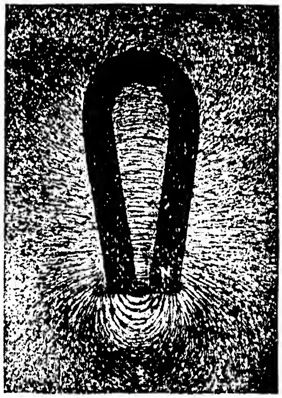


Fig. 264.

(vi) One horse-shoe magnet, without keeper (Fig. 264).

(vii) One cylindrical bar-magnet, fixed in a vertical position, and the paper supported horizontally over the upper pole (Fig. 265).

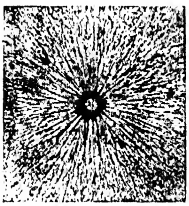


Fig. 265.

Symmetry of the magnetic field due to a single bar-magnet.—In Fig. 261 it will be seen that lines of force emerge from and re-enter the magnet at all points (with the exception of a small portion near the centre), and the distribution of the lines is densest in parts near to the extreme ends. The map does not indicate those lines of force which pass vertically through the paper, or those which pass vertically downwards through the table; in fact, the map is really a horizontal cross-section through the magnetic field. If it were possible to obtain a vertical map by the same methods, it would be found that the arrangement of the lines of force is identical

with that obtained in the horizontal maps. If the magnet is turned over on to its side the lines of force which were originally in a vertical plane will now be horizontal, and a map of the field of the magnet in this position will show that their contour and general distribution is identical with that obtained when the magnet was in its original position. In fact, the distribution of the lines of force is the same in all planes besides the horizontal and vertical. A bar-magnet may be imagined to be clothed completely on all sides in an invisible garment of lines of force. The lines of force in a vertical plane can be detected readily by means of a short magnetised needle supported from its centre by a silk fibre.

EXPT. 240.—**Vertical magnetic field.** Attach a silk fibre to a small sewing-needle and adjust the fibre so that the needle is exactly horizontal when swinging freely. Magnetise the needle by placing it inside a spiral of wire through which an electric current is passing. Clamp a large bar-magnet in a horizontal position and support the needle vertically over and under the magnet in a series of different positions (Fig. 266). It will be evident that the general contour of the vertical magnetic field is the same as the horizontal.

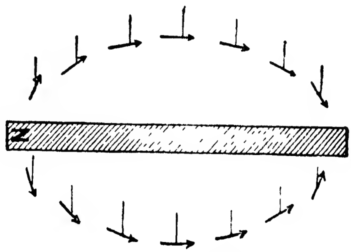


FIG 266—The vertical field of a bar-magnet mapped out by means of a suspended magnetised needle

Internal magnetic fields.—So far, only the magnetic phenomena in the region round a magnet have been considered. Each line of force may, however, be regarded as being continued through the substance of the magnet, so as to form a complete loop without free ends. That this is the case may be illustrated by breaking a magnet, when it will be found that lines of force proceed from one part of the fracture to the other.

Every small fragment of a magnet is, in fact, traversed to a greater or less degree by lines of force, which enter at its south-seeking pole and emerge at its north-seeking pole (Fig. 267).

Since each small fragment is a complete magnet in itself, a bar-magnet may be regarded as consisting of a large number of minute magnets arranged with like poles pointing in the same direction. There is no theoretical reason why this breaking

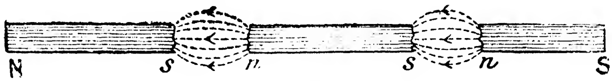


FIG. 267.—A broken magnet.

process should not be continued until the fragments are almost infinitely small, and each such fragment found to be still a complete magnet. Modern theory maintains that even the smallest physical quantity—the molecule—present in a bar-magnet is a minute magnet, though the bar may contain many millions of such molecules.

EXPT. 241.—**Effect of breaking a magnet.** Magnetise a piece of clock-spring about 10 cm. long. Break it into halves. Examine the pieces by a compass-needle. One half does not possess north-seeking polarity only, and the other half south-seeking polarity; each half is in itself a complete magnet, possessing two unlike poles. Lay the halves on the table in line with one another, and about 2 cm. apart. Sprinkle iron filings upon a sheet of paper placed over the parts of the clock-spring; there are evidently lines of force connecting the broken ends. Break the spring into still smaller fragments, and test the polarity of each. Observe that like poles of every fragment point in the same direction.

EXPT. 242.—**A steel filing magnet.** Fill a glass test-tube with steel filings loosely packed; cork up the tube, and notice that it behaves towards a test-needle like an ordinary piece of iron. Magnetise the tube by stroking it in one direction with one pole of a strong magnet, or better, by means of a spiral of wire and electric current (Expt. 223). Observe that the tube now has opposite polarities at the ends, and that the filings appear to some extent to have arranged themselves lengthwise. Each filing has been magnetised, just as small sewing-needles would have been magnetised by similar treatment. Each filing has its lines of magnetic force, which come from, and afterwards pass into, neighbouring filings, and only appear at the ends of the tube where they emerge into the surrounding space. Empty the filings upon a sheet of paper, mix well together, and pour them back into the tube; again test for polarity.

Theory of magnetisation.—In a bar of unmagnetised steel or iron, each molecule may be a magnet, but the molecules are grouped into numerous independent magnetic chains. These chains become broken up by the process of magnetisation, which causes all the molecular magnets to turn into line. With slight magnetisation the molecules merely turn through a slight angle, giving a slight excess of north-seeking polarity in the direction of the magnetising force, and slight excess of south-seeking polarity in the opposite direction. As the magnetisation proceeds, the molecules turn gradually more into line. When all the molecules point exactly in the direction of the force, an increase of the latter will not produce any further effect—in fact, the magnet is saturated. This explanation is known as **Weber's theory of magnetisation.**

Behaviour of soft iron in a magnetic field.—When a bar of soft iron is placed in a magnetic field, with its length coinciding with the direction of the magnetic lines of force, the molecular magnets of the soft iron are pulled partially or completely into alignment, according to the magnitude of the magnetising force; the iron temporarily becomes magnetised by induction. At all points where lines of force enter the iron we find a region of south-seeking polarity, and where they emerge a region of north-seeking polarity. If the iron be situated so that the lines of force pass through the iron from side to side perpendicular to its axis, then no lines of force traverse its length and no polarity is found at the ends; in such a case the polarity will be found distributed along the two sides. At the same time the presence of the iron disturbs the direction of the lines of force of the original field, the lines being drawn into the iron as though they preferred to pass through it rather than through the surrounding air.

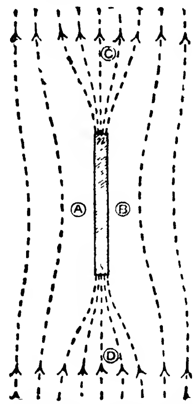


FIG 268.—A bar of soft iron in the earth's field.

EXPT. 243.—**Effect of soft iron on the earth's magnetic field.** Test a long strip of soft iron to ensure that it is entirely free from polarity. Place it on a sheet of plain paper, its

length coinciding with a north-south line. By means of a compass-needle map the lines of force in the region near to the iron (Fig. 268). The iron will have acquired S-seeking polarity at the end *s*, and N-seeking polarity at the end *n*; and the lines of force of the original field are no longer parallel and straight, as explained on p. 385, but have the appearance of being drawn into the iron. In the regions marked A and B, on each side of the iron, the lines are further apart than in the original field; in these regions, therefore, the field is weakened. Similarly, in the regions C and D, the field is strengthened.

When soft iron is arranged so as entirely to withdraw the lines of force from any region, the iron serves the purpose of a *magnetic screen* for that region. A region is screened most effectively by surrounding it with a thick cylinder of soft iron placed with its axis perpendicular to the direction of the field.

EXERCISES ON CHAPTER XXXI.

1. Several bar-magnets are placed on a table. How would you use a card and iron filings to determine how to place a nail, lying horizontally on the table with its centre at a given point, so that it may acquire (i) the largest, (ii) the smallest possible amount of magnetism by induction?

2. A strong bar-magnet is placed on a table with its axis lying in the magnetic meridian, and with its north-seeking pole towards the north. State in what direction a compass-needle points (i) when placed immediately over the centre of the magnet, (ii) when gradually raised vertically upwards.

3. What is meant by a line of force? Draw diagrams showing the general form of the lines of force when a small magnet is placed with its axis parallel to the lines of force of the earth's field, if the north pole of the magnet is turned towards (i) the north, and (ii) the south.

4. A short piece of soft iron is to be magnetised inductively. State how it should be placed relatively to (i) a bar-magnet, (ii) a horse-shoe magnet, in order to obtain a satisfactory result. Give diagrams.

5. Give a diagram of the lines of force due to a horse-shoe magnet (i) with the keeper on, (ii) with the keeper off.

6. Describe the difference between the magnetic properties of soft iron and hard steel. Which would you use (i) for the core of an electro-magnet, (ii) for a permanent magnet? Give reasons for your answer.

7. Iron filings are scattered on a piece of cardboard which is placed over a horse-shoe magnet and tapped. What differences would be observed in the arrangement of the filings when the ends of the magnet were joined in turn by bars of (i) steel, (ii) soft iron, and (iii) copper ?

8. If you were required to magnetise a circular ring of steel so that it should show no sign of magnetisation, how would you proceed ? And how, being allowed to deal with the steel in any way that you pleased, would you prove that it really was magnetised ?

9. An iron ball is held over a pole of a horse-shoe magnet. Will the attraction exerted on the ball be altered if the poles of the magnet are connected by a soft iron keeper, and, if so, in what way, and why ?

10. A long magnet and a piece of soft iron of the same size and shape are placed parallel to each other underneath a sheet of paper on which iron filings are strewed. How will the filings arrange themselves ?

11. A compass-needle is deflected by a bar-magnet placed some distance away from it. How is the deflection modified (if at all) when a bar of soft iron is placed parallel to, but not touching, the magnet ? Give reasons for your answer.

12. Explain the term *line of magnetic force*. What information can be supplied by a well-drawn diagram of the arrangement of these lines to any particular case ?

Explain carefully how it is that iron filings can be used to determine the arrangement of the lines of magnetic force round a bar-magnet. (Lond. Matric.)

13. Explain the following terms :—magnetic field, pole of a magnet, neutral point. Draw a diagram of the lines of force due to a bar-magnet in the magnetic meridian with its N-pole pointing (*a*) north, (*b*) south. Mark the positions of the neutral point in each case. (Cen. Welsh Bd.)

14. What do you understand by the terms *magnetic field* and *line of magnetic force ?*

A long bar-magnet stands vertically on a horizontal table, its lower end being of north polarity. Describe how you would map the magnetic field in the plane of the table, and explain with a clear diagram the result you would expect to obtain.

(Lond. Gen. Sch.)

15. The poles of a bar-magnet are 10 cm. apart. Indicate a geometrical construction for finding the direction of the resultant line of force at a point distant 6 cm. from one pole and 8 cm. from the other. How would you verify such a result experimentally ?

(Lond. Matric.)

CHAPTER XXXII.

TERRESTRIAL MAGNETISM.

The earth a magnet.—The characteristic manner in which a compass-needle swings to and fro and finally comes to rest pointing approximately north and south, even in the absence of any neighbouring magnet, suggests that the earth itself must be enveloped in a field of magnetic force. There are lines of magnetic force originating from a region of north-seeking polarity in the neighbourhood of the south geographical pole, and traversing the earth's surface towards a region of south-seeking polarity in the neighbourhood of the north geographical pole. This suggests that a piece of soft iron will become temporarily magnetised if held with its axis pointing in the same direction as that in which a compass-needle points.

EXPT. 244.—**Magnetisation by means of the earth's field.** Hold a strip of thin galvanised iron (about 30 cm. × 2 cm.) so that it is pointing approximately north and south. Tap it gently with the knuckles. Test its polarity by bringing its ends near to a compass-needle. The end pointing towards the north has acquired north-seeking polarity. Now hold the iron with its north-seeking pole pointing towards the south, and again tap it. Notice that its polarity is reversed. Finally, hold the iron in an east and west position and again tap it. Notice that all polarity has disappeared.

The tapping may even be dispensed with if the soft iron be simply kept in position and the compass-needle be brought near to its ends in order to detect the polarity.

Declination.—The **geographical meridian** at any point of the earth's surface is the vertical plane passing through that point and through the poles of the earth. The **magnetic meridian** at

any point is the vertical plane passing through the axis of a compass-needle placed at that point. In most localities on the earth's surface these two meridians do not coincide exactly. **The angle** between the magnetic meridian and the geographical meridian at any place is called the **Declination** at that place.

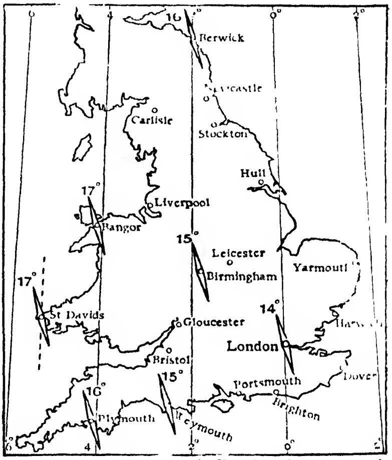


FIG. 269.—Magnetic declination is the angle between the geographical meridian and the direction in which a compass-needle points. The values shown are for 1925.

The fact that the compass-needle does not point to the true north was observed first by Columbus when on a voyage in 1492. He found that, at a point near the Azores, the compass pointed true north, but that in regions to the east of this it pointed west, and that in regions to the west it pointed east of true north.

In Great Britain, and in many other localities, the compass-needle points to the west of true north. Elsewhere the declination is easterly, and there are comparatively few localities where the needle points due north.

The magnitude of the declination in any locality is not constant, but changes slowly from year to year. The declination at Greenwich was 13° 24′ W. in 1924, and diminishes at the approximate rate of 10′ per annum. This **secular change**, as it is termed, was first observed in 1580 by Burroughs (comptroller in the Navy in the time of Queen Elizabeth). In that year the declination in London was 11° E.; this gradually diminished, and in 1657 the needle pointed due north. The declination then became westerly, and reached a maximum value of 24° 30′ W. in 1816. Since that date it has been slowly diminishing to its present value. It is estimated that 480 years are required for a complete cycle in the changes of the declination.

Measurement of declination.—In order to measure the declination at any point, it is necessary to determine the directions of both the geographical meridian and the magnetic meridian. The apparatus for obtaining an accurate measurement is elaborate; but the principles of the method can be explained by simple means.

The direction of the geographical meridian can be found only by observations of the sun, north star or other heavenly bodies; it cannot therefore be conveniently determined in a laboratory unless there is an open view to the south. A simple method of determining its direction approximately consists in fixing a rod upright on level ground where the sun can shine upon it (Fig. 270). About an hour or two before mid-day, the direction and length of the shadow is marked on the ground; and, taking this as a radius, a part of a circle is drawn on the ground. In the afternoon, when the shadow has exactly the same length as in the morning observation, the direction of the shadow is again marked. A line bisecting the angle between these two directions of the shadow is a true North and South line.

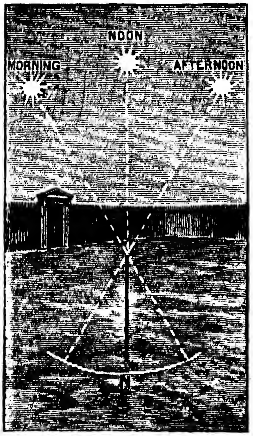


FIG. 270.—Method of finding the geographical meridian.

In order to determine the magnetic meridian, it is not sufficient to suspend a magnet and to observe the direction in which it points when at rest, because the *magnetic axis* of the magnet may not coincide with its *geometric axis*. This possible source of error is eliminated by recording the direction of the geometric axis when in one position and, without altering the position of the suspension, *inverting* the magnet and again recording the direction of the geometric axis. *The line bisecting the angle between these two recorded directions is the direction of the magnetic meridian.*

The principle of this method is explained in Fig. 271. A small piece of cardboard, pierced with a round hole with cross-

wires, is fixed to each end-face of the magnet. The direction *ab* of the intersection of the cross-wires is recorded on the bench by two long brass pins (Fig. 271 i.). The magnet is then reversed, top for bottom, the direction *a′b′* similarly recorded (Fig. 271 ii), and the acute angle between *ab* and *a′b′* is bisected.

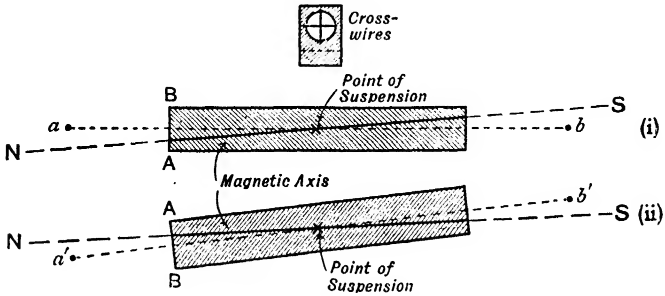


FIG. 271.—Method of finding the magnetic meridian.

To make the method quite clear, suppose that the magnetic axis of the bar-magnet is represented by the *thick black line*, in the diagram : the magnet will come to rest with this line coinciding with the magnetic meridian, whereas the direction of the line joining the cross-wires may be as shown by the broken line *ab*. When the magnet is reversed, the corners A and B of the magnet are interchanged, and the direction of the line joining the cross-wires will be in the direction *a′b′*. The magnetic meridian NS bisects the angle between *ab* and *a′b′*. The two diagrams in Fig. 271 are shown separately only for the sake of clearness : they *ought* to be superimposed.

An interesting modification of the above experiment is found by substituting for the bar-magnet a steel disc which is magnetised along a diameter not marked on the disc. The procedure is as follows :

Mark on each face of the disc a long arrow passing through the centre and pointing in the same direction. Suspend the disc horizontally, by means of silk cord, just above the table (Fig. 272). When the disc comes to rest mark on the table

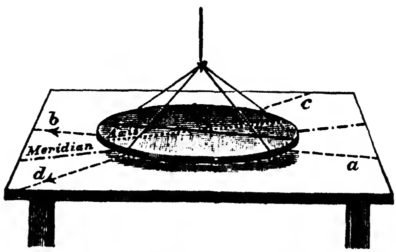


FIG. 272.—The magnetic axis of a magnetised disc.

the direction *ab* in which the arrow is pointing, and mark this direction with an arrow head. Invert the steel disc, and mark the direction *cd* in which the arrow now points. Remove the steel disc, and bisect the angle included between the arrow heads at *b* and *d*. This bisecting line is the magnetic meridian. Re-suspend the disc, and mark on its surface a line coinciding with the meridian ; this line is the magnetic axis of the disc.

Magnetic dip.—It does not follow that, because a compass-needle supported on a pivot remains horizontal, the lines of force acting upon it are also horizontal. Even if the lines of force are inclined to the horizontal, it may still be possible for them to have a directive action on the needle. In Fig. 273 let **ns** represent a compass-needle, and **nI**, **sI′** the forces due to the earth's field. The force **nI** may be regarded as the resultant of two separate forces—**nH** the **horizontal component**, and **nV** the **vertical component.** Similarly, **sI′** may be regarded as the resultant of the two forces **sH′** and **sV′**. The forces **nH** and **sH′** will pull the needle into the magnetic meridian, while **nV** and **sV′** will tend simply to tilt the needle out of the horizontal. The weight of the needle is usually sufficient to mask the effects of the latter forces. It is found that the earth's lines of force actually are inclined to the horizontal in most localities, and the tendency to tilt the needle is neutralised by making it slightly heavier at one end.

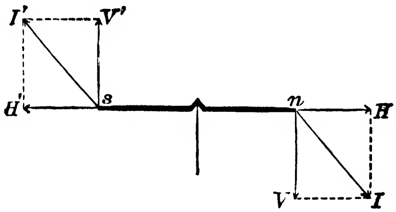


FIG. 273.—A compass-needle acted upon by magnetic forces inclined to the horizontal.

EXPT. 245.—**The dip of a magnetised needle.** Make a rigid axle for a long knitting-needle by holding two short pieces of thin copper wire, one on each side of the needle, at right angles to it, and *tightly* twisting the ends together by means of pliers. Adjust the position of the axle so that the needle, when supported by it, is in neutral equilibrium. Carefully magnetise the needle without disturbing the position of the axle. Observe that the needle now dips down with its north-seeking pole

downwards (Fig. 274). Since the needle naturally tends to take up a position along the lines of force, it follows that the latter must be inclined to the horizontal.

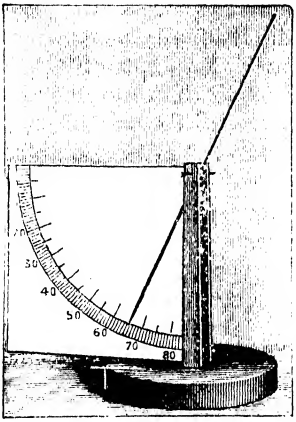


FIG. 274.—A simple form of dip-needle.

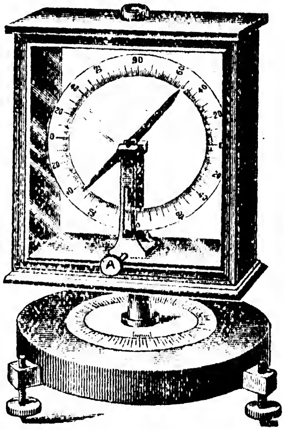


FIG. 275.—A dip-needle.

The Dip-needle.—The angle between the axis of a magnetised needle, which is free to move in the vertical plane of the meridian, and the horizontal line through its point of support, is called the Dip.

In order that a magnetised needle may move freely in a vertical plane, it must be supported on a rigid horizontal axle ; if it is to be influenced by magnetic forces only, and to be absolutely independent of the force of gravity, the axle must coincide with the centre of the needle. The construction of an accurate dip-needle is an extremely delicate operation, and it is only possible to obtain correct measurements with a costly apparatus. Fig. 275 represents a form of dip-needle used for determinations of magnetic dip.

To make an observation, the instrument is turned until the needle is vertical, in which case it is at right angles to the magnetic meridian. The stand is then rotated through 90° to bring the plane of movement of the needle into the magnetic meridian, and the angle which the needle then makes with the horizontal is the angle of dip.

Angle of magnetic dip in various localities.—The **dip**, like the declination, differs in different localities, and also changes from year to year. The dip at Greenwich, for the year 1920, was 66° 54′, and was 66° 51′ in the year 1924. Near the equator localities are found where the dip is nil. As the needle is conveyed northwards the dip gradually increases, and at a point in Boothia Felix (Lat. 70° 5′ N., Long. 96° 46′ W.) Sir John Ross found, in the year 1831, that the dip-needle was exactly vertical. This region must be one of south-seeking polarity ; it is one of the so-called magnetic poles of the earth. When the needle is conveyed southwards from the equator, the south-seeking pole of the needle dips downwards, and the amount of dip gradually increases as the south magnetic pole is approached. It was found in 1909, by the Shackleton Antarctic Expedition, that the South magnetic pole was situated in South Victoria land in latitude 72° 25′ S and longitude 154° E.

The secular change in the dip is far less in magnitude than that of the declination. Thus, in the year 1576 it was 71° 50′ at London, in 1720 it was 74° 40′, and at the present time it is decreasing at about the rate of 0·5′ every year.

The directive action of the earth's field.—The action of the earth's field is directive only, and not translatory also. Consider a small magnetic needle on a cork floating in water. The forces due to either of the earth's magnetic poles acting on the two poles of the needle are opposite in direction. The distance of the earth's magnetic pole from this latitude is several thousand miles, in comparison with which the length of the needle is infinitesimal. Hence the two poles of the needle may be regarded as being equally distant from the earth's magnetic poles, and the forces acting on the needle are therefore practically equal in magnitude.

When, however, the pole of a bar-magnet is held near the needle the length of the needle is no longer small compared with the distance away of the magnet-pole. One pole of the needle will be considerably nearer to the magnet-pole than the other, one force will be greater than the other, and the floating needle will move bodily in the direction of the greater force.

EXPT. 246.—**Action of the earth.** Fix a magnetised sewing-needle to a flat cork with wax so that the needle is horizontal when the cork is floating on water contained in a dish. Float the cork on the water so that the needle points east and west. Notice how the needle rotates into the magnetic meridian, but does not tend to move bodily towards the side of the dish.

EXPT. 247.—**Action of a magnet.** Hold the pole of a bar-magnet near the needle. Notice how the needle not only points in a definite direction depending upon the position of the magnet, but also moves bodily towards the magnet.

Simple hypothesis of terrestrial magnetism.—Both declination and dip may be explained roughly as being due to a huge

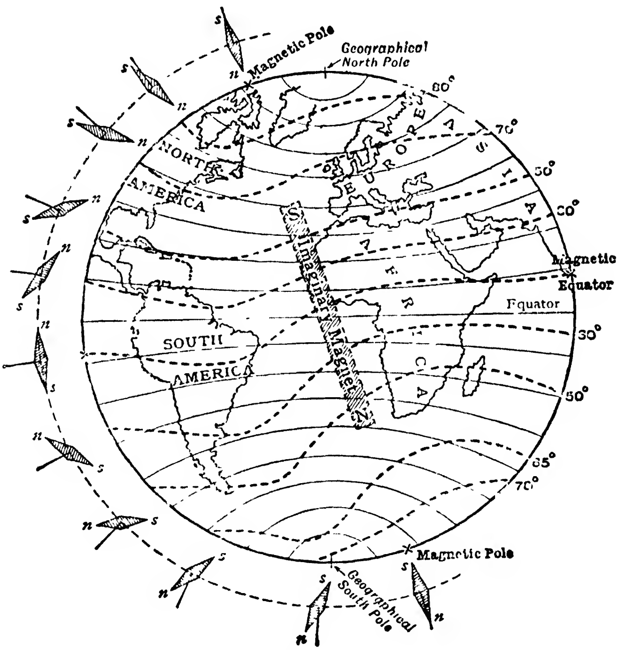


FIG 276.—MAGNETIC DIP. A magnet supported through its centre of gravity is horizontal only when near the equator. As it approaches the poles the angle of dip increases; and the localities where it is vertical are the earth's 'magnetic poles' A dotted line traverses localities where the dip is the same: in Great Britain the *magnetic dip* is 67° approximately.

imaginary bar-magnet passing through the earth's centre and slightly inclined to its axis, so that one end approaches the earth's surface at Boothia Felix and the other end approaches the surface in South Victoria land. At these points the dip-needle stands vertically, and they are called the **magnetic poles** of the earth. The directions of the lines of force of such a magnet would coincide approximately with the directions in which a dip-needle is observed to point. Fig. 276 indicates the relative positions of the north geographical and north magnetic poles; and the directions are shown of a dip-needle placed at various points on the earth's surface.

The ship's compass.—The simplest form of mariner's or ship's compass consists of a magnetised needle fastened underneath a circular card, the upper surface of which is divided by radii into thirty-two divisions. These divisions are called **the points of the compass.**

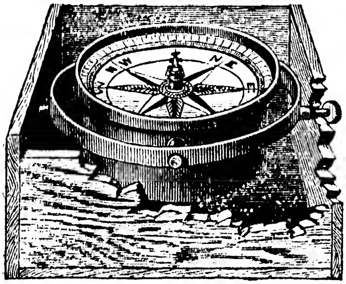


FIG. 277.—Method of supporting a compass-box on gimbals.

In order to prevent the rolling of the ship from disturbing the compass out of the horizontal position, the circular box (made of brass or copper) containing the needle is supported on **gimbals**, the nature of which will be understood from Fig. 277.

Kelvin and Chetwynd compasses.—Lord Kelvin's standard compass (Fig. 278), which until recently was adopted by the Admiralty, consists of eight thin steel needles, fastened together on two silk threads, and slung by other threads to the edge of a thin aluminium ring. A paper scale, with the points of the compass printed on it, is gummed to the ring. A small metal disc, with a sapphire cup at its centre, is attached by silk threads to the same ring. The sapphire cup rests on an inverted needle-point supported from the base of a compass-bowl.

In more recent times the above type of compass has been found to have insufficient steadiness under vibrations due to

heavy gun fire ; and Chetwynd's *liquid compass* (Fig. 279) is now generally used on ships of war. The compass-bowl is closed at the top and bottom by glass plates, forming a cham-

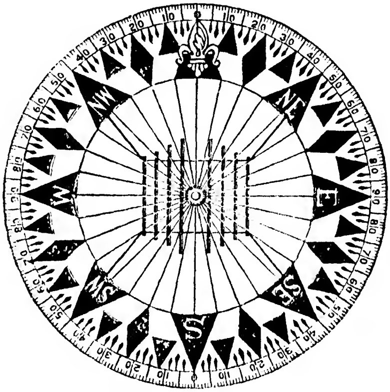


FIG. 278.—The Kelvin compass.

ber which is completely filled with a mixture of water and alcohol. The use of glass enables the card to be illuminated from beneath by a light fixed in the binnacle. The *card* is of thin sheet mica with the scale printed on its under surface by a

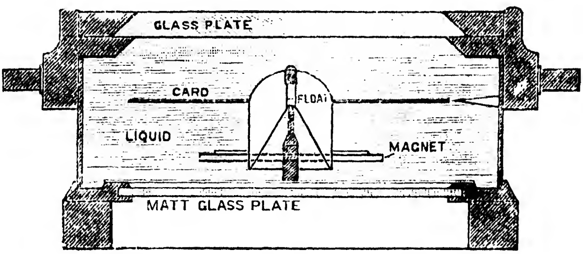


FIG. 279.—Chetwynd's liquid compass.

photographic process ; and the scale is protected by a coating of white paint. A hollow hermetically sealed float carries the card and also a pair of short cylindrical magnets. The size of the float is adjusted so that the total weight, when immersed in the liquid, is very small. A metal pointer, with its point

close to the edge of the card, is fixed horizontally inside the bowl, and with its length parallel to the keel of the vessel.

An Astatic pair.—It is sometimes desirable to use a magnetic needle which when suspended is unacted upon by the earth's field. Of the several available forms, Nobili's **astatic pair** (Fig. 280) is the most useful in practice; it consists of two magnetised needles, as identical as possible in dimensions and in degrees of magnetisation, fixed rigidly together as shown. When freely suspended in the earth's field the forces acting on the lower needle are neutralised by those acting on the upper needle. In practice it is almost impossible to obtain two magnets absolutely identical in every respect, but it is easy to obtain an arrangement which is sufficiently astatic for experimental purposes.

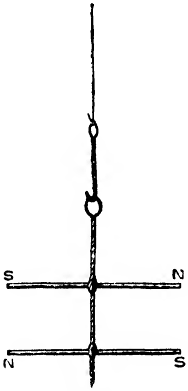


FIG. 280.—An astatic pair.

EXERCISES ON CHAPTER XXXII.

1. A strip of steel is bent about the middle point, so that the halves are inclined to each other at a right angle. It is then magnetised so that its extremities are south poles and the angular point a north pole, and is placed on a flat piece of cork floating in a basin of water. How will it set ?

2. A horse-shoe magnet lies flat on a sheet of brass which is supported by strings in such a way that it turns about a vertical axis, but always remains horizontal. How will it place itself ?

3. A bar-magnet is laid on a table perpendicularly to the magnetic meridian, and so as to point to the centre of a compass-needle. Describe and explain the behaviour of the needle.

4. A large soft iron rod lies on a table in the magnetic meridian, and a dipping needle is placed at some distance and at about the same level (i) due south, (ii) due north of it. How will the magnitude of the angle of dip be affected in each case ? (Neglect any inductive action between the needle and the bar.)

5. A tall iron mast is situated a little in front of the compass in a wooden ship. Explain the nature of the compass error when the ship is sailing in an easterly direction (i) in the northern, (ii) in the southern hemisphere.

6. A rod of iron when brought near to a compass-needle attracts one pole and repels the other. How will you ascertain whether its magnetism is permanent or is due to temporary induction from the earth ?

7. A bar of soft iron is held vertically over the centre of a dip-needle, but not near enough to have magnetism induced in it by the needle. Is the dip increased or diminished by the presence of the bar, and would the result be the same in each of the two hemispheres ?

8. A bar of soft iron lies on a table at right angles to the magnetic meridian, and a compass-needle is placed at some distance from the bar with its centre on the axis of the bar produced. The end of the bar nearest to the needle being kept in the same position, the bar is then turned round, upon the table, until it is parallel to the magnetic meridian, the fixed end of the bar being to the south. Describe the behaviour of the compass (i) before, (ii) during the rotation of the bar.

9. What is meant by magnetic dip ? Explain how you would proceed to determine this at any place. What is approximately the dip at Greenwich at the present time ?

10. How would you hold a rod of soft iron so that the influence of the earth's magnetic field upon it may be (i) as great as possible, (ii) as small as possible ? Give reasons for your answer.

11. Define *magnetic meridian.* How would you determine it *either* (*a*) with a bar magnet, *or* (*b*) with a dip-needle ?
Having found the magnetic meridian, how would you find the declination ? (Lond. Matric.)

12. Describe the construction of a dip-needle, and the method of its use for determining (*a*) the magnetic meridian, (*b*) the dip, at any given place. (Lond. Matric.)

13. Define *dip* and *declination.*
Describe and explain the behaviour of a dip-needle as the base supporting it is gradually turned through 360° in a horizontal plane. (Lond. Matric.)

CHAPTER XXXIII.

MAGNETIC MEASUREMENTS.

The unit of pole-strength.—All measurements in magnetic phenomena are based upon the system of **absolute units** proposed by C. F. Gauss (1777-1855) of Göttingen. This system has for its fundamental units the *centimetre*, the *gram* and the *second*: for this reason it is referred to universally as the **C.G.S. system.**

Gauss defined the **unit of force** as 'that force which gives unit velocity to unit mass in unit time.' In the C.G.S. system this unit force is termed the **dyne**: it is equal approximately to the weight at the earth's surface of 1 milligram (see p. 102).

Similarly, Gauss defined the **unit of magnetic pole-strength** as **that which exerts a force of one dyne on an equal pole situated at a distance of one centimetre.** When the pole-strengths are m_1 units and m_2 units respectively, the force will be $(m_1 \times m_2)$ dynes; and, by the law of inverse squares, when the distance apart is d cm., the force is expressed by the equation

$$f = m_1 m_2 / d^2. \quad \text{(Fig. 281.)}$$

This equation assumes that air, or some other non-magnetic substance, is the medium in which the poles are situated.

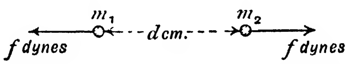


FIG. 281.—The repulsion between two similar magnet-poles.

If three long thin magnets* are available, it is possible to measure approximately the pole-strength of each of them by

* Magnets having exceptionally high pole-strength can be made from "Cobalt-crom" steel—a steel alloy containing 15 per cent. cobalt. The magnetisation must be carried out by means of a spiral traversed by a *strong* electric current.

a direct application of the above formula. The magnets having been given distinctive marks, such as A, B and C, Fig. 282 suggests how the force of attraction between opposite poles of magnet A and of magnet B may be measured by means of a simple balance. With B removed to a distance, A is weighed as accurately as possible ; B is then placed with its opposite pole vertically below the pole of A, and A is again weighed. The apparent increase in the weight of A measures

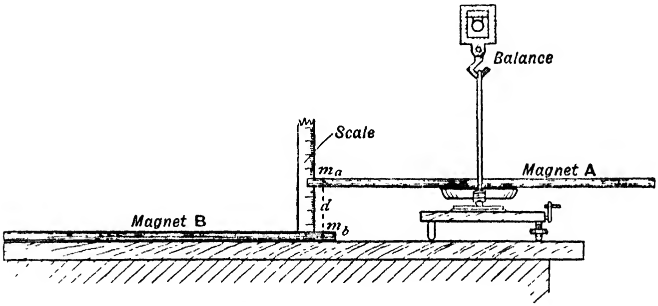


FIG. 282 —Measurement of the attraction between two opposite magnet-poles

the force of attraction between A and B : this increase in weight must be multiplied by 981, in order to convert it into *dynes* of force. The distance apart of the poles is measured by a millimetre scale supported vertically behind the poles. From this result, the product $m_a m_b$ is calculated. Similar experiments give the products $m_a m_c$ and $m_b m_c$. From these the individual pole-strengths can be calculated.

As the force of attraction, and the distance of the poles apart, are both small, the liable errors of a rough experiment are considerable ; and the results obtained may differ by as much as 10% from those obtained by direct methods, such as Expts. 249 and 250 (pp. 411-3) ; but the experiment here described is useful in giving a practical interpretation of the definition of the unit magnet-pole.

EXPT. 248.—**Measurement of the pole-strength of magnets (direct method).** Prepare three long magnets (50 cm. × 0·4 – 0·5 cm. diameter), and mark them A, B and C. The rods may be magnetised by inserting them inside a long solenoid (60 cm. long) consisting of two layers of copper wire wound on a narrow glass tube. Find, by the method of Fig. 256 (p. 382), the positions of the poles, and mark these clearly on the steel. Fix magnet A horizontally on

the pan of the balance, and weigh it accurately. Raise the beam of the balance, and place under the scale-pan two or more thin sheets of glass, adjusting this until, when the pan is resting on the glass, the pointer is at zero. Place *vertically* under the pole m_a the opposite pole m_b of magnet B, which is lying horizontally on the bench ; and fix magnet B in position by means of a lump of plasticene. Use a boxwood millimetre scale for measuring the distance between the poles. Add weights gradually to the other pan of the balance until the added weight is just sufficient to overcome the attraction between the poles. The following data are typical of an experiment :

(i) **Magnets A and B.**

Weight of magnet A 71·461
Weight of magnet A
(attracted by B) 71·800. Distance $(d) = 5·5$ cm.
Force = 0·339 gm.

Hence $m_a m_b = (0·339 \times 981) \times (5·5)^2 = 10{,}060$.

(ii) **Magnets A and C.**

Weight of magnet A 71·461
Weight of magnet A
(attracted by C) 71·867. Distance $(d) = 5·42$ cm.
Force = 0·406 gm.

Hence $m_a m_c = (0·406 \times 981) \times (5·42)^2 = 11{,}700$.

(iii) **Magnets B and C.**

Weight of magnet B 68·010
Weight of magnet B
(attracted by C) 68·358. Distance $(d) = 5·92$ cm.
Force = 0·348 gm.

Hence $m_b m_c = (0·348 \times 981) \times (5·92)^2 = 11{,}960$.

From (ii) and (iii) $m_a m_c / m_b m_c = m_a / m_b = 11700/11960 = 0·978$.
And $m_a m_b \times m_a / m_b = 10{,}060 \times 0·978 = 9838$,
or $(m_a)^2 = 9838$,
or $m_a = 99·2$ units.
Similarly, $m_b = m_a m_b / m_a = 10060/99·2 = 101·4$ units.
and $m_c = m_a m_c / m_a = 11700/99·2 = 118·0$ units.

It is interesting to compare these results with those obtained by either of the following methods :—(*a*) Determine the *moment* of the same magnet (by Expts. 249 and 250), and divide the moment by the *magnetic length*.

(*b*) Clamp the magnet vertically with its N-seeking pole touching the bench, and by means of a compass-needle find the position of the *neutral-point* (see Fig. 260B, p. 387). If its distance from the magnet pole is d cm., the intensity of the magnet's field at the neutral point is m/d^2; and this must be equal and opposite to the horizontal intensity (H) of the earth's field. Hence $m/d^2 = \mathrm{H}$. The method of obtaining an approximate value of H, by means of Expts. 249 and 250, is explained on p. 414.

Intensity of a magnetic field.—The intensity of a magnetic field is expressed numerically as the force (in dynes) with which it acts on a unit pole placed in the field. Hence, **a magnetic field has unit intensity when the force with which it acts on a unit pole placed in the field is equal to one dyne.** This unit of field intensity is termed **the gauss.**

The intensity of a magnetic field is expressed graphically by the number of lines of force supposed to pass through unit area of a section of the field drawn at right angles to the direction of the magnetic lines of force. Thus *the unit magnetic field* would be represented by *one line of force per square centimetre* (Fig. 283). Similarly, a field having an intensity of 25 units would be represented by 25 lines of force passing through each sq. cm.

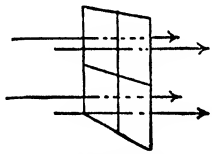


FIG. 283.—Magnetic field, of unit intensity.

It follows, from the definition of field intensity, that when a pole of strength m units is placed in a magnetic field of intensity H units, the force f acting on the pole is expressed by the equation

$$f = m \times \mathrm{H} \text{ dynes.}$$

The intensity of the earth's magnetic field.—It has been explained in the previous chapter that, at Greenwich, the angle of *dip* is 67° approximately; hence, in that district, the lines of force of the earth's field are inclined downwards at an angle of 67° to the horizontal (Fig. 284). A magnetic force, like any mechanical force, can be resolved into two *components*: thus, the magnetic force of the earth's field can be resolved into a *horizontal component* OH and a *vertical component* OV.

By a method which will be described in a later section, the *horizontal component* can be measured accurately; and the angle of *dip* can also be measured accurately. Hence, since

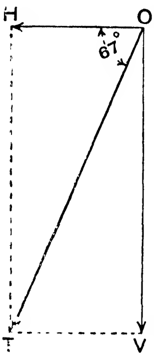


FIG 284.—The intensity of the earth's field resolved into two components

$$OH/OT = \cos 67°, \quad \text{and} \quad OV/OH = \tan 67°,$$

both the total intensity OT and the vertical intensity OV can be calculated. At Greenwich, in 1924, the horizontal intensity of the earth's field was about 0·184 dyne; hence, the total intensity was $0{\cdot}184/\cos 67° = 0{\cdot}471$ dyne, and the vertical intensity was $0{\cdot}184 \times \tan 67° = 0{\cdot}434$. It is important to remember that, as the latitude increases, the horizontal intensity becomes less, and becomes zero at the north magnetic pole. Similarly, at any locality on the magnetic meridian, the horizontal and the total intensities are identical, and the vertical intensity is zero.

Examples of magnetic fields of high intensity are found in dynamos and motors: in these, the intensity of the field in which the armatures rotate may amount to 15,000 gauss or more.

The deflection method of comparing magnetic forces.—This method has been used previously (Expt. 236) in order to verify the Law of Inverse Squares. In Fig. 285, a suspended magnetised needle *ns* is deflected through an angle θ by a distant bar-magnet placed to the west of the needle and with its axis pointing towards the point of suspension O. If m' is the needle's pole-strength, H the horizontal intensity of the earth's field, and F the intensity of the field due to the distant bar-magnet, then the two forces acting on each pole of the needle are m'H and m'F; and the needle comes to rest in such a position that the *moments* of these two forces round O are equal and opposite.

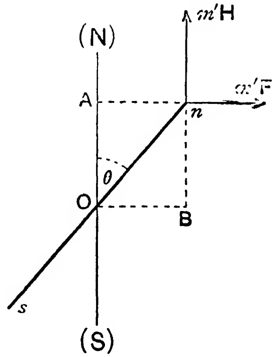


FIG. 285.—A suspended magnet, acted upon by two magnetic fields.

Hence, moment of $m'F$ = moment of $m'H$,

or

$$m'F \times OA = m'H \times OB$$
$$= m'H \times An.$$

Therefore $F = H \times An/OA = H \times \tan\theta$.

Thus the intensities of the different magnetic fields (due to bar-magnets placed appropriately) may be compared by allowing them, consecutively, to act upon a suspended-needle, and observing the deflection due to each field. **The intensities are proportional to the tangents of the angles of deflection.**

The moment of a magnet.—Imagine a bar-magnet, of pole-strength m and magnetic length $2l$, suspended in a uniform

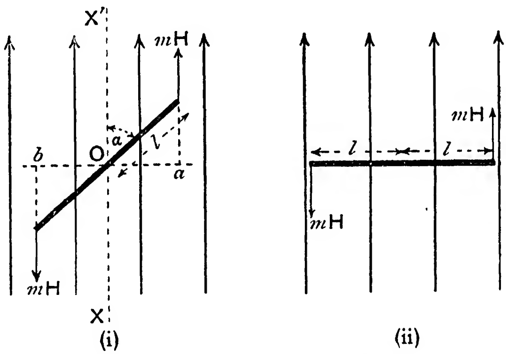


FIG. 286.—The 'moment' of a magnet.

magnetic field, the horizontal intensity of which is H, and the direction of which is XX′ (Fig. 286 i). Each pole is acted upon by a force mH; and the sum of the moments of these two forces, tending to pull the magnet back into a direction parallel to XX′, is

$$(mH \times Oa) + (mH \times Ob) = 2mH \times Oa$$
$$= 2mH \times l \sin\alpha.$$

If the deflection is increased until $\alpha = 90°$ (Fig. 286 ii), the sum of the moments is

$$(mH \times l) + (mH \times l) = mH \times 2l.$$

Finally, when $H = 1$, the sum of the moments is equal to $2lm$. This quantity is termed the **moment of the magnet,** and

is usually denoted by the symbol M. It may be defined as **the product of the pole-strength and the magnetic length of the magnet.**

The intensity of field, due to a bar-magnet.—(i) *At a point on the axis of the bar-magnet produced.* In order to obtain a formula for calculating the intensity of field due to a bar-magnet, at a point on its axis produced, let NS (Fig. 287) be a

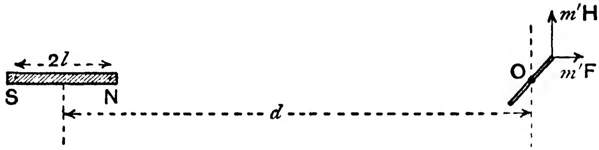


FIG. 287.—A suspended magnet, deflected by a distant bar-magnet.

bar-magnet of pole-strength m and magnetic length $2l$. Let O be a point at a distance d cm. from the centre of the magnet.

Imagine a single N-seeking pole, *of unit strength*, to be placed at O. The force acting upon it, and due to the pole N, will be $m/(d-l)^2$; and the force due to the pole S will be $-m/(d+l)^2$. Hence, the resultant force is

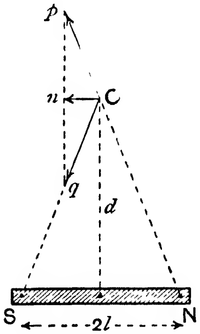


FIG. 288.—The intensity of the field at O, due to the magnet.

$$\frac{m}{(d-l)^2} - \frac{m}{(d+l)^2} = \frac{m \cdot 4ld}{(d^2-l^2)^2}$$
$$= \frac{2Md}{(d^2-l^2)^2}.$$

This is the **intensity** of the field at O, due to the magnet NS; and it is represented by the symbol F in Fig. 287. (When l is very small, relatively to d, the intensity may be written $2M/d^3$.)

(ii) *At a point on the line drawn through the centre of the magnet, and at right angles to the axis.* Imagine a single N-seeking pole *of unit strength* to be placed at O (Fig. 288). The force acting on it due to N will be m/ON^2; this may be represented to scale by the line Op. The force due to S will be m/OS^2, and represented by the line Oq. The resultant of these two forces will be 2On.

It is a useful geometrical exercise for the student, to prove that this resultant is equal to

$$\frac{M}{(d^2+l^2)^{\frac{3}{2}}}.$$

This is the **intensity** of the field at O, due to the magnet NS; and when l is very small, relatively to d, this intensity of field may be written M/d^3.

It will be noticed that, with a *short* magnet in position (i), called the *end-on position,* the intensity at a point is *twice* as great as it is at an equally distant point in position (ii), called the *broadside-on position.* It can be proved experimentally that the intensities at such a pair of points are in the ratio of 2 to 1. And, as the formulae have been derived *on the assumption that the Law of Inverse Squares is true,* the verification of the above simple ratio is a rigorous proof of the truth of the Law.

EXPT. 249.—**Comparison of the moments of two bar-magnets (deflection method.)** Mark the two magnets with the letters A and B. Find, by the method of Fig. 256 (p. 382), the positions of the poles of the magnets, and measure their *magnetic lengths.* Adjust a magnetometer so that its arms are perpendicular to the magnetic meridian, and place the bar-magnet A on the scale to the East of the suspended needle, with its N-seeking pole towards the needle, and with its *centre* at a distance of 20 cm. from the needle. Note the scale-readings of *both* ends of the pointer. Reverse the magnet, pole for pole, and again note the scale-readings of the pointer. Transfer the magnet to the other arm of the magnetometer and, with the centre of the magnet at the same distance from the needle, repeat all the above observations. Thus, *eight* readings of the pointer are obtained, and the mean of these is taken as the angle of deflection (θ). In the deflected position of the needle,

$$m'F = m'H \times \tan\theta, \quad \text{(See Fig. 285, p. 408.)}$$

or

$$m' \times \frac{2M_A d}{(d^2-l^2)^2} = m'H \times \tan\theta,$$

or

$$\frac{M_A}{H} = \frac{(d^2-l^2)^2}{2d}\,.\tan\theta.$$

Calculate the value of M_A/H. Remove the magnet A to a distance, and repeat all the above observations with the other magnet B: calculate the value of M_B/H.

Finally, find the value of the ratio

$$\frac{M_A}{H}\Big/\frac{M_B}{H}=M_A/M_B.$$

Enter your observations thus :

(i) **Magnet** 'A.' Magnetic length $(2l)=11\cdot3$ cm.

Distance.	Deflection of pointer.	Mean deflection (θ).	$\frac{(d^2-l^2)^2}{2d}\cdot\tan\theta$.
30 cm.	(*a*) $\begin{cases}26\cdot3\\24\cdot0\end{cases}$ (*b*) $\begin{cases}23\cdot8\\26\cdot5\end{cases}$ (*c*) $\begin{cases}23\cdot9\\26\cdot2\end{cases}$ (*d*) $\begin{cases}27\cdot0\\23\cdot5\end{cases}$	$25^\circ\cdot15$	$\frac{(\overline{30}^2-\overline{5\cdot65}^2)^2}{60}\cdot\tan 25^\circ\cdot15$ $=$**5897.**

(ii) **Magnet** 'B.' Magnetic length $(2l)=11\cdot2$ cm.

Distance.	Deflection of pointer.	Mean deflection (θ)	$\frac{(d^2-l^2)^2}{2d}\cdot\tan\theta$.
30 cm.	(*a*) $\begin{cases}30\cdot5\\27\cdot0\end{cases}$ (*b*) $\begin{cases}26\cdot4\\28\cdot6\end{cases}$ (*c*) $\begin{cases}26\cdot0\\28\cdot0\end{cases}$ (*d*) $\begin{cases}31\cdot2\\27\cdot8\end{cases}$	$28^\circ\cdot19$	$\frac{(\overline{30}^2-\overline{5\cdot6}^2)^2}{60}\cdot\tan 28^\circ\cdot19$ $=$**6740.**

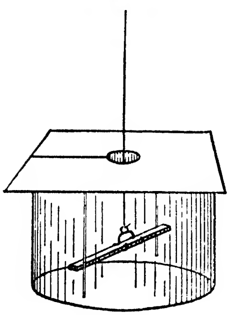


FIG. 289.

The ratio $M_A/M_B=\frac{M_A}{H}\Big/\frac{M_B}{H}=\frac{5897}{6740}=\frac{0\cdot875}{1}.$

The rate of vibration of a suspended magnet.—The mode of vibration of a suspended magnet (Fig. 289) is quite analogous to that of a simple pendulum. In the case of a pendulum, the time of one complete vibration can be calculated from the equation $t=2\pi\sqrt{l/g}$, where l is the length of the pendulum, and g is the acceleration due to gravity.

The time of vibration of a suspended magnet is calculated from the equation

$$t=2\pi\sqrt{\frac{I}{MH}},$$

where H is the horizontal intensity of the field in which the magnet is suspended, M is the ' moment ' of the magnet, and I is its ' moment of inertia ' : this latter can be calculated from the weight and dimensions of the magnet.*

EXPT. 250.—**Comparison of the moments of two magnets (vibration method).** In order to screen the magnet from draughts, it is suspended inside a glass dish with vertical sides. A sheet of thick paper, with a central hole and a radial slit, is placed over the top of the dish after the magnet has been suspended and brought to rest. The suspension consists of a bundle of *unspun* silk fibres. Two pieces of thin white cotton are fixed vertically on the outside of the dish, and at opposite ends of a diameter : similarly a piece of cotton is fixed vertically down the middle of each end-face of the magnets.

Mark the two magnets (A and B), and suspend the magnet A. Bring it finally to rest (by means of an external magnet temporarily held in the hand), and adjust the glass dish so that all the cotton threads are in the same straight line when viewed with one eye. Set the magnet swinging through a *small* angle and, with a stop-watch, determine the time required for 50 complete vibrations. Calculate the time (t_1) required for 1 vibration. Remove the magnet, find its weight and dimensions, and calculate its moment of inertia (I_A).

Then calculate M_AH, from the equation

$$M_AH=4\pi^2I_A/t_1^2.$$

Repeat the experiment with the other magnet, marked B, and calculate the value of M_BH.

From these two equations, the ratio M_A/M_B is obtained, since

$$M_AH/M_BH=M_A/M_B.$$

* For a rectangular magnet, $I=\left(\frac{(\text{length})^2+(\text{breadth})^2}{12}\right)\times\text{mass}$;

for a cylindrical magnet, $I=\left(\frac{(\text{length})^2}{12}+\frac{(\text{radius})^2}{4}\right)\times\text{mass}$.

Enter your observations thus :

(i) **Magnet 'A.'**

(*a*) *Moment of Inertia* : Dimensions : Length $=12\cdot70$ cm.
Width $=1\cdot437$ cm.
Weight $=87\cdot042$ gm.

$$I=\frac{(\text{length})^2+(\text{width})^2}{12}\times\text{weight}=\frac{(12\cdot70)^2+(1\cdot437)^2}{12}\times87\cdot042=\mathbf{1185.}$$

(*b*) *Time of vibration* :

62 complete vibrations in 17 m. 16·4 sec., or

$t=1036\cdot4/62=\mathbf{16\cdot71}$ sec.

(*c*)
$$M_AH=\frac{4\pi^2I}{t^2}=\frac{4\pi^2\times1185}{(16\cdot71)^2}=\mathbf{167\cdot54.}$$

(ii) **Magnet 'B.'**

(*a*) *Moment of Inertia* : Dimensions : Length $=12\cdot64$ cm.
Width $=1\cdot389$ cm.
Weight $=85\cdot87$ gm.

$$I=\frac{(12\cdot64)^2+(1\cdot389)^2}{12}\times85\cdot87=\mathbf{1157.}$$

(*b*) *Time of vibration* :

68 complete vibrations in 17 m. 33·2 sec., or

$t=1053\cdot2/68=\mathbf{15\cdot49}$ sec.

(*c*)
$$M_BH=\frac{4\pi^2\times1157}{(15\cdot49)^2}=\mathbf{190\cdot37.}$$

(iii) **Ratio** M_A/M_B.

$$M_AH/M_BH=167\cdot54/190\cdot37=0\cdot875/1.$$

[*N.B.*—In Expts. 249 and 250, the same pair of magnets were used ; and it will be noticed that the two values of M_A/M_B are identical. In view of the fact that only simple apparatus was available, this identity of result is quite chance ; and results differing by 2 – 4% may be regarded as quite satisfactory.]

The horizontal intensity of the earth's field.—If the same magnets have been used in both Expt. 249 and in Expt. 250, the results obtained provide the data for calculating the earth's horizontal intensity (H), since the ratio

$$M_AH\bigg/\frac{M_A}{H}=H^2.$$

Unless elaborate apparatus is used the result must be regarded as only approximate ; but the experiments demon-

strate the principle of the method frequently used in magnetic observatories. The correct value in Great Britain does not differ widely from 0·184; but the value found in a room in which there are iron pillars or pipes may differ from this considerably; and it may have very different values in different parts of the same room.

The correct value can best be obtained by taking the measurements in a brick or wooden building, in which all metal work, such as pipes and screws, are of copper or brass, and erected well away from a town, so as to avoid underground waterpipes, electric tram routes, etc.

Using, for the calculation of H, the data obtained in the above experiments, we have

(i) **Magnet** 'A':

$$M_A H \Big/ \frac{M_A}{H} = H^2 = \frac{167 \cdot 54}{5897} = 0 \cdot 0283 \text{ approximately};$$

or $$H = \sqrt{0 \cdot 0283} = 0 \cdot 168.$$

(ii) **Magnet** 'B':

$$M_B H \Big/ \frac{M_B}{H} = H^2 = \frac{190 \cdot 37}{6740} = 0 \cdot 2824 \text{ approximately},$$

or $$H = \sqrt{0 \cdot 2824} = 0 \cdot 168.$$

Comparison of the horizontal intensity in different parts of the same room.—From the equation $t = 2\pi\sqrt{I/MH}$, it is evident that the time of vibration of a suspended magnet is inversely proportional to $\sqrt{H}$. The simplest experimental method of using this fact is to measure the time occupied by a suspended magnet in describing an observed number (say, 50) of complete vibrations, and calculating from this result the number (n) which it would describe in exactly one minute; the measurements are taken in each of the selected positions in the room. The number of vibrations n, of course, is inversely proportional to the time occupied by one vibration,

or $$n \propto 1/t.$$

But $$t \propto 1/\sqrt{H}$$

hence, $$n \propto \sqrt{H},$$

or $$H \propto n^2.$$

If H_1 and H_2 are the horizontal intensities at the two selected positions, and n_1 and n_2 are the calculated numbers of vibrations described in one minute, then

$$H_1/H_2 = n_1^2/n_2^2.$$

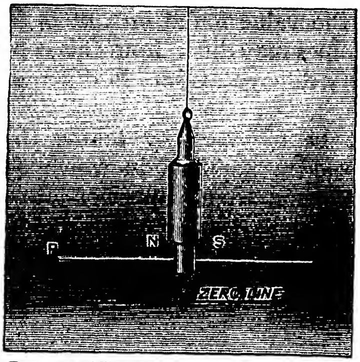


FIG. 290.—Vibration magnetometer (Searle's).

For the experiment, a very short magnet (about 1 cm. long) made from a piece of knitting-needle or a piece of watch-spring is convenient; and to make the rate of vibration slow, the magnet is rigidly fixed to a piece of non-magnetic metal, such as zinc rod. A pointer, made from drawn-out glass, attached to the needle assists in taking observations accurately. The arrangement is suspended by a single silk fibre; and, to protect it from draughts, hangs inside a beaker. An efficient apparatus, shown in Fig. 290, consists of a heavy brass cylinder carrying the magnet NS and a pointer P. For taking the observations, a pencil-line is drawn on a sheet of paper supported below the apparatus, and the paper is adjusted so that the line is vertically under the pointer in its position of rest.

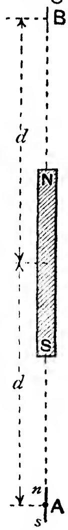


FIG. 291.

The magnetic intensity at different points in the field of a bar-magnet.—In order to investigate the intensities at different points of the magnetic field due to a bar-magnet, it is necessary to eliminate the effect of the earth's field on the rate of vibration of a suspended magnetised needle. Suppose that it is desired to obtain a measure of the intensity of the field of a bar-magnet at a point A (Fig. 291) on the axis produced, and at a distance d cm. from its centre. The method consists in determining how many vibrations are described in one minute when the suspended needle is influenced by the earth's field only: if the number of vibrations is n, then H is proportional to n^2. The magnet is

then placed in position with its axis in the meridian and pointing directly towards the point A. As the S-seeking pole is pointing towards the South, both fields are acting in conjunction, and the *resultant intensity* at A may be expressed as (F + H), where F is the intensity due to the magnet alone. Suppose that the suspended needle now makes n_1 vibrations in one minute; then (F + H) is proportional to n_1^2. Hence

$$\frac{F+H}{H}=\frac{n_1^2}{n^2},$$

or

$$F/H=(n_1^2-n^2)/n^2.$$

If the value of H is known, then F can be calculated.

When the bar-magnet is reversed, pole for pole, the two fields will be in opposition to each other, and the resultant intensities will be (F − H). If the needle now makes n_2 vibrations in one minute, then (F − H) is proportional to n_2^2. Hence

$$\frac{F-H}{H}=\frac{n_2^2}{n^2},$$

or

$$F/H=(n_2^2+n^2)/n^2.$$

If the bar-magnet NS (Fig. 271) is uniformly magnetised, and its two poles have equal strength, the rate of vibration of the needle will be the same at point B as at point A. This would serve as a means of demonstrating experimentally that the two poles of the bar-magnet have equal strength.

EXERCISES ON CHAPTER XXXIII.

1. Explain the meanings of the numbers in the following statements:

(*a*) The magnetic moment of a magnet is 100.
(*b*) The strength of a magnetic field is 0·18.

What couple would be required to hold the magnet of (*a*) at an angle of 60° with the field of (*b*)? (Lond. Matric.)

2. What quantities must be known in order to determine completely the earth's magnetic field in any place? Describe in general terms how the readings of the dip-circle would vary as you travelled from England to the S. magnetic Pole. (Camb. S.C.)

3. State what is meant by saying that in London the horizontal component of the earth's field is 0·18 gauss (or dyne per unit pole). Explain briefly the system of units used.

(Lond. Matric.)

4. An unmagnetised steel needle in this country is pivoted so that it rests in a horizontal position. It is then magnetised and will no longer rest horizontally. How must it be loaded to be level again. Suggest a method of finding the magnetic moment of the needle by measurement of this load. What other quantities will be needed ? (Lond. Matric.)

5. Given the strength of the horizontal component of the earth's magnetic field, how would you determine experimentally the strength of the magnetic field due to a bar-magnet at a point on its axis ? (Camb. S.C.)

6. The pole strength of a magnet is 10 C.G.S. units, and the moment of the magnet is 200 units. Find the strength of the magnetic field due to the magnet at a point on the axis produced, 60 cm. from the middle point. (Camb. S.C.)

7. (i) A long cylindrical magnet is fixed in a vertical position with its N-seeking pole touching the table. It is found, by means of a sensitive compass-needle, that the 'neutral-point' is 15 cm. distant from the pole, when measured along the surface of the table. The horizontal intensity of the earth's field at that point is 0·17 gauss. Calculate the pole-strength of the magnet.

(ii) Suppose that the compass-needle is moved into a position below the table, but in the same vertical plane as before, and situated somewhere along the line passing through the pole and inclined 30° to the horizontal. What is now the distance of the 'neutral-point' from the pole ?

8. Define *unit magnetic pole.*

If the pole strength of a bar-magnet is 30 units and the distance between its poles 12 cm., calculate the magnetic force at a point on the axis distant 18 cm. from the nearer pole.

(Lond. Gen. Sch.)

9. Explain in detail how you would use a deflection magnetometer to compare the field due to a magnet with that of the earth.

Two magnets, each 10 cm. long, are placed end-on to a compass needle, and there is no deflection when the near poles are 15 cm. East and 20 cm. West of the needle respectively. Compare the moments of the magnets. (Camb. S.C.)

10. Explain what is meant by the strength of a magnetic field.

How would you compare experimentally the fields produced by a bar-magnet at equal distances from its centre (*a*) at a point on the axis, (*b*) in a direction through the centre of the magnet at right angles to the axis ? What important deduction can be drawn from the results of this experiment ?

(Oxf. and Camb. S.C.)

11. What is meant by the statements (*a*) that the strength of a magnet pole is *m units*, (*b*) that the strength of a magnetic field is H *units* ?

The distance between the poles of a bar-magnet is 15 cm., and the strength of each pole is 100 units. Find in magnitude and direction the field due to the magnet at a point distant 15 cm. from each pole. (Lond. Matric.)

12. Explain the term *moment of a magnet.*

A small bar-magnet oscillates 10 times per minute at a certain point in the earth's field. It then comes into contact with another magnet and afterwards makes only 7 vibrations per minute at the same place. Compare the final value of its magnetic moment with the original one. (Lond. Gen Sch.)

13. A small magnet oscillates in the earth's field (0·18 dyne per unit pole). A bar-magnet, placed end-on to it and East of it (*i.e.* at right angles to the earth's field), deflects it through 60°. What will be the strength of the resultant field ? If the rate of oscillation in the earth's field was 10 per minute, what will be the new rate of oscillation ? (Lond. Matric.)

14. How would you show that the two poles of a magnet are always equal and opposite ?

A bar-magnet is found to have a N-pole at each end. How is this explained, and how would you test the truth of the explanation ? (Lond. Matric.)

15. Describe experiments for comparing the intensity of two uniform magnetic fields (*a*) when they are quite separate, (*b*) when they are superimposed at right angles to each other. (Lond. Matric.)

16. Two small magnets A and B are alike in every respect, except that one is more strongly magnetised than the other. At one end of the laboratory A oscillates 10 times per min. and B 6 times per min. In another part A oscillates 6 times per min. Find how many oscillations per min. B would make, and the ratio of the moments of the two magnets. Describe apparatus which might be used to carry out this experiment. (Lond. Matric.)

17. Define *unit pole, strength of magnetic field.*

How could you determine the effect of interposing various thicknesses of sheet iron on the strength of the field at a point in the neighbourhood of a strong magnet ? (Lond. Matric.)

18. Explain the method of comparing the intensities of magnetic fields by observations of the times of oscillation of a magnetic needle.

A small magnet vibrating in a horizontal plane in the earth's field has a period of 5 seconds. When another magnet is brought near, the period is reduced to 4 seconds. Compare the strength

of the field due to the magnet with that of the earth, assuming that the two fields are (*a*) in the same direction, (*b*) in opposite directions. (Camb. S.C.)

19. Two magnetised needles are rigidly connected, first, with their opposite poles pointing in the same direction, as in an astatic galvanometer, and secondly, in the same manner but with like poles together. In each case the system is suspended by a fine silk thread, and the times of vibration in the earth's field are observed. These are found to be 8·9 and 2·8 sec. respectively. Compare the moments of the two magnets. (Lond. Matric.)

20. How would you magnetise a bar of steel so that it has three poles ?

How would you show experimentally that the sum of the pole-strengths of the two like poles is equal to that of the unlike pole ? (Camb. S.C.)

21. Describe a method for finding the distribution of magnetism on a bar-magnet. (Camb. Junior.)

22. A small suspended magnet is placed at a point A on the axis of a bar-magnet set in the magnetic meridian, and its time of swing is found to be 1·5 sec. In the earth's field alone the time of swing is 3 sec. If the strength of the horizontal component of the earth's field is 0·2 dyne per unit pole, find the strength of the field at A, produced by the bar-magnet. Find also the time period of the suspended magnet at A, when the bar-magnet is turned end for end. (Bristol S.C.)

23. Why is an ordinary compass of but little use in polar expeditions ?

Assuming that the horizontal intensity of the earth's magnetism is 0·197 C.G.S. unit at Paris and 0·368 C.G.S. unit at Bombay; what would be the ratio of the times of vibration of the same vibration magnetometer at these two places ? (Camb. S.C.)

24. In a two-storied building the upper storey is supported by vertical iron pillars, and it is found that the magnetic field at a certain point on the upper floor is very much weaker than at the corresponding point on the ground floor. Describe the situation of this point relatively to one of the iron pillars, giving reasons for your answer, and indicate a method of making the necessary observations. (Lond. Matric.)

PART VII.

STATIC ELECTRICITY.

CHAPTER XXXIV.

ELECTRIFICATION AND THE ELECTRIC FIELD.

ELECTRIFICATION.

Electrification by friction.—The ancient Greeks observed that when amber was rubbed with wool it acquired the property of attracting light objects. This is mentioned in the writings of Thales of Miletus (B.C. 600). Our word electricity is derived from the Greek word for amber (ἤλεκτρον). Until the year 1600 A.D. it was thought that amber was the only substance capable of exhibiting these phenomena, but in that year Dr. Gilbert found that many other substances were capable of affording similar results, *e.g.* resin, sulphur, glass, etc., and these substances he called **electrics**. When a substance is rubbed with a suitable material, and is then found to possess the property of attracting light objects, it is said to be **electrified** (or to possess a charge of electricity).

In order to produce these effects of attraction actual force is required, and this force can only be due to some peculiar condition which the substance has acquired when electrified. Such forces are called electric forces. The space around the substance, extending as far as the forces are evident, is called the **electric field.**

EXPT. 251.—**Attraction of light objects.** Rub a rod of vulcanite on the coat-sleeve. Notice that the rod has acquired the peculiar

property of picking up small fragments of paper, cork, or cotton fibre when brought near to them. Also notice that actual contact is not necessary, but that the effects take place when the rod is still some distance away.

EXPT. 252.—**Attraction of balanced lath.** Balance a long wooden lath (*e.g.* a metre scale) on an inverted round-bottomed flask. Bring a piece of vulcanite which has been rubbed as in Expt. 251 near the end of the lath, and notice the attraction which takes place.

The forces of electric attraction are mutual, just in the same way that the forces of magnetic attraction between a magnet and a piece of soft iron are mutual.

EXPT. 253.—**Mutual attraction.** Rub a piece of well-dried flannel* (or brown paper) with a clothes-brush, and notice how it will cling to the walls of the room.

Two kinds of electrification.—Whenever a substance is electrified by friction, a mutual force of attraction is set up between it and unelectrified bodies. An electrified body may, however, attract or repel another electrified body. Thus, a rubbed rod of vulcanite will repel another rubbed rod of the same kind, and a rubbed rod of glass will repel another glass rod which has been rubbed with the same material, but will attract a rubbed rod of vulcanite.

EXPT. 254.—**Electrified vulcanite rods.** Suspend an electrified rod of vulcanite, and bring near one end of it another rod of vulcanite which has been similarly electrified. Notice the **repulsion** which takes place.

EXPT. 255.—**Electrified glass rods.** Repeat Expt. 254, using, instead of the vulcanite, glass rods which have been dried in the oven and rubbed with silk. Notice the **repulsion.**

EXPT. 256.—**Electrified vulcanite and glass.** Suspend an electrified rod of vulcanite. Bring near to it a rod of glass which has been rubbed with silk. Notice the **attraction.**

* **A Simple Form of Drying Oven.**—It is often expedient to dry artificially the appliances used in experiments on Static Electricity. A portable drying oven may be constructed in the following manner. Fill a shallow baking-tin (about 40 cm. × 20 cm.) with sand, and cover it with a sheet of thin iron (about 40 cm. × 35 cm.) bent into the form of a semicircle, so as to form a hood over the sand-bath. The bath is supported on tripods, and heated by Bunsen-burners placed underneath. Glass rods may be placed in the sand, and paper, flannel, silk, etc., may be spread over the hood.

The terms **vitreous** and **resinous** electricity were formerly used to express the two different kinds of electrification produced by rubbing glass and vulcanite or sealing wax respectively. It was found, however, that the kind of electrification depends upon the substance used as a rubber; for instance, glass when rubbed with fur becomes charged with resinous electricity. For this reason, the terms **positive** and **negative**, which were first suggested by Benjamin Franklin in 1747, are now adopted. Using this nomenclature, the results of experiments show that:

(i) Glass rubbed with silk is charged **positively**.

(ii) Vulcanite (or resin) rubbed with fur (or flannel) is charged **negatively**.

(iii) Bodies with **like** charges **repel**, and bodies with **unlike** charges **attract** each other.

(iv) A charged body always attracts an uncharged body.

In these experiments, rods or tubes of *lead-glass* will be found more efficient than when of soda-glass. Also, considerable care is required in using glass rods; for, without any evident reason, the glass will sometimes become negatively charged when rubbed with silk. If the glass rods have been dried in a sand-oven, the charge generated upon them is undoubtedly positive; but if the rods have been dried by passing them through a Bunsen flame, they become negatively charged when rubbed with silk. The cause of this is obscure. If, however, the rods are allowed to become quite cold, and then warmed again in the sand-oven, they recover their normal property of becoming positively charged.

Tubes of *fused silica* are now frequently used instead of glass. Apparently, this material is not affected by a damp atmosphere, and the preliminary warming of the tubes is seldom necessary.

Conductors and insulators.—Dr. Gilbert found that many substances, chiefly metals, did not show any signs of electrification when rubbed—these he called **non-electrics**. It is now known that this depends upon the manner in which the experiment is conducted.

EXPT. 257.—**Loss of charge.** Suspend an electrified rod of vulcanite. Bring near to it a second electrified rod of vulcanite. Notice the repulsion. Pass the latter rod gently * through the hand, taking care that all parts of the rod are touched by the hand, and again test. Attraction shows that the rod is no longer charged. Again electrify the rod, and afterwards pass it through the flame of a Bunsen burner. Attraction shows that the rod has lost its charge in this case also.

Objects like the hand and a flame, which can take away the charge from an electrified body, are called **conductors.** Vulcanite is evidently not a conductor, since the charge on one portion of its surface is not conveyed to the end which is held in the hand of the experimenter. Vulcanite, and all substances which Dr. Gilbert called electrics, are now termed **insulators.** If metals are conductors of electricity, then it is seen readily why Dr. Gilbert was unable to detect any electrification on the surface of a metal which had been rubbed ; any charge which the metal acquired would be conducted away immediately by the hand in which the metal is held. When, however, a piece of metal is held by means of some insulating material, so that any charge produced upon it cannot escape, electrification can be produced upon it by friction. Held in this way the metal is said to be **insulated.** By adopting similar precautions it can be proved that almost all substances become electrified when rubbed with suitable material.

EXPT. 258.—**Electrification of a metal.** Fix a short brass or iron tube (or a square piece of sheet brass or zinc) on the end of a rod of vulcanite, or on the end of a piece of clean dry glass-tubing. Flick the metal with a piece of fur. Bring it near to a suspended rod of vulcanite which has been electrified. Notice the repulsion. The metal is evidently charged negatively.

Prolonged exposure to light and air causes vulcanite to lose much of its insulating power. This is due to the fact that the substance contains a high percentage of sulphur ; the exposure causes some of the sulphur to oxidise, thus forming a thin film of sulphuric acid on the surface ; and sulphuric acid is a

* If vulcanite is passed vigorously through the hand it is – ly electrified.

good conductor. When the defect becomes apparent, it can be remedied by wiping the surfaces with a duster moistened with dilute liquid ammonia, and afterwards drying the surfaces with a dry clean duster.

Electroscopes.—Any appliance which is devised so that, by means of it, it becomes possible to detect very weak electrical forces, and also small changes in the magnitude of such forces, is termed an **electroscope.**

The **pith-ball electroscope** shown in Fig. 292 consists of two pith-balls suspended by thin aluminium wire from a thick metal wire supported horizontally from the top of a vulcanite rod. When electrified the balls repel each other in the manner shown. If desired, a pith-ball may be gilded by moistening the surface with weak gum and, when nearly dry, rolling it in Dutch-metal leaf.

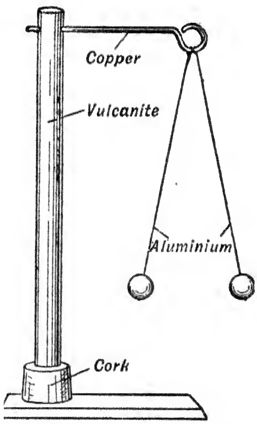


FIG. 292.—Pith-ball electroscope.

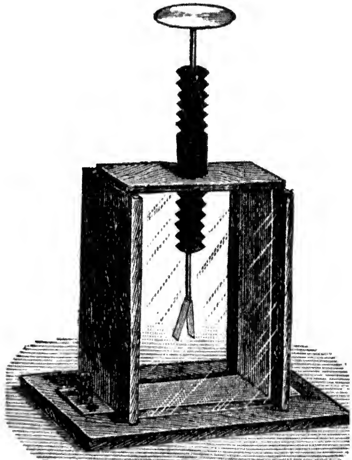


FIG. 293.—Gold-leaf electroscope.

A more usual form of instrument is the **gold-leaf electroscope** (Fig. 293) in which use is made of the fact that two similarly charged bodies repel one another. Two narrow strips of gold-leaf are suspended from the lower end of a stout wire at the top of which a metal disc is fixed. The wire is supported vertically by a plug of insulating material (*e.g.* ebonite or

sulphur), and the leaves are protected from air-currents by means of a case, the front and back of which are of glass. The sides of the case should be lined on the inside with strips of metal which are earth-connected. When a charge of electricity is given to the metal disc, the leaves will diverge, and the degree of divergence depends upon the magnitude of the charge.

Relative power of conductivity.—It has been seen that the hand, a flame, and metals are conductors, and that sealing-wax and glass are insulators. With the aid of an electroscope the conducting or insulating power of any substance can be determined roughly.

EXPT. 259.—**Tests of conduction.** Flick the metal disc of a gold-leaf electroscope with a small piece of fur, thus imparting to the leaves a negative charge. Touch the disc with the finger, and notice the *instantaneous* collapse of the leaves. Charge the electroscope again, and touch the disc with a strip of dry paper held in the hand. Notice the *gradual* collapse of the leaves. Repeat the observations, using dry glass, dry and wet threads (both cotton and silk), charcoal, wood, shellac, paraffin-wax, etc.

Experiments with various substances suggest the following classification :

Conductors—metals, the body, water, charcoal.

Partial conductors—paper, cotton, wood, stone.

Insulators—fused silica, glass, sealing-wax, shellac, vulcanite, silk, wool, sulphur, oils.

It is evident that when a conductor is required to retain a charge of electricity it is necessary to insulate the conductor on a support of silica, dry glass, sealing wax, or vulcanite, or to suspend it by silk threads.

Simultaneous production of both kinds of electrification.—When glass is rubbed with fur it acquires a negative charge of electricity. Does the fur acquire any charge in the process? and if so, is it positive or negative? To answer this question by experiment, it is necessary to insulate the fur by attaching a disc of cardboard to the end of a rod of vulcanite and cover-

ing the disc with a piece of fur about the same size. A small square of glass should be mounted on a similar handle. If these precautions are taken to prevent the loss of electricity it is found that **when electrification is developed by friction the two kinds are developed in equal quantity.**

EXPT. 260.—**Equality of opposite charges.** Holding glass and fur by insulating handles, rub them together. Keeping them in contact, bring them near to an uncharged pith-ball. No effect is seen. When the fur is removed, the glass alone attracts the pith-ball. The fur alone will also attract it. Evidently both the glass and the fur are charged; but since they have no effect when together, the charge on the fur must be equal and opposite to the negative charge on the glass. To verify that the fur is charged positively, bring it near to a pith-ball charged positively and notice the repulsion.

Theories of electrification.—When two bodies are rubbed together the electricity which may be generated cannot be of the nature of a substance (solid, liquid, or gaseous), for an electrified body weighs just the same when electrified as it does when unelectrified. The difference between these two conditions may be compared more satisfactorily to the difference between a clock-spring when wound up and when run down (*i.e.* when in a condition of strain and when free from strain), or to a piece of elastic thread when stretched and when unstretched (*i.e.* when in a state of tension and when free from tension),—the difference is simply one of physical condition. But the question still remains unanswered as to where the seat of the tension or strain exists, and we may not assume that this is confined necessarily within the limits of the electrified body.

Two theories were propounded many years ago which may be mentioned briefly here. Symmer suggested a **two-fluid theory**; according to it there are two electric fluids of opposite kind present in all substances, and the process of electrification involves the complete or partial withdrawal of one of them. At a later date Franklin suggested the more feasible **one-fluid theory**, according to which all unelectrified bodies contain a

normal amount of an electric fluid; the process of electrification involves either an increase or a diminution of the amount of the electric fluid present. In the former case the body was said to be charged **positively**, and in the latter case charged **negatively**. If the words 'positive' and 'negative' are interchanged, Franklin's theory is in close agreement with the remarkable results which have led to the modern **electron theory**.

Electrons.—Experiments in recent years have shown that the 'atom' of a substance consists of one or more small particles of 'negative' electricity rotating rapidly round a nucleus, or core, carrying a 'positive' charge of electricity—exactly like the planets moving in their orbits round the sun. These particles of negative electricity, called **electrons**, are vastly smaller than the atom itself, their diameter being perhaps 25,000 times less than the diameter of the atom: the atom itself is so small that a single row of fifty millions of hydrogen atoms would be only 1 centimetre long.

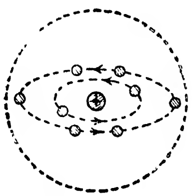


FIG. 294.—A MODEL OF AN OXYGEN ATOM. It consists of eight negative electrons rotating in circular orbits round a positively charged nucleus.

Atoms of different elements have different weights, and the weight of the atom of any element may be expressed numerically as the number of times that the atom is heavier than that of the lightest element known, viz. hydrogen. These numbers are called the **atomic weights** of the elements. All the elements may be tabulated in the order of their atomic weights, beginning with the lightest (hydrogen), and ending with the heaviest (uranium). The position of any element on this Table is called its **atomic number**; thus the 'atomic number' of hydrogen is 1, and that of uranium is 92. The remarkable discovery has been made that the number of electrons around the nucleus of the atom of an element is the same as its atomic number: for example, the element oxygen comes eighth on the list, its 'atomic number' therefore is 8, and its atom consists of eight electrons rotating round a nucleus which has a positive charge correspondingly great (Fig. 294).

Structure of atoms.—As the atomic number of hydrogen is 1, it is supposed that its atom consists of a single positive nucleus with one electron rotating round it (Fig. 295, i). This single positive nucleus of the hydrogen atom is sometimes called the *proton*; and, since the normal hydrogen atom is electrically neutral, it is assumed that the positive charge of a proton is exactly equal to the negative charge of an electron. Practically the whole weight of the atom is concentrated in the nucleus, because the weight of an electron has been proved to be only about 1/1845 of that of an atom of hydrogen. The gas *helium* comes next in the series of elements, and has an

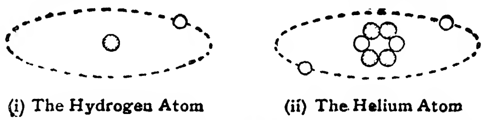


FIG. 295.—THE STRUCTURE OF THE ATOM. (i) The hydrogen atom consists of a 'positive' nucleus with a single electron rotating round it. (ii) The helium atom has a more complex nucleus, and two electrons rotate round it.

atomic number $=2$; it is assumed, therefore, to have two electrons rotating round the nucleus; and as its atomic weight is 4, its nucleus consists of four protons. But this would have an excess of positive charge in the atom; so, to give a neutral atom, it is assumed that the nucleus also contains two electrons (Fig. 295, ii). It would seem, therefore, that the structure of the atom becomes more and more complicated as the atomic weight becomes greater.

The same electrons do not always remain in the same atoms—some of them are interchangeable, and pass from one atom to another. When, at any instant, an atom is deprived of one or more of its electrons, the remainder of the atom has an excess of 'positive' electricity, and it is *positively charged*; but when the atom has one or more electrons above the normal number it has an excess of 'negative' electricity, and it is *negatively charged*. For some reason unknown, when glass touches silk, electrons pass from the glass to the silk, giving a negative charge to the silk and leaving the glass positively charged. We usually *rub* the glass with the silk: simple contact only is necessary, but the rubbing increases the effect because, thereby, the contact is more complete and extensive.

Electric Fields of Force.

Analogy of magnetic fields of force.—Experiments on Magnetism have shown that like poles repel and that unlike poles attract, that the space separating such poles is a field of force through which magnetic forces are acting in definite directions (called the lines of force), and that if we conceive these lines of force to have properties similar to those possessed by stretched elastic threads (viz. tending to contract lengthwise and to expand crosswise), it is possible to explain all the experimental phenomena observed. It has been seen, also, that bodies with like electric charges repel, and with unlike charges attract, one another. Moreover, these forces are transmitted through the intervening space in a similar manner to that observed in magnetic phenomena. These analogies suggest that an electrified body must be surrounded by an electric field, at all points of which it is capable of exerting electric force upon other bodies. If such a field of force exists, then the force at any point in it must act necessarily in a definite direction, which may be regarded as the direction of the electric line of force at that point. Consequently, **an electric field may be traversed by electric lines of force, in the same way that a magnetic field is traversed by magnetic lines of force.**

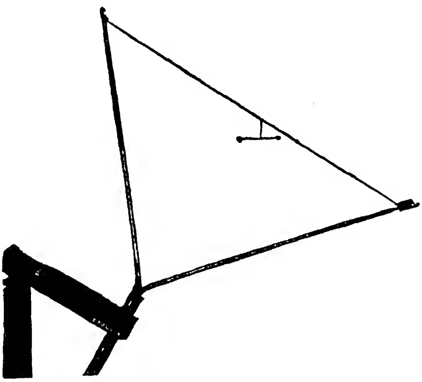


Fig. 296.—Apparatus for exploring an electric field.

Exploration of an electric field.—Unfortunately, it is difficult experimentally to map out a field of electric force either so satisfactorily or so simply as in the case of a magnetic field. Nevertheless, it is possible to construct a simple appliance which, when placed at different points in an electric field, will indicate the direction of the force at each point. In this

manner, not only is the existence of the forces verified, but their general distribution in space is also proved to be comparable to that which has been found to exist in magnetic fields of force. The following device may be used :

Fix two long pieces of glass rod in a cork, and bend the rods so that they form a large V. Bore a hole in a small cork, so that it will fit tightly on the end of one of the rods. Attach one end of a silk fibre to this cork, and the other end to the free end of the other glass rod. The fibre may be tightened by rotating the small cork. To the centre of the fibre attach another short fibre (about 2 cm. long), which carries the pointer. The pointer consists of a piece of fine copper wire (5 cm. long), on the ends of which are threaded two small gilt pith-balls. Adjust the pith-balls so that the pointer hangs freely in a horizontal position (Fig. 296).

It will be found that the electric forces due to bodies charged directly by friction are weak, and far more satisfactory results will be obtained by using large insulated brass spheres which are connected by wires to a Wimshurst machine (see p. 459).

EXPT. 261.—**Lines of force of a single sphere.** Charge a single insulated sphere, and hold the pointer in various positions in the surrounding space. Observe how the lines of force appear to radiate outwards from all points of the sphere's surface (Fig. 297).

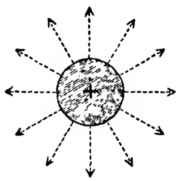


FIG. 297.—Lines of force due to a positively-charged sphere

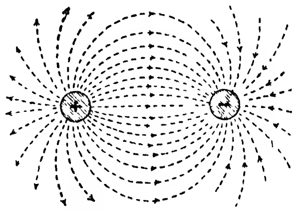


FIG. 298.—Lines of force due to two electrified spheres.

EXPT. 262.—**Lines of force between two spheres.** Place two insulated brass spheres about 50 cm. apart, and charge them oppositely by connecting them to the poles of a Wimshurst machine. Verify the general distribution of the lines of force as shown by dotted lines in Fig. 298.

If the electric lines of force are considered to have properties similar to those possessed by stretched elastic threads, it can be understood at once why oppositely charged bodies attract.

Strength of electric field and properties of lines of force.—The more strongly two insulated metal spheres are charged with opposite kinds of electrification the stronger is the electric field which is generated ; usually this is regarded as being due to an increase in the number of lines of force passing through the field, and generally is indicated so in diagrams. As +ve and −ve electrifications are generated always in equal quantities, there will be the same number of lines leaving the +ly charged surface as there are lines entering the −ly charged surface. No line of force will end blindly in space—at opposite ends of it will always be found equal quantities of opposite electrification, whatever its path may be. This is exactly analogous to the magnetic lines of force between unlike magnetic poles.

The direction of the force in a magnetic field is chosen arbitrarily as that in which a single north-seeking pole would tend to move. **In an electric field of force the direction of the force is chosen arbitrarily as that in which a +ly charged body will tend to move.** Hence the lines of electric force may be regarded as proceeding from a +ly charged body towards a −ly charged body.

Electric potential.—In Fig. 299 let A represent an insulated sphere +ly charged. Let V_1 represent a small +ly charged sphere (which we will term a test-charge) which is free to move. The force of repulsion $\mathbf{F}_1$ due to A will tend to make the test-charge move further away from A. In order to move the test-charge from V_1 to V_2 work must be done on it, so that its **potential energy** is greater at V_2 than at V_1. In the same way its potential energy is greater at V_3 than at V_2, and it is greatest at a point as near as possible to the surface of A. The space round A is a region of **electric potential** which gradually diminishes in value as the distance from A increases, and we say that the electric potential is greater at V_3 than at V_1.

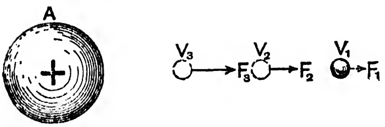


FIG. 299.—The potential is greater at V_3 than at V_1.

No force will act on the test-charge when it is removed to

a great distance from A, consequently the potential at a great distance from A is zero.

No electric forces originate from uncharged bodies, hence the region round them (in the absence of any charged bodies) is one of zero electric potential. An uncharged body has zero potential, and since the earth may be regarded as a huge spherical conductor which is uncharged, it is usual in experimental work to take the potential of the earth as our zero or starting-point for measurement. In the same way gravitational potential is measured from sea-level.

If the charge on A is −ve, then work must be done in withdrawing the positive test-charge from the neighbourhood of A, and most work would be required if the test-charge is almost touching A. Hence the potential is least at points near to A, and gradually increases as the distance increases, and finally becomes zero at a great distance from A. The field round A in this case is said to be one of **negative potential.**

From this reasoning we derive the following important rules :—(i) **A positively-charged body tends to travel from a point of higher electric potential to a point of lower electric potential.**

(ii) **Since the force acting on the body is in the direction of the lines of force at the point where the body is situated, the movement, if any, will trace out the path of the lines of force.**

(iii) **The forces acting on a negatively-charged body are opposite in direction to those acting on a positively-charged body. Hence a negatively-charged body tends to travel from points of lower potential to points of higher potential.**

The electrostatic units of quantity and of potential.—(i) The electrostatic unit of *quantity* is defined in the same manner as the unit of magnet pole-strength : **the Unit of Quantity is**

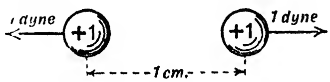


FIG. 300.—The unit of quantity (electrostatic).

that which exerts a force of one dyne on an equal quantity situated at a distance of one centimetre, when air is tho medium separating the charges (Fig. 300).

It is important to remember that the force exerted depends upon the medium. When some other insulating medium, such as paraffin-oil or turpentine, is substituted for air, the force of repulsion (or of attraction, if the charges are unlike) is less than one-half of that which occurs when air is the medium.

Suppose that each of the charges is greater than one unit, say q_1 and q_2 units respectively, and that the distance apart is d cm.; then, when the medium is air, the force of repulsion (or attraction) is given by the equation

$$f = q_1 q_2 / d^2 \text{ dynes.}$$

The electrostatic unit of quantity must not be confused with another unit, the *practical electromagnetic unit,* which is called the **coulomb.** This is defined as the quantity conveyed by a current of one *ampere* in one second; and its definition is based upon the magnitude of the magnetic effect of the current at points near to its path. The coulomb and ampere are explained on pp. 495 and 503. This unit, the coulomb, is (3×10^9) times as great as the electrostatic unit.

(ii) **The potential at any point near to a positively charged conductor is equal to the work (in ergs) which is required in order to convey a unit of positive electricity up to the point from an infinite distance.** When the conductor is negatively charged, the potential at a point near to it is equal to the work required to convey a unit of positive electricity *from* the point *to* an infinite distance away. In this case, the potential is expressed as a *negative* quantity.

Based upon the preceding paragraph, **the difference of potential between two points is equal to one unit when one erg of work is required to convey one unit of positive electricity from the point of lower potential to the point of higher potential.** Of course, when the unit charge is free to move, it moves in the opposite direction, and the same amount of work is done *by the electrical forces.*

The **volt** (p. 513), which is the practical unit always used in expressing potential-differences in a circuit conveying an electric current, is totally different from the electrostatic unit both in origin and in magnitude: one electrostatic unit is equal to 300 volts.

Flow of electricity.—In the previous sections we have imagined the positive test-charge to be conveyed from point to point on a small insulated sphere surrounded by a non-conducting medium—air. Suppose that the small sphere containing the test-charge is fixed rigidly at some point in an electric field of force, and that a mass of conducting material (such as a metal) is brought into contact with the small sphere, then the charge will leave it, if, by so doing, it can move into a region of lower potential. The charge would subsequently be found on that portion of the conductor which is situated in the region of lowest potential. **Electricity is said to flow in a conductor from points of higher to points of lower potential.** Difference of potential and flow of electricity are allied to one another as cause and effect, but the cause will only produce the effect when the medium is a conductor. No current can traverse a perfect insulator although there may be a difference of potential within it; but such a medium is thrown into a condition of strain.

When a +ly charged insulated conductor is connected to earth by means of a wire the charge rapidly 'flows' along the wire, and the conductor is discharged quickly. The field of force originally surrounding the conductor has disappeared —in fact, each portion of the charge in its passage along the wire has been accompanied by the lines of force associated with it. The so-called 'flow of electricity' may therefore be regarded as a disappearance of lines of force, with the result that the surrounding medium is relieved from its condition of strain.

It might be said that the charge from a −ly charged sphere would flow along the wire to earth, and pass from a lower to a higher potential, thus disobeying the conclusion arrived at above. In this case, however, the electricity is regarded as flowing along the wire from the earth to the body until the conductor is raised to zero potential. This may be expressed in another manner by saying that the transference of a −ve charge in one direction is the same thing as the transference of a +ve charge in the opposite direction.

The recent discoveries concerning electrons would appear to

render obsolete the nomenclature used in the preceding paragraphs to describe the 'flow of electricity.' Thus, it would be more correct to describe the discharge of a +ly charged insulated conductor as 'a flow of negative electrons from the earth to the insulated conductor'; similarly, it would be correct to describe the discharge of a −ly charged insulated conductor as a flow of electrons from the conductor to the earth. But, the earlier mode of describing the phenomena has become so firmly established, and used so universally, that the 'flow of electricity' is still described as taking place *from* regions of higher potential *to* regions of lower potential.

Hydrostatic and thermal analogies.—The ideas of electric potential may be grasped by means of analogies, which, although unsound in principle, are often useful in imparting the main facts of the subject. The following analogies are often adopted:

(i) The difference of potential between two charged bodies is compared to the difference of level of water in two cisterns, which are connected by means of a narrow pipe. The difference of level is termed generally the head of water. The head of water causes water to flow along the connecting-pipe from the higher to the lower level, and the flow ceases as soon as the level of water is the same in both cisterns. The equality of level in the cisterns is analogous to the equality of potential of two charged conductors connected by means of a wire.

(ii) Heat will pass from a hot bodv to a cold body placed in contact. The flow of heat depends upon the difference of temperature, and will cease when both bodies are at the same temperature. The difference of temperature is analogous to a difference of potential between two charged conductors.

These analogies particularly fail in not directing special attention to the field of force between the charged bodies; moreover, the analogies hold good only up to a certain point. The student is recommended not to place too much reliance upon what may, at first sight, be an easy means of understanding the principles of potential.

EXERCISES ON CHAPTER XXXIV.

1. How would you show that a brass rod is capable of being electrified? Explain why on rubbing a brass rod and a glass rod the latter only ordinarily appears to be electrified by the friction.

2. How would you prove that glass and silk when rubbed together are charged equally and oppositely?

3. If you want to find out whether a body is electrified by seeing how it acts on an electrified pith-ball hung by a silk thread, why is it a surer test that the body is electrified if it repels the pith-ball than if it attracts it?

4. An electrified pith-ball is hung by a cotton thread attached to a glass rod. An electrified rod of sealing-wax is found to repel the pith-ball at first, but the repulsion gradually diminishes, and finally becomes an attraction. What conclusion would you arrive at from this?

5. What is the simplest method of removing completely the charge from an electrified rod of sealing-wax? What precaution must be adopted if the hand is used for the purpose?

6. How would you charge a gold-leaf electroscope negatively by means of a piece of fur only?

7. A charged gold-leaf electroscope is required for a certain experiment, and the divergence of the leaves is observed to be greater than is required. How would you remove a portion of the charge without completely discharging the instrument?

8. Two metal spheres of equal size, standing on insulating supports, are oppositely and equally electrified, one positively, the other negatively. They are then placed near together, but not so near as to produce a spark between them. Describe the general distribution, when so placed, of the charges upon them, and of the electric lines of force in the field between them.

9. Two equal metallic spheres charged with equal quantities of electricity of the same sign are placed near together, but not in contact. Give a sketch, showing the way in which the electricity is distributed over the spheres.

10. An insulated brass ball without charge is hung near a negatively-charged conductor. It is connected with the charged conductor momentarily. Is its potential altered thereby, and if so, how? It is connected momentarily with the earth. How does this affect its potential?

11. Discuss the analogies between differences of level, temperature, and electrical potential respectively.

12. A small insulated uncharged sphere has positive potential if placed near a positively-charged conductor. How would its potential be modified if it already possessed a slight negative charge? How would the result of connecting it momentarily to the earth then depend upon the distance between the sphere and the conductor?

13. A glass rod is rubbed with a silk handkerchief, and a piece of sealing-wax is rubbed with flannel. Describe exactly how you would show that the state of electrification of the glass rod is different from that of the sealing-wax.

14. State the law of force between electric charges.

Two equal small metal balls are given charges of electricity equal to $+10$ and -20 respectively. They are then allowed to touch and are again separated to the same distance as before. Compare the forces between the two balls in the two cases.

(Camb. S.C.)

CHAPTER XXXV.

ELECTROSTATIC INDUCTION.

Electrostatic induction.—A bar of soft iron, when supported in a magnetic field, temporarily acquires polarity (p. 391B); is there any analogous phenomenon when a conductor is supported in an electric field of force?

EXPT. 263*.—**Induced charge on a cylinder.** Support on an insulating stand a cylinder of wood, with rounded ends, and coated with tinfoil or black-lead. A suitable stand may be made from a rod of unpolished vulcanite, which is fixed vertically in a hole bored in a wooden base. Hold a glass rod, which has been rubbed with silk, near one end of the cylinder (Fig. 301). Hold a proof plane † with its flat side in contact with the end A of the cylinder. Convey the proof plane to a pith-ball electroscope which is charged $-$ly, and observe that the proof plane also is charged $-$ly. While holding the glass rod in the same position as before, touch the distant end of the cylinder with the proof plane, and test the charge on the latter by means of a $+$ly charged electroscope. Observe that the proof plane is charged $+$ly. When the proof plane touches the cylinder it becomes part of one

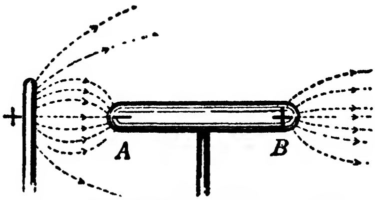


FIG 301.—Inducing charges on a cylinder.

* The student is reminded that the terms 'positive' and 'negative' continue to be used, not because we assume that there are two different kinds of electricity, but because it is accepted generally as the only convenient form of nomenclature. Whenever the term *negatively charged* is used it must be assumed tacitly that this implies more than the normal amount of electricity, while *positively charged* implies less than the normal amount.

† A proof-plane consists of a thin disc of copper or brass (about 2 cm. diameter) fixed to the end of an insulating handle.

and the same conductor, and therefore acquires a portion of the electrification which may be present at the ends of the cylinder.

Induced charges.—When a positively charged rod is held near one end A of an insulated conducting cylinder, it is found that the cylinder is charged –ly at A and +ly at B. Fig. 301 represents the distribution of the lines of force in the experiment. The end A is nearer to the glass rod than B, and is consequently at a higher potential. The cylinder is a conductor, therefore electricity flows from A towards B, and the flow continues until the potential of the cylinder is uniform. Lines of electric force proceed from the + ve charge on the glass rod to the – ve charge on A ; lines of force also proceed from the + ve charge at B towards the walls of the room. Notice how the lines of force appear to converge towards A, and to diverge outwards from B, and how this suggests the idea that the cylinder is a better conductor than the surrounding air for the lines of force. It is instructive to compare this with the flow of magnetic lines of force through soft iron. We say that **the charges on the cylinder are due to induction from the electrified glass rod.**

When the glass rod is removed the electric field goes with it, and no lines of force remain to influence the cylinder. The + ve and – ve charges at A and B have become distributed over the entire cylinder, and have neutralised each other exactly, and therefore must have been present in equal quantity.

Expt. 264.—**Uncharged cylinder.** Remove the charged glass rod to a distance, and again test with the proof plane the electrification of the insulated cylinder. No charge is found on the cylinder.

Expt. 265.—**Induced opposite charge.** Again hold a charged glass rod near A. Touch the cylinder with the finger, and again test the electrification at A and B. A is charged –ly, and B is uncharged.

Fig. 302 indicates the result of touching an insulated cylinder while under the influence of a charged glass rod. The potential is reduced to zero, consequently no lines of force pass from the end B to the walls of the room, and the + ve charge formerly distributed over the end B has disappeared. The few lines of force from the glass rod which formerly traversed the greater distance to the walls of the room (or beyond), to terminate there in their equivalent – ve charge, are able now to find this equivalent charge by traversing

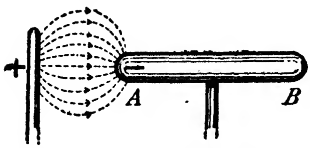


Fig. 302.—The induced charge on a cylinder.

the less distance to the end A of the earth-connected cylinder; this preference for a shorter path is not a new property, but simply the result of the fundamental tendency to shorten shown by all lines of force. We consequently find that rather more lines of force now terminate on the end A than was the case before connecting to earth, resulting in the −ve charge at A being slightly greater after being earthed than it was before being earthed.

The fact that the cylinder can be at zero potential while still in a region of +ve potential may be understood more clearly if it is remembered that the cylinder itself has a −ve charge, which would, in the absence of any charged body, give it a −ve potential. The external field, however, tends to give the cylinder a +ve potential. The two effects are equal and opposite, thus giving the cylinder an apparent zero potential.

EXPT. 266.—**Charging negatively by induction.** Bring a charged glass rod near one end of an insulated cylinder; while it is there touch the cylinder for a moment, and then remove the glass rod to a distance. Test the charges on the ends A and B. Both are charged −ly. So also are all parts of the cylinder's surface. **The cylinder has been charged −ly by induction.*** The lines of force (which essentially coexist with the −ve charge) now proceed from all directions towards the cylinder, and they must originate from an equivalent +ve charge; they cannot come from the glass rod, because this has been removed to a distance, and we shall be able to prove in a subsequent experiment that they come from the boundaries of the room (Fig. 303).

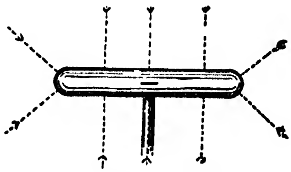


FIG. 303.—Cylinder charged by induction.

EXPT. 267.—**Charging positively by induction.** Repeat Expt. 266, using electrified vulcanite instead of the glass rod. Prove that A is electrified +ly, and B electrified −ly, and verify the results shown in Fig. 304.

In Fig. 304 (i) the point B is in a region of higher potential than A, consequently electricity flows from B to A. In Fig. 304 (ii) B has been connected to earth, and electricity has passed up

* **Free and Bound Charges.**—These terms are sometimes used in order to distinguish the charge which disappears on touching with the finger from that which still remains on the insulated conductor. Thus, in Expt. 263, the +ve charge on the end B would be termed the free charge, and the −ve charge on the end A would be termed the bound charge.

into the cylinder until its potential has been raised to zero; the lines of force entering B have been destroyed. In Fig. 304 (iii) the vulcanite has been removed, the +ve charge formerly at A is distributed all over the cylinder now, which is consequently charged +ly by induction.

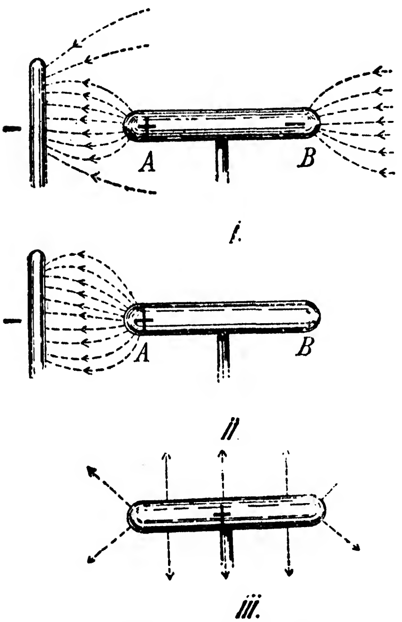


FIG. 304.—Charging an insulated conductor +ly by induction.

Attraction of uncharged bodies due to induction.—When a cylinder or a lath is supported so that it can turn freely, and a charged rod is brought near the right or left of one end, the lines of force would cause the cylinder to approach the rod; for the same reason, if the rod be held above (or below) one end, then the latter tends to rise (or fall). Experimental results similar to this have been obtained already with a long wooden lath (Expt. 252) and by using a proof plane it is easy to verify the induced charges at the ends of the lath.

The attraction of light objects (Expt. 251) is due to the same effect. Each fragment is acted upon inductively before attraction takes place. But if the fragments are lying on the table they are earth-connected, so that the field of force is analogous to that of a cylinder which has been touched when a charged rod is held near it (Expt. 265).

Each stage of the action of a charged glass rod on a pith-ball electroscope is represented in Fig. 305. (i) shows attraction of the pith-ball. (ii) shows the pith-ball drawn up into contact with the rod, thus destroying the lines of force between

the rod and the near side of the ball. The lines of force from the distant side of the ball now tend to pull it away from the rod, with the result shown in (iii). This is a simple case of

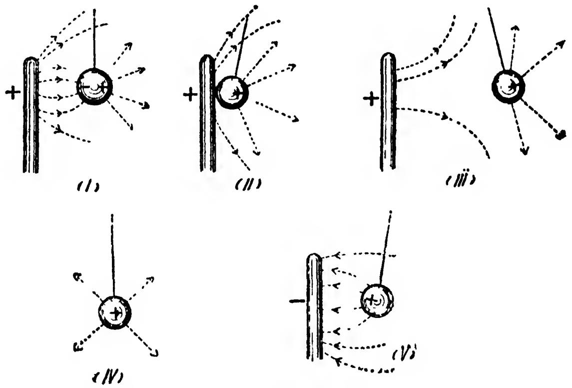


FIG. 305 —Action of a charged glass rod on a pith-ball electroscope.

repulsion between similarly charged bodies. (iv) represents the ball charged +ly after the rod has been removed to a distance. (v) represents the effect produced by bringing a −ly charged rod near to the +ly charged pith-ball.

Theory of the gold-leaf electroscope.—The theory of Expts. 266 and 267 is directly applicable to the gold-leaf electroscope. Instead of the insulated cylinder we have an insulated conductor, with a flat metal disc at its upper end, and a pair of metal leaves at its lower end.

EXPT. 268.—**Charging an electroscope by induction.** (i) Hold a −ly charged rod of vulcanite over the disc. The leaves are at a higher potential than the disc, consequently electricity passes from the leaves to the disc, giving the former a −ve and the latter a +ve charge. The charge on the leaves induces a +ve charge on the tinfoil. Lines of force (see Fig. 306 (i)) proceed across from each tinfoil strip to the nearest metal leaf, resulting in the leaves being pulled apart. The same number of lines of force also pass from the disc to the vulcanite. The degree of divergence will depend upon the number of lines of force passing between the leaves and the tinfoil.

(ii) Hold the vulcanite still in same position, and touch the disc with the finger. The potential of the leaves is raised to zero, the lines of force between the tinfoil and the leaves disappear, and the leaves collapse (Fig. 306 (ii)).

(iii) Remove the vulcanite to a distance. The +ve charge distributes itself uniformly over the conductor, a portion going into the leaves and inducing a −ve charge on the tinfoil. The lines of force thus brought into play cause the leaves to diverge (Fig. 306 (iii)). *The electroscope has been charged +ly by induction.*

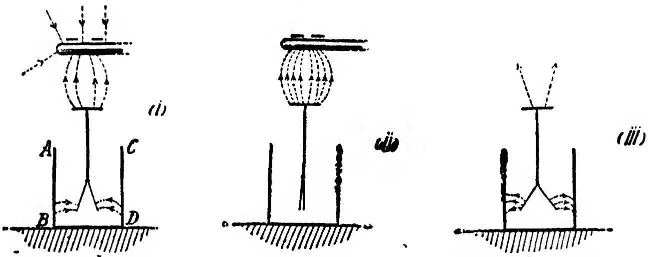


FIG. 306.—Stages in the action of a charged rod on a gold-leaf electroscope.

(iv) Hold a +ly charged glass rod over the disc. The potential of the disc is raised above that of the leaves. More electricity enters the leaves, and the increased number of lines of force *causes the leaves to diverge more.*

(v) Hold a −ve charged rod of vulcanite over the disc. The potential of the disc is lower than that of the leaves. Electricity passes from the leaves to the disc, thus *diminishing* the number of lines of force between the leaves and the tinfoil, and therefore the divergence.

(vi) Repeat Expt. 268 (i)-(iii), using a +ly charged body instead of the charged vulcanite rod. Explain how, after touching the disc and removing the external +ve, the instrument has acquired *a negative* charge.

Hold in succession above the disc a +ly charged and a −ly charged body, and observe that the divergence of the leaves is diminished and increased respectively.

(vii) Hold the hand, or any other earth-connected conductor, immediately over the disc of a charged electroscope. Notice and explain the change produced in the divergence of the leaves.

It is evident that a charged electroscope may be used to determine the sign of the charge on any body held over the disc. The rules to observe are as follows:

Electroscope charged +ly { *Increased divergence implies +ve charge. Diminished divergence implies −ve charge (or an earth-connected conductor.)*

Electroscope charged −ly { *Increased divergence implies −ve charge. Diminished divergence implies +ve charge (or an earth-connected conductor).*

It is important to remember that, in *all* possible circumstances, the divergence of the gold-leaves is a measure of *electric potential* or, more correctly, of the *difference of potential* between the leaves and the foil strips which line the sides of the instrument.

In general use, the instrument stands on a table, and the foil strips therefore are at zero potential. If the potential of the leaves is raised slightly, they diverge; but they will diverge to the same extent if they are given a −ve potential of the same magnitude as the previous +ve potential. The divergence therefore measures, not the actual potential of the leaves, but the *difference of potential* between the leaves and the case of the instrument.

The same point may be investigated by placing the instrument on an insulating stand—*e.g.* a slab of paraffin-wax. Suppose that a small +ve charge is given *to the case of the instrument*, instead of to the leaves: the leaves will diverge, although no charge has been directly given to them. The same divergence would be observed if a −ve charge, of equal magnitude, were given to the case of the instrument.

Finally, with the instrument still on an insulating stand, if the disc is connected temporarily by a thin metal wire to the case, and a charge be given to the instrument, there will be no divergence of the leaves, however great the charge, *because there is no difference of potential between the leaves and the case.*

Another method of demonstrating the same fact is to place in contact two conductors (*e.g.* metal spheres) of different size, each on an insulating stand. Give a charge to the combined conductors, and then separate them. They have been charged to the *same potential*; but the larger conductor has

more of the charge then the smaller conductor, because its 'capacity' (p. 453) is greater. Connect one end of a long thin wire to the cap of an electroscope, and wrap the free end round a stick of sealing-wax, so that the wire can be carried without becoming earth-connected. Touch the larger conductor with the free end of the wire, and note the divergence; and, without discharging the instrument, convey the end of the wire to the smaller conductor. On making contact, *there is no change in the divergence,* although the charge on the smaller conductor is much less than that on the larger one.

The electrophorus.—This is a convenient appliance for obtaining larger charges of electricity than can be obtained from electrified glass rods or vulcanite rods. It was devised by Volta in 1775. The instrument consists essentially of a circular slab of vulcanite, sealing-wax or shellac, and a flat metal disc with an insulating handle. (Fig. 307).

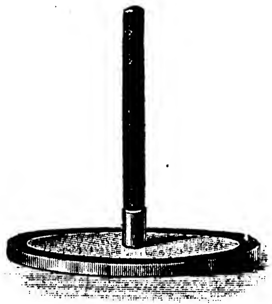


FIG. 307.—An electrophorus.

EXPT. 269.—**Use of an electrophorus.** Charge the electrophorus plate −ly by rubbing with fur or flannel. Place the metal disc resting on the top of the plate. Touch the disc. Raise the disc away from the plate. Test the charge on the disc by holding it over the disc of a +ly charged gold-leaf electroscope; an increased divergence shows that the electrophorus disc is +ly charged. Bring the finger near the disc; when sufficiently near, a small spark is seen to pass from the disc to the hand. Completely discharge the disc by touching it with the hand. Again place it on the plate, and repeat the experiment. The disc may be charged many times without it being necessary to re-charge the plate.

The spark obtained from the charged disc represents the expenditure of a certain small quantity of energy—it generates slight heat, and light and sound waves. Where does this energy come from? A spark cannot be obtained while the metal disc is in contact with the charged plate; but it can be obtained after the disc is lifted. In addition to the *work*

which must be done in order to lift the disc, a small additional amount of work must be done in stretching the electric lines of force which join the +ve charge on the disc to the −ve charge on the plate: it is this additional amount of work which re-appears in the energy of the spark.

When an electrophorus is used for charging an insulated conductor, it is not possible to increase the charge indefinitely. Each time the charged disc is brought near to the conductor, a spark will be observed; but, soon, these sparks can no longer be obtained. This condition arises when the potential of the conductor has been raised to that of the charged disc, because no charge can pass from the disc to the conductor unless the potential of the former is higher than that of the latter.

Potential of a conductor.—That the potential is the same at all points of a conductor may be verified by deduction from the fundamental facts of static electricity. When two points on the surface of a conductor are at different potentials, electricity will continue to pass between them until they are at the same potential. Hence, in an electric field which is not changing, all points of a conductor must be at the same potential. The same conclusion may be arrived at experimentally in the following manner:

EXPT. 270.—**Equality of potential.** Charge the insulated cylinder (as used in Expt. 263) by means of the electrophorus. Connect the disc of a proof plane to the disc of a gold-leaf electroscope by means of a thin copper wire. (It is convenient to bore two small holes through the discs.) Holding the proof plane by its insulating handle, bring it into contact with the cylinder and observe the divergence produced (Fig. 308). The degree of divergence is a measure of the potential of the point on the cylinder which is in contact with the proof plane. Move the proof plane to other points of the cylinder, and notice that the divergence remains unaltered.

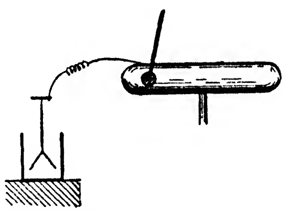


FIG. 308.—All points on a charged conductor are at the same potential.

Hollow conductors.—We have now to consider whether lines of force are present inside, as well as outside a charged

body. One way to test this in the case of a hollow conductor is to introduce a proof-plane within the conductor, with the object of removing a portion of any charge which may be present. When this is done no charge is found to be taken by the proof-plane, thus indicating that electrification is absent.

EXPT. 271.—**Absence of charge inside a hollow conductor.** Place a coffee-tin (or calorimeter) on an insulating stand. Charge the tin by means of an electrophorus. Touch the outside of the tin with a proof plane, and verify the charge with a golf-leaf electroscope. Discharge the proof plane, and with it touch the inside of the tin; carefully remove the proof plane without touching the edge or outer surface of the tin. Test it by means of the electroscope; it is uncharged. Hence there is no charge inside a conductor (solid or hollow).

If a charged hollow vessel could be turned inside out, would the outer surface be charged still, although now inside the vessel? Or, would the charge leave it and pass to the surface which is now outside? These questions may be answered experimentally by using a cotton net, supported on an insulated handle, and capable of being turned inside out by means of a long silk thread attached to the end of the net (Fig. 309). This arrangement is known as **Faraday's butterfly net.** When

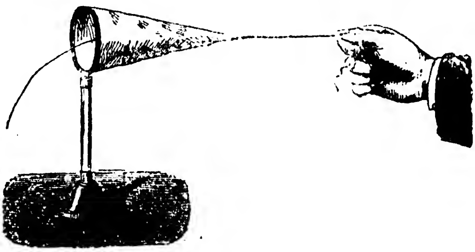


FIG. 309.—Faraday's butterfly net.

such a net is charged, the electrification is found on the outside surface even after the net has been turned inside out. It is, in fact, a fundamental property of an electric charge to distribute itself on the outer surface only of a conducting body.

EXPT. 272.—**Faraday's butterfly net.** Charge a Faraday butterfly net by means of an electrophorus. Test for the charge outside

and inside by means of a proof plane. The charge is entirely on the outside. Now turn the net inside out, taking care not to touch the cotton net with the hand. Again test the inner and outer surfaces. The charge is found again on the outside only.

Faraday's ice-pail experiment. The only method of setting up lines of force inside a hollow-conductor is to suspend *inside* the conductor another conductor which is insulated and charged ; or, as an alternative, the hollow conductor may be placed on an insulating stand and charged, and an earth-connected conductor then suspended inside. In either case, the presence of a charge on the inner surface of the hollow conductor can be proved by means of a proof-plane and an electroscope.

When the earth-connected hollow conductor is sufficiently deep, and the open top is sufficiently narrow, *all* the lines of force proceeding from a charged body suspended inside it will be intercepted by the inner surface of the hollow conductor : in other words, **the induced charge is equal and opposite to the inducing charge** ; and there will be no charge on the outside. But, when the hollow conductor is insulated, an equal and similar charge will be found on its outer surface. Finally, if the suspended charged body be lowered until it touches the inside of the hollow-vessel, the inducing charge and the induced opposite charge neutralise each other, and the suspended body is completely discharged, while the charge on the outside of the hollow conductor remains unaltered. *This is the only method by which the whole of the charge on an insulated conductor can be transferred to another conductor* : if contact is made in any other way, the charge is only *shared*, and not completely transferred.

These facts can be demonstrated by means of an experiment which is known as *Faraday's ice-pail experiment* (so-called because Faraday first performed the experiment, and used an ice-pail as a hollow conductor).

EXPT. 273.—**Faraday's ice-pail experiment.** Place a deep metal can on an insulating stand, and connect the can to the disc of an electroscope (Fig. 310). Support inside the can an insulated conductor which is charged +ly. The induced +ve charge is

distributed partly over the outer surface of the can and partly over the electroscope. Observe the amount of divergence of the

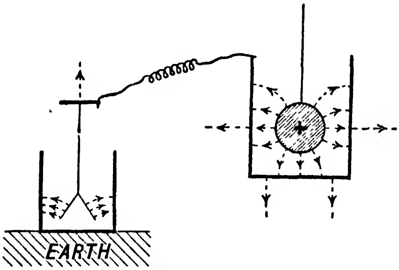


FIG. 310.—Faraday's ice-pail experiment.

leaves. Allow the insulated conductor to touch the inside of the can, and notice that *the divergence remains unaltered.*

Remove the insulated conductor, and prove (by means of another electroscope) that it is completely discharged. Evidently the conductor's charge has been exactly neutralised by the −ve charge induced on the inside of the tin, without leaving an excess of either—for, had there been an excess of either, it would have proceeded to the outside of the can, and so modified the divergence of the leaves. Moreover, as the induced +ve charge must be equal in quantity to the induced −ve charge, it follows that the charge which finally remains on the can must be equal in quantity to the charge which was originally on the insulated conductor.

Distribution of a charge on the surface of a conductor.—Although the potential of a conductor is the same at all points, it does not follow necessarily that the charge is distributed uniformly over the surface. The quantity of electricity on each square centimetre of the surface of a conductor is not necessarily the same. Usually this quantity is termed the density of the charge. Hence, **although the potential of a charged conductor is uniform, the electric density is not necessarily so, but depends upon the shape of the conductor.**

EXPT. 274.—**Sphere.** Charge a large insulated sphere. Touch the surface with the flat side of a proof-plane, and bring the proof-plane into contact with the disc of an uncharged electroscope. Notice the degree of divergence. Discharge the proof-plane and electroscope. Test other portions of the sphere's surface in the

same way. The divergence is the same in each case. When the proof-plane is touching the surface of the sphere it becomes a portion of the sphere's surface (so far as the distribution of the electricity is concerned, because the charge is entirely on the outer surface); the removal of the proof-plane is equivalent to removing a portion of the sphere's surface equal in size to the proof-plane, and the divergence of the leaves therefore measures the quantity of electricity on such a portion.

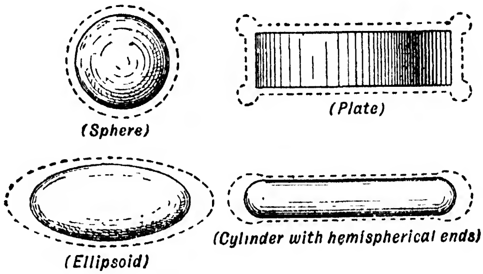


FIG. 311.—Distribution of charge on conductors

The distribution of electricity on the surface of a sphere is uniform (Fig. 311).

EXPT. 275.—**Cylinder.** Repeat this experiment with a large insulated cylinder with hemispherical ends. The greatest divergence is obtained when the proof plane has touched either end, and the least divergence when the proof plane has touched the straight sides (Fig. 311).

EXPT. 276.—**Plate.** Repeat the experiment, using a charged flat metal plate. More electricity is obtained from the edge than from the sides (Fig. 311).

EXERCISES ON CHAPTER XXXV.

1. Describe how to arrange an experiment so that a conductor charged all over with negative electricity may nevertheless receive a further charge of negative electricity on being connected with the ground by a conducting wire.

2. (i) Under what conditions is it possible for a negatively-charged insulated sphere to have (i) zero potential, (ii) positive potential?

(ii) How is the potential of a positively-charged insulated sphere modified by bringing another positively charged body near to it?

3. Explain how the surface densities and potentials at different points on a charged pear-shaped conductor may be compared. Illustrate your answer by diagrams of the apparatus used. (Camb. S.C.)

4. Describe the gold-leaf electroscope. Explain fully how you would employ it (*a*) to find the sign of the charge on a charged body, (*b*) to show the distribution of charge over the surface of the body. (Cen. Welsh Bd.)

5. Point out the chief resemblances and differences between magnetic and electrostatic phenomena. (Lond. Matric.)

6. Describe an electroscope. What is meant by the statement that an electroscope indicates potential rather than charge? Describe experiments in illustration. (Camb. S.C.)

7. Describe an experiment to prove that two parts of the same conductor may be electrified differently although they are at the same potential.

8. The caps of two gold-leaf electroscopes, A and B, are connected by a long wire, and a positively charged sphere is brought near A. What will be the indications of the electroscopes, and how will they alter if either A or B is touched?

9. What is meant by the electrical potential of an insulated conductor? If an electroscope is connected by a wire to the carrier plate of an electrophorus, describe, and explain in terms of potential, the movements of the leaves which may be observed when the electrophorus is worked. (Camb. S.C.)

10. Describe the electrophorus and explain its mode of action. Is there any limit to the magnitude of the charge which can be accumulated on an insulated conductor by means of an electrophorus? (Bristol S.C.)

11. Describe experiments to illustrate electrostatic induction. A small metal ball A is charged positively. How would you give to a nearly closed conductor B (*a*) a positive charge, (*b*) a negative charge, each equal to that of A? (Camb. S.C.)

12. Describe two experiments which show that the electric charge on a hollow conductor resides on the outside of the conductor. How would you arrange matters so that when a charged body is put in contact with an insulated body, it will part with *all* of its charge to the latter? (Cen. Welsh Bd.)

13. Explain what is meant by the potential of a conductor.
A positively charged conductor A is brought near (*a*) an insulated conductor B, (*b*) an earth-connected conductor C. What changes, if any, will be produced in the potentials of the three conductors, and what charges will be produced on B and C?
How would you test your conclusions experimentally? (Camb. S.C.)

CHAPTER XXXVI.

CONDENSERS. ELECTRIC MACHINES.

Capacity of a conductor.—It has been seen that when two insulated conductors, one of which is charged, are brought into contact, the charge spreads over both conductors. The uncharged conductor becomes charged, but as yet we have not described what fraction of the origonal charge has been transferred to it. The amount transferred depends evidently upon the size of the uncharged conductor—a large conductor receiving a larger fraction of the original charge than is the case when the conductor is small.

The potential of the two conductors becomes the same as soon as they are brought into contact, but it does not follow that the quantity of electricity is the same on each. The final potential would be expected, however, to be less than that of the charged body before contact was made, since the same number of lines of force which formerly originated from the charged conductor will be distributed now over a larger area.

EXPT. 277.—**Capacity and size.** Obtain two or three metal spheres of different sizes, each mounted on an insulating support. (Instead of the spheres, bottles of different sizes and coated with tin-foil may be used.) Place a hollow can on the top of the electroscope disc. Charge one of the spheres by means of an electrophorus, and touch it with an uncharged sphere. Both spheres are now charged to the same potential. Convey the larger sphere to the electroscope, lower it into the can, and allow it to touch the inner surface. The whole charge is now transferred to the can and electroscope. Withdraw the sphere, and observe the divergence of the leaves. Discharge the electroscope. Proceed in the same manner with the smaller sphere. Notice that the divergence is much less. Hence

the larger portion of the charge was on the larger sphere. We say that the spheres have not the same **capacity for electricity.**

The capacity of a conductor evidently depends upon its size; therefore a larger conductor requires more electricity to raise it to a given potential than a smaller conductor.

The capacity of a conductor is measured by the quantity of electricity which must be given to it in order to raise its potential to a given amount.

$$\text{Or, } \textbf{Capacity} = \frac{\textbf{Quantity of electricity (Q)}}{\textbf{Potential to which it is raised by Q}}.$$

From this definition it is seen that if the capacity of a conductor increases while the quantity of electricity on it remains constant, its potential will become less.

EXPT. 278.—**Quantity and potential.** Connect a large insulated sphere to the electroscope by means of a long thin wire. Give a small charge to the sphere by means of the electrophorus. Notice the divergence of the leaves. Bring an insulated uncharged sphere into contact with the charged sphere. Observe the diminution of the divergence, showing that, although the total quantity of electricity is the same, the potential is less. Repeat the experiment, using a larger uncharged sphere, and observe the greater diminution of divergence.

Capacity affected by neighbouring conductors.—So far only the relationship between the capacity and the size of a conductor have been considered. The capacity is increased by the presence of neighbouring conductors, either insulated or earth-connected. If the hand or any other conductor be brought near to a charged electroscope, the divergence of the leaves is decreased. The conductor (*i.e.* the disc and leaves of the electroscope) does not change in size, nor does the quantity of electricity on the conductor alter, yet the potential is reduced. Evidently the 'capacity' of the conductor is increased by holding the hand over the disc. The diminution of potential is explained by remembering that the +ve charge on the electroscope induces a −ve charge on the under-surface of the hand. This induced −ve charge creates a region of −ve potential in its neighbourhood, thus causing a reduction of the +ve potential of the electroscope.

EXPT. 279.—**Action of neighbouring conductor.** Charge an electroscope +ly. Observe the divergence of the leaves. Hold the hand just above the disc, and observe how the divergence is less than before. When the hand is removed the divergence increases to its original value.

The lines of force tend to accumulate on the side of a charged conductor facing an earth-connected conductor.

EXPT. 280.—Charge an insulated sphere; the density of the charge is uniform. Hold a metal plate in the hand and near to the sphere. Touch the near side of the sphere with a proof plane, and test the density of the charge by means of an electroscope. Observe the divergence, and discharge the electroscope. Test the distant side of the sphere in the same manner, and observe that the density is much less. Hence the charge has become accumulated on the side facing the earth-connected conductor.

This tendency of a charge to accumulate (or to become piled up) owing to the presence of a neighbouring earth-connected conductor is termed the condensing of electricity. **Any arrangement by which the capacity of a conductor is increased artificially is termed a condenser.** The capacity of a condenser depends directly upon the area of surface of the two conductors and is inversely proportional to the distance separating them. It also depends largely upon the medium through which the lines of force pass. This medium is generally termed the **dielectric** because the electrical forces between the conductors are transmitted through it.

The fundamental facts concerning condensers may be demonstrated by means of the apparatus shown in Fig. 312; this consists of two vertical metal plates (about 15 cm. square) supported on sealing-wax rods, which have been fixed to the plates by heating. Handles of sealing-wax are also attached. The plates are readily made of sheet zinc.

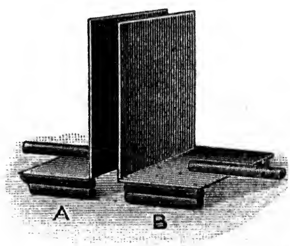


FIG. 312.—A simple form of condenser.

EXPT. 281.—**Capacity of a condenser depends upon (i) the distance apart, and (ii) the medium.** Connect plate A to an electroscope by means of a long thin wire, and connect plate B to earth (by keeping a finger in contact with it). Charge the plate A, and observe the divergence of the leaves when B is about 20 cm. distant from A. Slowly move B towards A, and observe the diminution of the divergence. Slowly remove B, and observe the increase of divergence.

(ii) Place B about 3 cm. distant from A, and charge A. Carefully insert between the plates a square slab of paraffin-wax about 2 cm. thick, and slightly larger in area than the plates, and notice the diminution of divergence. The effect is much more evident when a slab of thick plate glass is used instead of paraffin-wax.

[*N.B.*—It frequently happens that the surface of a paraffin-wax slab is already charged, probably due to handling it with the fingers; and this would spoil the experiment. When this condition is suspected, the surface of the wax is completely discharged by passing it *rapidly* through the flame of a Bunsen burner.]

It would seem, from the latter experiment, that the effect of replacing a portion of the air between the plates by paraffin-wax or glass is the same as if the plates had been brought still nearer together: the wax and the glass seem to transmit the force more readily than air. This varying power of transmitting the lines of electric force is termed the **Specific Inductive Capacity (S.I.C.)**. Nearly all insulating solids have a higher **S.I.C.** than air.

Usual form of condenser.—The most usual form of condenser consists of a large number of sheets of tinfoil separated from each other by sheets of paraffin paper; alternate sheets of the foil are connected together, so that the area of surface of the two conductors is many times greater than that of a simple two-plate condenser (Fig. 313).

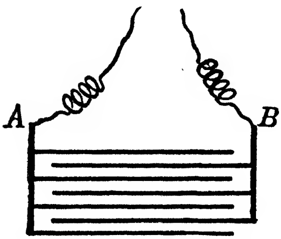


FIG. 313.—Diagram of ordinary form of condenser.

The Leyden jar is a simple form of condenser which derives its name from the fact that it was first used by Van Musschenbroek, a professor at Leyden, in Holland. It consists of a glass jar, coated outside and inside with tinfoil

to within a short distance of the top. It may therefore be regarded as a condenser consisting of two parallel plates separated by a glass dielectric. A brass rod, terminating above in a brass knob, stands in a collar fixed to a tripod foot in contact with the inner lining. The tinfoil lining serves as the insulated conductor, which may be charged conveniently through the knob; the jar is placed either on a table or held in the hand, so that the outer coating is consequently earth-connected (Fig. 314).

EXPT. 282.—**Charge and discharge of Leyden jar.** Place a Leyden jar on the table. Bring the charged disc of an electrophorus into contact with the knob; repeat this five or six times. The jar is charged now. Hold the knuckle near to the knob, and observe the slight shock which is felt when the spark passes.

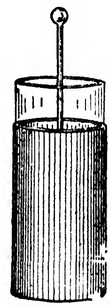


FIG 314—A LEYDEN JAR is a form of condenser consisting of a glass jar coated, inside and outside, with tinfoil. The metal rod facilitates the charging of the inner coating.

As a rule it is advisable not to discharge a Leyden jar through the body by touching the knob, since a powerful discharge may have serious consequences. A safe method of discharging the jar is afforded by **discharging tongs**, which consist of a jointed brass rod with brass balls at each end, and provided with glass handles. To use the tongs, one knob is placed in contact with the outer coating, and the other knob is brought towards the knob of the jar.

A much simpler method of charging a Leyden jar than by using an electrophorus is to bring the knob into contact with a terminal of a Wimshurst machine (see Fig. 315), the other terminal of which is connected to the nearest gas or water pipe.

Units of capacity.—An insulated conductor has unit capacity when one unit of quantity will raise its potential by unity. When the charge and the potential are expressed in electrostatic units, the magnitude of the unit of capacity may be realised by the fact that a metal ball, radius $=1$ cm., suspended in space, and at a distance from any earth-connected conductors, has a capacity of one unit.

Similarly, a condenser has unit capacity when one unit of

quantity imparted to the insulated plate raises its potential above that of the earth-connected plate by one unit. A condenser of unit capacity could be made from two square flat metal plates, the length of each edge equal to 3·5 cm., and placed 1 cm. apart in air. It is evident that this unit is extremely small.

The practical unit of capacity, called the **Farad**, is based upon the *coulomb* and the *volt* as units of quantity and of potential-difference respectively: a condenser is defined as having a capacity of one farad when one coulomb of electricity imparted to the insulated plate will raise its potential by one volt.

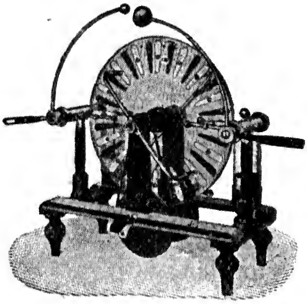


FIG. 315.—Wimshurst influence machine.

In practice, the *farad* is inconveniently great: even condensers of large size have a capacity which is but a small fraction of one farad; and it is usual to express their capacities in *micro-farads* (or, millionths of a farad). The small condensers used in wireless receiving apparatus have a capacity of the order 0·0001-0·001 microfarad. Even the microfarad is 900,000 times greater than the electrostatic unit of capacity.

Electric Machines.

An electric machine.—It has been seen that a body may become electrified either by friction or by induction. Any mechanical appliance designed to produce these effects on a large scale is termed an electric machine. The electrophorus may be regarded as a simple example of an electrical machine, depending for its action upon the principles of Static Induction, but it is unsuited for the generation of large electrical charges.

The earliest forms of electric machines were mere elaborations of the simplest experiment in which a rod of sulphur or resin is charged −ly when rubbed with the dry hand; at a later date glass was substituted for the sulphur, and suitable rubbers were used instead of the hand. Such machines may be termed

frictional electrical machines, as distinct from **induction (or influence) machines**, which for all experimental purposes have replaced almost entirely the former type.

The Wimshurst induction machine.—The early types of machine designed for generating electric charges on a large scale were based on the principle that glass or sulphur become electrified when rubbed with a suitable material. These are now superseded by **induction machines**; and, of these, the Wimshurst type is more generally used. It consists of two circular plates of varnished glass, placed as close together as possible, and geared so as to rotate in opposite directions. On the outer surface of each plate are fastened an even number of thin metal sectors, which serve both as **inductors** and **carriers.** Across the front is fixed a diagonal conductor terminating in metal brushes which touch the sectors as they pass; a similar diagonal conductor is fixed across the back plate, but sloping in the opposite direction. The insulated collecting combs are placed at opposite ends of the horizontal diameter, and each comb has teeth projecting towards the sectors on both front and back plates. Two Leyden jars are supported often on the base-board of the machine, with their knobs connected to the collecting-combs by movable wires. The combs are supplied with adjustable discharging knobs which are placed above the machine.

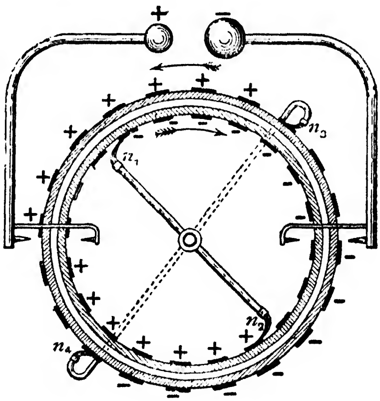


FIG. 316.—Action of Wimshurst influence machine.

The action of the machine is explained best by means of a diagram (Fig. 316), in which the two plates are represented as though they were two cylinders of glass rotating opposite ways as shown by the arrows. The neutralising brushes are represented by n_1n_2 and n_3n_4. In order to start the machine it is sufficient if one of the sectors has a slightly different potential from that of the others; as a rule this is the case, and the machine

is then self-starting. Imagine that one of the back sectors at the top of the diagram has a slight +ve charge. When it comes opposite the brush n_1 it acts inductively on the sector touching n_1, giving to it a slight −ve charge, and simultaneously giving a +ve charge to the sector touching n_2. These sectors, with their induced charges, leave the brushes and rotate into positions opposite the brushes n_3 and n_4; the sectors touching n_3 and n_4 now receive induced +ve and −ve charges respectively, which they retain after leaving the brushes. Thus, after one or two revolutions, all sectors approaching the left-hand comb have +ve charges, and all sectors approaching the right-hand comb have −ve charges. The sectors are neutralised by the combs, the knobs connected to which acquire +ve and −ve charges respectively.

If the machine is found not to be self-starting it is sufficient to hold a piece of electrified vulcanite near the front plate opposite the brush n_3.

The Electric Discharge.

Action of points.—When a needle is connected with the charged conductor of an electric machine the surface-density at the point of the needle becomes so great that the air in contact with the point becomes charged with similar electrification, and is repelled forcibly away from the needle. This action continues until the conductor is discharged.

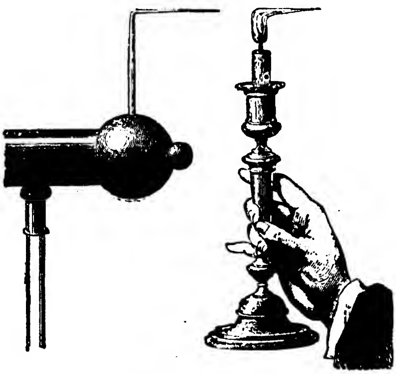


Fig. 317.—Discharge from points.

Expt. 283.—**Discharge from points.** (i) Attach a sewing needle or a piece of copper wire with sharpened end to the terminal of a Wimshurst machine by means of soft wax, taking care that the needle is in metallic connection with the terminal. Connect the other terminal to earth. On turning the machine, hold the hand near to the point of the needle, and notice the current of air which appears to be driven from the point.

Hold a candle flame near to the point, and observe how it is blown aside (Fig. 317).

(ii) Transfer the needle to the other terminal, and earth-connect the terminal which carried the needle in Expt. (i). Observe that the phenomena observed with the −ve terminal are the same as with the +ve terminal.

(iii) Allow the current of air from the point to impinge on a small insulated metal plate or sphere. Verify by means of an electroscope that the plate is charged with the same kind of electricity as that of the terminal to which the point is attached. Verify this statement by transferring the needle to the other terminal and testing the charge acquired by the metal plate. Evidently the stream of air which is repelled from the point is charged electrically.

When the point of a needle is held near the charged conductor of an electric machine it becomes charged with opposite electrification by induction, and produces similar effects to those observed when a needle is connected directly with the machine. This action illustrates the use of lightning conductors. During a thunderstorm the clouds are charged electrically and induce an opposite charge on the earth's surface immediately beneath the cloud; when the potential difference is sufficiently great a spark discharge (in the form of lightning) takes place between the cloud and any conductor projecting above the earth's surface. By fixing an earth-connected metal point (*i.e.* a lightning conductor) over the building to be protected, any +ly charged cloud will cause an induced −ve charge on the metal point, resulting in the partial or complete neutralisation of the cloud's charge.

EXPT. 284.—**Principle of lightning conductor.** (i) Hold a needle in the hand, with its point towards the terminal of the machine. Interpose a candle flame between them, and observe how the flame is blown away from the point.

(ii) Hold an insulated metal plate between the point and the terminal, and verify that the plate is charged now with the opposite electrification to that which is found on the terminal.

Spark discharge.—When the knobs of an electric machine are not far apart, the sparks pass between them quickly and almost in straight lines. Upon separating the knobs, however, the sparks are less frequent and follow irregular paths. The

diminished frequency of the sparks with increased distance of the knobs apart is due to the fact that a greater difference of potential is necessary in order to overcome the dielectric strength of the air, and a greater interval of time is required to charge the knobs to the required potential. The discharge always follows the path of least resistance, and dust particles floating in the air are sufficient to divert the discharge from a straight path into a variable and zigzag path.

EXPT. 285.—**Character.** Notice the sharp intermittent sparks which pass almost in a straight line between the knobs of the machine. Separate the knobs still farther apart, and notice that the sparks are less frequent and trace out a zigzag path (Fig. 318).

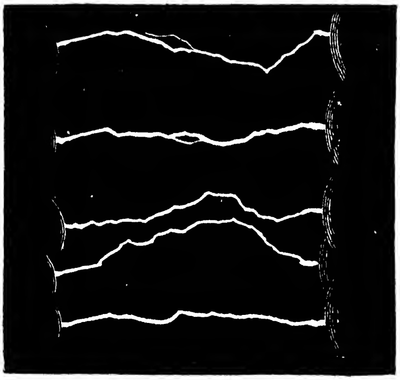


FIG. 318.—Photographs of the electric spark.

EXPT. 286.—**Quantity.** Connect the two Leyden jars to the terminals of the machine, and notice that the sparks are less frequent but far more violent than before. The capacity of the knobs is increased considerably now by being connected to the Leyden jars, and a far greater quantity of electricity must be collected in order to raise the potential of the knobs to a sufficient degree to cause a discharge between the knobs. Carefully bring the knobs together before completing the experiment.

EXPT. 287.—**Duration.** What seems to the eye to be a single electric spark is really a succession of discharges from one knob to the other, yet the total duration is extremely brief, being about the twenty-four thousandth part of a second.

Attach a few small pieces of gummed paper to the surface of one of the glass discs of the machine, as near to the edge as possible. Darken the room, and observe the scraps of paper when illuminated by successive sparks between the knobs. Notice that the paper seems to be absolutely at rest although they are really revolving at a high speed. The duration of the spark is so brief

that the discs do not move appreciably during the passage of a spark discharge.

The spark discharge has considerable penetrative power, and is capable of piercing holes through solid dielectrics.

EXPT. 288.—**Penetrative effect.** Hold a sheet of cardboard between the discharging knobs, and observe that each spark pierces a small hole through the cardboard; notice also that each hole appears to have a slight burr on both sides, as though the discharge had simultaneously passed in both directions.

Discharge through conductors.—We see that the electric field of force between the terminals of a Wimshurst machine when in action may be destroyed rapidly by means of the spark discharge. A sequence of sparks is accompanied by the equally rapid destruction and remaking of the electric field of force. The energy used up in this process is derived from the mechanical work done in turning the machine.

The field of force may be destroyed also by connecting the terminals together by means of a conductor. When a good conductor (such as copper wire) is used, the field is destroyed almost instantaneously, even before it acquires any considerable intensity. In fact, we have two opposing tendencies, (i) the machine tending to create a field of force, and (ii) the conductor tending to destroy it, with the result that there is a steady 'flow' of electricity along the wire, which continues so long as any potential difference is maintained between the ends of the conductor.

When the machine is in action there is a gradual fall of potential between consecutive points of the wire, and the end in contact with the +ve terminal has the highest potential. But copper is such a good conductor that the charges are not able to accumulate in the terminals sufficiently to make the potential differences at all great. When a poor conductor, such as string or cotton, is used instead of copper, the discharge is sufficiently slow to enable the machine to maintain a considerable potential difference between the terminals. The potential at various points along the string might be compared by connecting the points momentarily with a gold-leaf electroscope

and observing the divergence of the leaves, but this instrument is too sensitive for such high potentials. The experiment may be conducted more satisfactorily by using several pairs of pith-balls suspended by cotton threads from various points of the string; the degree of repulsion of the pith-balls will indicate the potential of that point of the string to which they are attached.

EXPT. 289.—**Change of potential.** (i) Stretch a piece of thin string AE (1 metre long) between two vertical glass rods (40 cm. high).

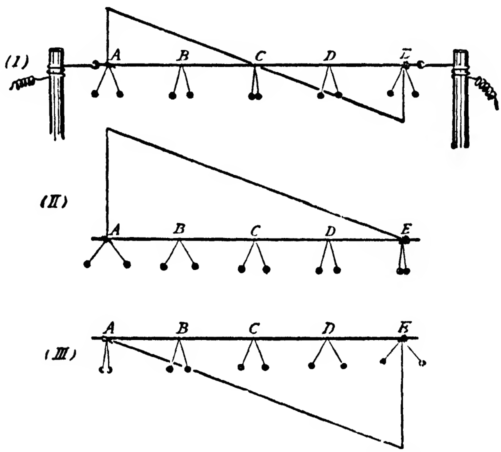


FIG. 319.—Difference of potential along an electrified string.

Connect the ends of the string to the terminals of a Wimshurst machine by means of copper wires. Suspend five pairs of pith-balls (on cotton threads) from equidistant points of the string. When the machine is in action notice how the greatest divergence is at A and E, less at B and D, and nil at C (Fig. 319 (i)). Verify that the pith-balls at A are charged +ly by bringing near to them the charged plate of an electrophorus, and that the pith-balls at E are −ly charged by holding near to them an electrified rod of sealing-wax. The sloping line indicates the gradual fall of potential along the string. Place the finger on the string at C. The divergence of the pith-ball is in no case altered, since C is already at zero potential.

Place the finger at E. The pith-balls at E collapse, those at C now diverge, and the divergences at A and B increase. This would be

anticipated from the change in position of the potential line (Fig. 319 (ii)). The potential at E has been raised to zero, and the potential at all other points will be raised to a corresponding degree, since the machine will maintain the same potential difference between the terminals quite independently of the actual values of the potentials.

Place the finger at A. The result is represented in Fig. 319 (iii). The potential at A is reduced now to zero, and the potential at all other points is reduced to a corresponding degree.

While the actual potentials are modified by connecting a point on a conductor to earth, yet the potential differences and the consequent 'flow' are not altered in any way. But this ceases to hold good if two different points of an electrified string are connected simultaneously to earth (*e.g.* the two ends); in this case the two ends are both at the same potential, and the flow does not take place along the conductor, but is diverted through the hands and arms (assuming, of course, that the human body is a much better conductor than that which is connecting the terminals; an assumption which holds good if cotton or string, and not metals, are used).

Chemical, heating, and magnetic effects of an electric discharge.—The mechanical effect of an electric discharge has been illustrated already (Expt. 288). The discharge is also able to produce chemical, thermal, and magnetic effects.

When an electrical machine is working, the characteristic smell of ozone is often evident; the chemical action by which oxygen is converted into ozone being due to the electric discharge.

If a piece of white filter paper, previously soaked in an emulsion of starch and potassium iodide, be laid on a sheet of glass and fixed just below the terminals of an electric machine, patches of blue coloration will be developed where the discharge strikes the paper, owing to the liberation of iodine from the potassium iodide. The same chemical change is produced when the paper is touched by the terminals of a voltaic battery, but in this case the coloration is only evident round the positive terminal.

The heat effect of the discharge may be shown by connecting two insulated metal balls by a short length of very fine wire; on discharging a Leyden jar battery through the wire it is volatilised with explosive violence. If the discharge be made to pass through a small heap of gunpowder, it is scattered simply, but not ignited; this is owing to the discharge

being so brief that the mechanical scattering takes place before the powder is heated to its temperature of ignition. But if the discharge be slowed-down by including a poor conductor —*e.g.* a piece of wet string—in the circuit the powder will be ignited.

The discharge from a Leyden jar will ignite ether. Also, if the experimenter stands on an insulating stand with one hand on a terminal of an electrical machine, the other terminal being earth-connected, and holds a finger near to a gas burner from which coal-gas is escaping, the gas will be ignited by the spark passing from the finger to the burner.

A steel sewing-needle may be magnetised by placing it inside an insulated open spiral of thick guttapercha-covered copper wire through which a discharge is passed from a Leyden jar battery.

Such effects as those now described can be obtained only by **electricity in motion,** and none of them can be obtained when an electric charge is stationary. An electrically charged body may indeed indicate a force of attraction upon either pole of a suspended magnet; but this is not due to any magnetic phenomenon, for the same effect could be observed if the magnet were replaced by a strip of any metal or other material.

As will be seen in subsequent chapters, similar chemical, heating, and magnetic effects can be observed when an **electric current,** generated by means of a voltaic cell, is passing along a metal wire. The only difference is that in the case of a so-called electric discharge the passage of electricity is either momentary or intermittent, while in the case of an electric current it is steady and continuous.

EXERCISES ON CHAPTER XXXVI.

1. A Leyden jar is held in the hand by its outer coating, and the knob is presented to the prime conductor of an electrical machine in action. Describe the resulting charged condition of the jar, and explain why it is safe to put the charged jar down on the table. Explain why you receive a shock on touching the knob when the jar is standing on the table, but not when you or the jar stand on a dry cake of resin.

2. An electrified drop of water, supported by a non-conductor. evaporates. Assuming that the vapour is not electrified, what changes will the potential of the drop undergo?

3. Two similar vertical insulated plates, A and B, are placed parallel to each other and about an inch apart. Each is connected to the cap of a separate gold-leaf electroscope. State and explain the indications of the electroscope when (i) a positive charge is given to A, and afterwards (ii) B is touched.

4. A sheet of tinfoil is suspended by a dry silk thread and charged as highly as possible by an electrical machine, but on discharging it a slight spark only is obtained. If the tinfoil is placed on a sheet of dry glass lying on the table, a bright spark can be obtained after the tinfoil has been charged by the machine. Explain the cause of the difference.

5. How do you explain the fact that a Leyden jar cannot be charged highly unless its outer coating be earth-connected?

6. How could you show that electricity gathers most at points and corners of a conductor? Give two practical applications of this property.

7. An orange, into which a sewing-needle has been stuck, point outwards, is suspended by a dry silk thread. A charged body is brought near to it (i) opposite the point of the needle, (ii) opposite the side remote from the needle. State and explain the electrical effect in each case.

8. A sharp point attached to a conductor A is held near an insulated charged conductor B. What will be the effect on B if A is (1) insulated, (2) uninsulated?

9. The cap of a gold-leaf electroscope is connected by a wire to an insulated metal plate A, and a second insulated metal plate B is placed parallel to A and about 1 cm. from it. The gold-leaf electroscope is charged positively. What would be the effect of placing a slab of paraffin-wax between the plates?

If the plate B is now touched with the finger, what happens? Give reasons for your answer in each case. (Durham S.C.)

10. Describe some apparatus for supplying large quantities of electricity by induction. (Lond. Matric.)

11. Define *unit electric charge, electric potential, capacity of a condenser.*

A metal plate is connected by a wire to a gold-leaf electroscope, and the system is insulated and given an electric charge. Describe and explain what happens when an earthed plate is brought to the first plate, but without touching it. (Camb. S.C.)

12. Describe an experiment showing the discharging effects of a sharp point. How is it employed in the protection of buildings from lightning? (Lond. Matric.)

PART VIII.

VOLTAIC ELECTRICITY.

CHAPTER XXXVII.

VOLTAIC CELLS. MAGNETIC EFFECTS OF AN ELECTRIC CURRENT.

VOLTAIC ACTION.

Chemical action.—The energy represented by an electric current obtained by means of a voltaic cell is derived from the chemical action proceeding within the cell. It is therefore essential that the phenomenon of chemical action should be understood clearly. The following experiments are typical, and closely allied to the changes which go on in several types of cells.

EXPT. 290. (i).—**Chemical change.** Hold the end of a strip of thin sheet-zinc in a hot gas-flame (such as that obtained with a mouth-blowpipe). Notice how the metal burns with a bright blue-green flame, and becomes changed to a white powder. This powder is oxide of zinc, and has been formed by the chemical combination of zinc and oxygen.

(ii) Try to obtain the same effect with a strip of sheet copper. The metal does *not* burn, but it becomes coated with a black film (oxide of copper).

(iii) When a strip of platinum is treated in the same manner, no change can be perceived.

EXPT. 291. (i).—**Action of metal on acid.** Drop a small strip of commercial zinc into a test-tube containing dilute sulphuric acid (1 in 8). Notice that bubbles of a gas are given off from the surface

of the zinc. Close the end of the test-tube with the thumb for a few moments, so as to prevent the gas from escaping. Remove the thumb and hold the open end of the tube at the side of a gas flame. The gas in the test-tube burns with a dull blue flame. The gas obtained by this means is called hydrogen. At the same time observe that the zinc disappears gradually.

(ii) Repeat the above experiment with copper, and notice that the dilute acid has no action upon it, even if the acid is heated. Even with *strong* sulphuric acid no action is perceived, unless the acid is heated.

(iii) When platinum is tested in the same manner no action takes place even with hot strong sulphuric acid.

A simple voltaic cell.—Expts. 290-1 have shown that metals are not acted upon chemically with equal readiness; and, of the metals zinc, copper, and platinum, zinc undergoes change most readily and platinum least readily. Any pair of these metals may be used for the construction of a simple voltaic cell; but since copper and zinc are the most available, and for other reasons, these metals are selected generally for the purpose.

EXPT. 292.—**Electric current.** Cut two rectangular pieces of sheet-copper and sheet-zinc (10 × 4 cm.); solder a short piece of thick copper wire to the upper edge of each. Fig. 320 represents a convenient method of supporting the plates in a beaker of dilute sulphuric acid. Connect the terminals by means of a fairly long thin copper wire. Place a compass-needle on the table and hold just over it a straight piece of the connecting wire, the wire being so adjusted that it lies in the magnetic meridian. Notice how the needle is deflected. This experiment is a simple test for the presence of an electric current, and the theory of it will be explained in a subsequent paragraph. A wire through which an electric current is passing not only has a magnetic action upon a compass-needle, but also it is capable of magnetising a piece of steel. When such a wire (providing that it is cotton-covered) is wound in a close spiral round a piece of narrow glass-tubing inside which a steel sewing-needle is supported, the needle is found to acquire slight permanent magnetisation.

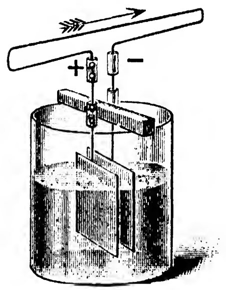


FIG. 320.—A simple voltaic cell.

While a voltaic cell is in action, bubbles of gas are seen to form on the surface of the *copper* plate, and when commercial zinc is used, the same effect can be observed on the surface of the zinc. Even when the wire joining the zinc and copper plates is disconnected from the cell so that no electric current is being generated, bubbles of gas are still liberated from the surface of the zinc (as in Expt. 291 (i)); the zinc is therefore being consumed wastefully without the generation of the equivalent amount of energy being developed in an external conductor. This is known as **Local Action.** If the zinc plate be removed from the cell and its surface smeared uniformly with a few drops of mercury and then replaced within the cell, it will be seen that this local action is prevented, and the zinc is used far more economically.

It can be shown readily that the consumption of the zinc is the source of the energy represented by the current passing along the wire. Thus, when the zinc plate is dried carefully and weighed, connected up as shown in Fig. 320, and, after the current has passed for some time, again dried and weighed, it is found to weigh less than before, the loss in weight being proportional approximately to the time during which the current has passed. In the case of the copper plate its weight is found to remain practically constant.

The difference of potential between the terminals of a voltaic cell.—Previous observations (Expt. 223 and p. 466) have shown that a steel needle may be magnetised if a spiral of wire surrounding it is traversed either by the discharge obtained from a Leyden jar or by a current derived from a voltaic cell. The chief difference between the two experiments is that in the case of the Leyden jar discharge the current is quite *momentary*, and consists of a small quantity of electricity passing between the coatings of the jar previously charged to a high potential difference, while in the case of the voltaic cell the current is *continuous* and consists of the transference of a large quantity of electricity between the plates of the cell, which, by the chemical action within the cell, are maintained at a slight potential difference.

The difference of potential between the plates of a simple

voltaic cell is so slight that it is impossible to detect it by connecting the plates directly to the disc and to the outside of a gold-leaf electroscope. Whereas, if an attempt were made to use the same means for detecting the potential difference between the coatings of a charged Leyden jar, probably the leaves would be shattered owing to the potential difference being so great. In order to demonstrate effectively the condition in the case of a voltaic cell, it is necessary to use a considerable number of the cells connected together *in series* (*i.e.* with alternate zinc and copper plates joined together) and to increase the sensibility of the electroscope by using it in conjunction with a condenser of variable capacity—such an arrangement is termed usually a **condensing electroscope.**

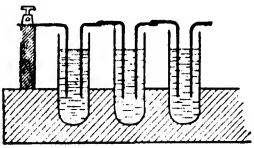


FIG. 321.—Simple electric cells in series.

EXPT. 293.—**Potential condition of a voltaic cell.** Construct a series of simple cells (Fig. 321) by fixing a row of small glass tubes (2 inch × ½ inch) in a block of wood. Solder together a number of copper and zinc strips, and bend each compound strip so that the copper dips into one tube and the zinc into the next. Nearly fill each tube with very dilute sulphuric acid. Connect the zinc plate of the first cell to earth, attach a copper wire to the opposite terminal, and wrap the free end of this wire round a vulcanite rod, which will serve as a convenient handle. Fit up the condensing electroscope* (Fig. 322). Connect plate A to earth, and touch plate B *momentarily* with the free end of the copper wire; A and B now have the same difference of potential as that between the terminals of the cells. On raising plate A the capacity of the lower plate is diminished, and the potential of B is increased

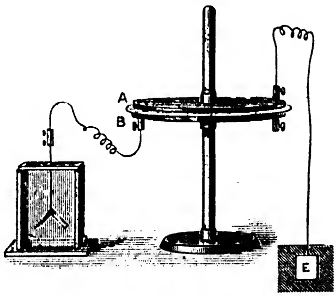


FIG. 322.—A condensing electroscope.

* A condensing electroscope consists of two brass discs, about 20 cm. diameter The lower disc is supported on a vertical rod of unpolished vulcanite, and is connected to a gold-leaf electroscope; the upper disc is supplied with a vulcanite handle, and is earth-connected. The plates are separated from each other by a thin sheet of paraffined paper.

correspondingly; the potential of B is now sufficient to cause a divergence of the leaves. Verify that the charge on the leaves is positive. Discharge the condenser, reverse the battery, and repeat the observations; verify that the plate B now acquires a negative charge.

By experiments with a condensing electroscope it is proved that there is always a difference of potential between the terminals of a voltaic cell—just as in the case of the terminals of an electric machine when in operation—and the zinc plate is always at a lower potential than the other plate, whatever the other metal may be. Hence, **when the plates are connected externally by means of a metal wire, a current traverses the wire from the end joined to the copper to the end joined to the zinc plate.**

Electro-motive force.—The force which serves to maintain the potential difference between the metal plates is called the **electro-motive force** of the cell (usually written **E.M.F.**). Since the degree of potential difference between the plates is dependent upon the degree of **E.M.F.** inside the cell, it follows that the numerical value of either expresses at the same time the value of the other, and it is customary to refer to the potential difference of the metal plates as the **E.M.F.** of the cell. So long as the potential of the copper is maintained, a discharge of +ve electricity will pass along the wire, or, in other words, a **current of electricity** is obtained. The current will only cease when either all the zinc or all the acid has been used up.

The **E.M.F.** of cells is expressed usually in terms of a unit which is called the **volt.** The student may realise the magnitude of this unit by the fact that the **E.M.F.** of a simple voltaic cell, such as described on p. 470, is approximately 1 volt. The **E.M.F.** of a Daniel cell (p. 475) is about 1·07 volts, and that of a Leclanché cell (p. 475) is about 1·43 volts.

Polarisation.—Attention has been directed already to the fact that, when a simple voltaic cell is in action, bubbles of gas (hydrogen) collect on the surface of the copper plate. Each small portion of the copper plate to which a bubble of hydrogen adheres is protected from the acid, and thereby the effective area of the copper plate is reduced. The accumulation of hydrogen is injurious for another reason. Hydrogen is oxidised readily, and behaves, in this sense, similarly to zinc; if

present in a voltaic cell it behaves somewhat similarly to a zinc plate, and tends to send a current through the acid from the copper to the zinc. In this manner the primary effect of the metal plates is reduced by the opposing tendency of the hydrogen. The current passing through the cell and the connecting wire is reduced consequently. This effect, due to the hydrogen accumulating on the copper plate, is termed **polarisation** of the cell.

Removal of the hydrogen by mechanical means is not convenient, but it is possible to prevent its accumulation by chemical means, *e.g.* by oxidising it. It cannot be burnt in the air in this case, but other substances besides air afford a supply of oxygen available for this purpose (*e.g.* permanganate of potash, manganese dioxide, or bichromate of potash). These substances contain much oxygen, which they readily give up when dissolved in water; they are termed **oxidising agents.**

Other chemical methods are available for preventing polarisation; and of the many modifications of the simple cell which have been devised, the chief differences are due to the various methods adopted for preventing polarisation.

There are alternative methods of observing the phenomenon of polarisation: if a suitable *ammeter* (for measuring current strength) or *voltmeter* (for measuring potential-difference) is not available, the effect can be observed by means of a simple voltaic cell (Fig. 320) and a tangent galvanometer.

EXPT. 294.—**Polarisation** (i). Fit up a simple voltaic cell, using *very dilute* sulphuric acid; and connect in series with it a rheostat and tangent-galvanometer. Adjust the rheostat until a deflection of 50°-60° is obtained. Remove the plates, dip the *copper* plate for a moment into a solution of copper sulphate (contained in a separate beaker), replace the plates, and read the deflection as *quickly as possible.* Note the deflection every half-minute, for several minutes. The film of copper sulphate adhering to the copper plate delays the setting-up of polarisation, and thus the needle may become steady before polarisation becomes evident.

Polarisation in a simple voltaic cell may be considerably delayed by adding to the dilute acid a small quantity of a concentrated solution of potassium bichromate.

(ii) If a Leclanché cell and a voltmeter (reading 0-3 volts) are available, connect the cell to the voltmeter, and note the reading: it should be about 1·43 volts. Disconnect the cell from the voltmeter, and join its poles to a rheostat of low resistance (or a short length of German-silver wire). Allow the current to pass for a few minutes, then break this circuit and again join-up to the voltmeter. Notice that the reading is now much less than 1·43 volts, *and that it slowly recovers its normal value* (see next paragraph).

(iii) An ammeter (reading 0-3 amperes) joined in series with a Leclanché cell and a rheostat, will show a gradual diminution of current. If the circuit be broken, left for several minutes, and the circuit be again closed, it will be observed that the cell has partly, or completely, recovered from the polarisation.

The Leclanché cell.—In this cell, known by the name of the physicist who devised it, the materials are zinc, carbon, and a concentrated solution of ammonium chloride (sal-ammoniac). Manganese dioxide is used as a depolarising agent. The carbon plate (C, Fig. 323) is placed in the centre of a cylindrical porous pot which is packed closely with a mixture of carbon and manganese dioxide. The zinc rod Z dips into the solution of sal-ammoniac contained in the glass jar. When the cell is in action, ammonia and hydrogen are produced; the ammonia gas, being very soluble in water, does not tend to produce polarisation. The manganese dioxide is but a very slow oxidising agent, and the cell consequently soon becomes polarised if used continuously; it soon becomes depolarised, however, if allowed to remain unused for a short time.

Fig. 323.—A Leclanché cell.

The Daniell cell.—In a Daniell cell copper and zinc are used as the metals, and sulphate of copper (blue vitriol) is used as a depolariser. The outer vessel (Fig. 324) is of copper, and serves as the copper plate. The porous pot is surrounded by a strong solution of sulphate of copper, the strength of which is maintained by placing crystals of the sulphate on a perforated copper shelf near the top of the outer vessel. The zinc rod and dilute sulphuric acid are contained by the porous pot.

When the cell is in use the hydrogen generated by the action of the zinc on the sulphuric acid passes through the porous pot, and, instead of appearing on the surface of the copper, displaces copper from the copper sulphate. The result is, that pure copper, and not hydrogen, is deposited on the copper plate. Hydrogen may be supposed to act on the copper sulphate to form sulphuric acid and copper; or, expressed as a chemical equation,

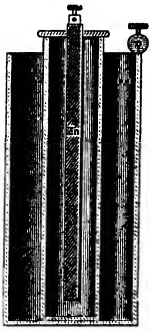


FIG. 324.—A Daniell cell.

$$H_2 + CuSO_4 = H_2SO_4 + Cu.$$

When the cell is left standing for a long time, some of the copper sulphate passes through the porous pot and is decomposed by the zinc forming zinc sulphate and copper, the latter being deposited on the zinc rod. This effect reduces the power of the cell, and it is consequently necessary to remove the liquids to separate bottles as soon as the experiments are completed.

The Dry Cell.—The **dry cell** (Fig. 325) is identical with the Leclanché cell, except that a *paste* is used instead of the solution of sal-ammoniac. The cell, therefore, is more portable than the Leclanché. The carbon plate is surrounded by a thick layer A of oxide of manganese, carbon, sal-ammoniac and zinc chloride; and round this is a paste B of plaster of Paris, sal-ammoniac and zinc chloride. The whole is contained in an outer vessel of thin sheet zinc, and is covered with a layer of pitch through which is fixed a small tube for the escape of any gases generated inside. The failure of a dry cell is often due to its becoming too dry; and it may be restored by boring a number of small holes through the outer zinc vessel, soaking it in water, and afterwards closing the holes with some wax.

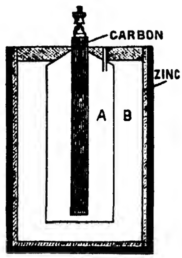


FIG. 325.—A DRY CELL. Similar to a Leclanché cell; but the solution of sal-ammoniac is replaced by a damp paste.

Accumulators, or storage cells.—Small portable accumulators are a very convenient source of current for experimental work in the laboratory; and they are widely used on motor-cars for the head-lights, and for starting the engine. In these cells it is *chemical energy* which is stored, and not electricity.

To charge the cells a current is sent through them in one direction by means of another battery, or by a dynamo, and this current is converted into chemical energy within the cells. This energy is available for supplying a current later on; and the current so obtained passes through the cells in the opposite direction to that of the current used in charging them. The theory of the accumulator is described in more detail in a later chapter (p. 507).

The nature, and direction, of an electric current.—It has been stated previously that the atom of any substance consists of a positively-charged nucleus round which are rotating one or more negative electrons. The electrons in an atom containing several of them are not necessarily moving along one single path or orbit; but they are divided into groups, the orbit of one group being quite near to the nucleus, and the orbits of other groups being more distant. Nor are the different groups all rotating in the same *direction* round the nucleus; also, the orbits are not necessarily in the same plane.

The same electrons do not always remain in the same atoms; those which traverse an orbit near to the outer boundary of an atom are less firmly held within the atom, they easily become detached, and, after wandering in all directions for a while, they become attached temporarily to another atom which is at the moment deficient in electrons. This wandering of electrons is particularly characteristic of metals; and, for this reason, metals are good conductors of electricity. Just as a swarm of bees may be moved onwards by a breeze, so these wandering electrons are moved onwards when an electric potential-difference is set up between two different points of the metal. Apply to the ends of a wire an electric force, such as can be done by joining it to the terminals of a voltaic cell, and the free

electrons will be urged to travel along the wire in a direction *from* the negative terminal of the cell *to* the positive terminal. This drift-motion of electrons along a wire is called an **electric current.** Even when the current is comparatively small, the number of electrons involved is enormous; thus, a 16 candle-power metal filament lamp may require a current of only one-tenth of an ampere[1]; the number of electrons passing any one point of the filament in each second is probably 600,000 million million!

It has become a universal custom to say that an electric current consists of a flow of *positive* electricity along a wire, *from* the +ve pole of a cell *to* the −ve pole; but, in reality, a current is a **flow of negative electrons along the conductor from the −ve pole of a cell to the +ve pole.** In spite of this recent discovery it is probable that the older method of stating the direction of a current will continue to be used. "Insulators" insulate because they have no wandering electrons.

How to distinguish the terminals of a battery.—Sometimes only the terminals of a cell, or of a battery of cells, are visible, and the terminals are not marked; or, it may be desired to take current from terminals connected to the electric-light supply (where the supply may be *direct-current*, and not *alternating-current*). In order to find which terminal is *positive*, the magnetic effect of the current on a neighbouring compass-needle (Expt. 295, p. 481) may be used. But when current is to be taken from the electric light mains, this method must be used with great care: the wire joining the terminals must include in its circuit a 30-50 candle-power lamp, otherwise too much current will traverse the wire, and the main fuses will be destroyed. A safe method to adopt is as follows: Lay on a glass sheet a small strip of *red litmus paper*, which has been wetted. Take two lengths of insulated wire, and connect one of these to each terminal. Hold the free ends of these wires about half an inch apart and in contact with the litmus paper. *The paper in contact with the negative terminal is turned blue.*

[1] The practical unit of current strength, called the *ampere*, is defined on pp. 495 and 503.

Commutators and rheostats.—It is advisable to have a simple appliance for reversing the direction of a current in a wire without interchanging the connections of the wire to the battery; any appliance designed for this purpose is termed

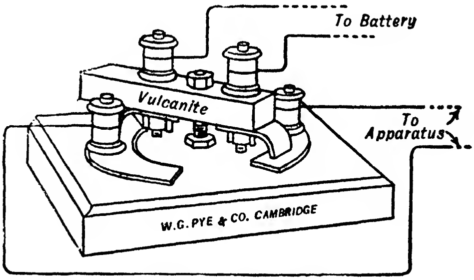


FIG. 326.—A commutator.

a **commutator**. One of the several available forms is shown in Fig. 326. It consists of a square block of wood on which are mounted two short circular arcs of thick sheet copper, each with a binding-screw. To these screws the wires leading to the apparatus are connected. Another pair of binding-screws, joined to the battery terminals, are mounted on a vulcanite rod which can be rotated (round a central pivot) to the right or left, so as to bring the metal brushes joined to the binding-screws into contact with either of the circular arcs. When the rod is in a mid-way position, the brushes do not touch the metal arcs, and the circuit is then "broken."

It is desirable to include in all experimental circuits a **rheostat**, which may be described as an adjustable unknown resistance: this is a precautionary measure for preventing too

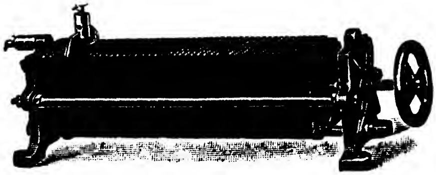


FIG. 327.—A carbon-block rheostat.

strong a current from being taken from a cell or battery. The rheostat shown in Fig. 327 consists of a number of thin slabs

of hard carbon, which can be squeezed together more or less tightly by means of a hand-screw. The more tightly the slabs are compressed the less is the resistance offered by the slabs to a current passing through them, and the stronger therefore is the current.

Methods of connecting cells.—The conventional method of representing, in diagrams, one or more accumulators or voltaic cells is shown in Fig. 328. A long thin line represents

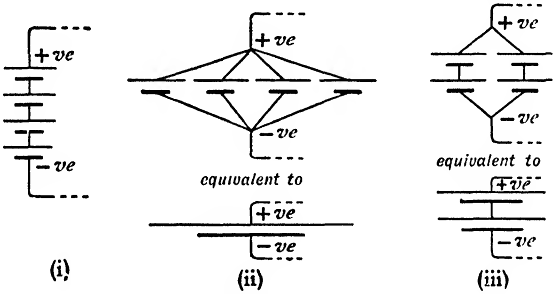


Fig. 328.—Methods of connecting cells

the +ve element or plate, and a short thick line the −ve element. Fig. 328, i, represents four cells joined **in series**, *i.e.*, the −ve plate of one cell is joined to the +ve of the next, and so on. The potential-difference between the +ve terminal at one end of this 'battery' and the −ve terminal at the other end will be four times as great as that which would be obtained by using only one cell. In this arrangement, *all* the current traversing the circuit has to pass through each cell.

An alternative method of connecting the cells is shown in Fig. 328, ii, where all the +ve elements are joined together, and all the −ve elements: this arrangement is termed **in parallel**. The potential-difference between the terminals of the battery will be the same as if one cell only had been used; in fact, the group of cells is equivalent to one large cell, with plates four times the size of those in one of the constituent cells. Indeed, the same potential-difference would be obtained if a very small cell, no larger than a thimble, were used; but a larger cell has other advantages which will be explained in a subsequent chapter. The potential-difference depends only

upon what metals and liquid are used in the construction of the cell ; and it is quite independent of the size of the cell.

Fig. 328 (iii) represents another method of grouping the four cells. The potential-difference between the terminals is equal to twice that obtained with a single cell ; and it is the same as that obtainable from two cells, each with plates of twice the area of those present in the individual cells.

Magnetic Effects of an Electric Current.

Oersted's experiment.—The effect which an electric current has upon a neighbouring compass-needle has been used already (Expt. 292) as a means of detecting the current. This effect was first observed by Oersted, of Copenhagen, in 1819.

Expt. 295.—**Action of the electric current on a magnetic needle.** Connect the poles of an accumulator, or large dry cell, to one pair of terminals of a commutator, and including a rheostat in series. Join the other terminals of the commutator by a long length of thin copper wire (cotton-covered). Make a simple diagram of the

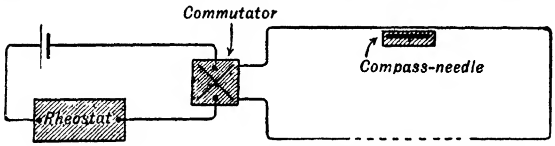


Fig. 329.—Diagram of a simple circuit, for the demonstration of Oersted's experiment.

circuit, such as Fig. 329. Stretch out a length of the wire horizontally and in a North-South direction. Close the circuit, place a compass-needle just below the wire, and observe in what direction the N-seeking pole is deflected. Reverse the direction of the current, and note that the same pole is deflected in the opposite direction. Repeat the observation with the needle just *above* the wire. Verify the results tabulated below :

Current passing from	Needle above (or below) wire.	North-seeking pole deflected towards
South to North	below	West
North to South	below	East
South to North	above	East
North to South	above	West

Ampère's Rule.—These effects of an electric current on a neighbouring magnetised needle are included in what is known as **Ampère's Rule :—Suppose a man to be swimming in the wire in the same direction as the current, and with his face towards the needle : the N-seeking pole is deflected towards his left hand.** This rule is extremely useful for determining the direction of a current the source of which cannot be seen (see p. 478).

Notice that, in the above experiment, a current from North to South *under* the needle has the same effect as a current from South to North *above* the needle. Hence the deflection, due to a current above the needle, is increased when the same wire is doubled back under the needle ; and evidently the deflection is still greater when the wire is wrapped over and under the needle several times (Fig. 330). This arrangement, by which an extremely weak current can be detected, explains the principle upon which some forms of galvanometer are constructed.

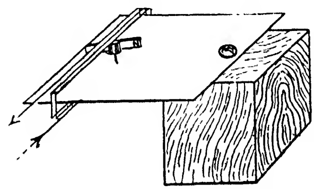


FIG 330.—HOW TO DETECT A VERY WEAK CURRENT. Several turns of the wire conveying the current surround the needle. Each turn of wire helps to deflect the needle.

It is important to notice that when the current ceases to pass along the wire the deflection of the needle simultaneously ceases. Hence the magnetic field is dependent for its maintenance upon the flow of the electric current, and it affords a distinct characteristic of the flow of electricity as compared with electricity at rest—for example, an insulated charged conductor has no effect on a neighbouring magnet.

Evidently an electric current sets up a magnetic field in the region near to it ; and the next matter to investigate is the nature of the field : are the magnetic lines of force circular, or have they any less simple form ?

EXPT. 296.—**Magnetic field due to a current.** A strong current (15-20 amperes) is necessary for this experiment ; and it can be carried out successfully only if a battery of large accumulators is available. The circuit should include an *ammeter* (for observing the strength of the current) and a rheostat. Bore a small circular hole through a piece of smooth white cardboard, and clamp it in

a horizontal position. Thread through the hole a length of thick copper wire, and clamp it, above and below the cardboard, so that it is vertical. Sprinkle fine iron filings on the cardboard, complete the circuit, and gently tap the cardboard. Break the circuit, and notice that the filings are arranged in circles concentric with the wire (Fig. 331).

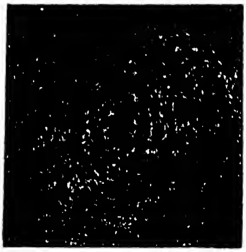


FIG. 331.—Map of the magnetic field perpendicular to a wire conveying a current.

Which is the positive direction of the circular lines of force around a wire through which an electric current is passing? In other words, would a single north-seeking pole appear, to an observer looking down on the experiment, to travel round the wire in the same direction as the hands of a clock or in the opposite direction? It can be shown by experiment that **the positive direction of the lines of force appears to be clockwise to an observer looking along a wire which is conveying a current away from him.**

EXPT. 297.—**Direction of magnetic force due to a current.** Place a compass-needle on the cardboard (Expt. 296) and near to the wire. Complete the circuit and observe the direction in which the needle points when placed to the north, south, east, and west of the wire. Reverse the direction of the current and notice that, in each position, the direction in which the needle points is reversed. Adjust the connections so that the current is passing *down* the wire, observe the direction of the needle, and verify the rule expressed above.

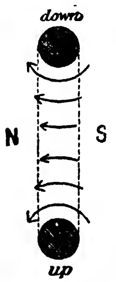


FIG. 332.—Magnetic field due to a current in a circular wire.

Magnetic field due to a current in a circular wire.—When a current is sent through a wire bent into the form of a circle the space enclosed by the wire is traversed by lines of force all travelling in the same direction. A horizontal cross-section through the centre of the circle is similar to Fig. 332, which represents the current passing down through the paper at A, and returning up through the paper at B. The lines of force shown in Fig. 332 are those due to short lengths of the wire

near to A and B, and they are all in the direction from right to left. Outside the wire circle the direction of the lines are from left to right. The lines of force due to all other portions of the wire proceed in the same direction—in fact, the diagram may be regarded just as readily either as a vertical or an inclined cross-section of the wire.

The magnetic field of the wire circle closely resembles that of a magnetised disc of steel, of which the thickness is equal to the diameter of the copper wire, the diameter equal to that of the wire circle, and magnetised so that the opposite faces of the disc have opposite polarity.

As a single turn of wire conveying a current behaves like a magnetised disc, then several turns of wire placed face to face, and each turn conveying the same current in the same direction, would be expected to show magnetic properties similar to a row of magnetised discs placed with faces of opposite polarity in contact—in other words, a spiral of wire conveying a current should resemble an ordinary bar-magnet.

EXPT. 298.—**Magnetic properties of a spiral carrying a current.**—Wind a close spiral of cotton-covered copper wire on a cardboard tube (5 cm. diameter and 20 cm. long). Support a sheet of paraffined paper horizontally so that its plane coincides with the axis of the tube, having previously cut away portions of the paper so as to fit symmetrically round the tube. Sprinkle iron filings over the paper, and obtain a map of the field due to a current passing through the spiral (Fig. 333).

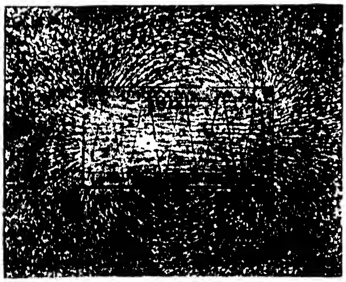


FIG. 333.—Magnetic field due to a spiral conveying a current.

Observe how closely this magnetic field resembles that of a bar-magnet. The hollow spiral enables us to obtain a map of the complete magnetic circuit, and the map indicates that the lines of force inside the spiral are parallel to the axis.

By means of a compass-needle verify that, when the end of the spiral is looked at directly, and the direction of the current round the turns of wire appears to the observer to be *clockwise*, the

polarity of that end is *South-seeking*: if the direction of the current is anti-clockwise, the polarity is North-seeking.

A simple method of remembering the relationship between the polarities of the ends of a spiral and the direction of the current is obtained by adding arrowheads to the ends of the capital letters S and N (Fig. 334); the arrows point in the direction of the current.

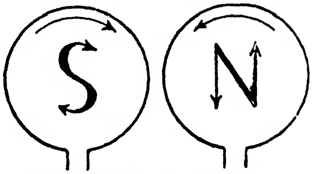


FIG. 334.—THE ENDS OF A SPIRAL CONVEYING A CURRENT EXHIBIT POLARITY. When the current is clockwise, the polarity is South-seeking.

When the space inside the spiral is filled with iron, the iron becomes strongly magnetised, and the lines of force entering and emerging from the spiral include both those due to the current traversing the spiral and those due to the magnetisation set up in the iron. This phenomenon, of the magnetisation of iron when placed in a magnetic field, has been mentioned previously (pp. 391B and 392), where a similar effect was obtained when iron was placed in a suitable direction in the earth's magnetic field. The electro-magnet, and the method of magnetising steel for permanent magnets (p. 368), are applications of the same principle.

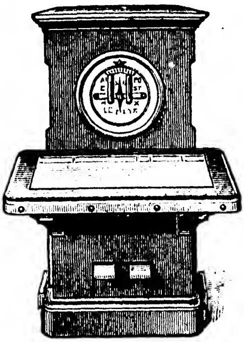


FIG. 335.—A single-needle telegraph instrument.

Application of Oersted's experiment to telegraphy.—The instrument represented in Fig. 335 is frequently to be seen in the telegraph offices attached to railway stations. In front of the disc of the instrument a vertical pointer is moving rapidly to and fro, and a tinkling noise is heard as long as the pointer continues to vibrate. This is the single-needle telegraph instrument, first introduced by Cooke and Wheatstone in the year 1837; and it is used for the purpose of transmitting messages between distant localities.

The principle of the instrument is similar to that of the simple experiment shown in Fig. 330, p. 482), but the coil of wire and the magnetised needle are fixed vertically instead of horizontally. The coil and magnetised needle are fixed inside the instrument; the end of the axle on which the needle is

mounted passes through the front of the instrument and carries the pointer.

One end of the coil is connected to a metal plate buried in the earth, and the other end is joined to a long insulated wire supported on telegraph poles leading to the distant station where there is a battery and a commutator. One terminal of the commutator is connected to the telegraph wire, and the other terminal is connected to a metal plate buried in the earth (Fig. 336). The two metal plates are always at the same

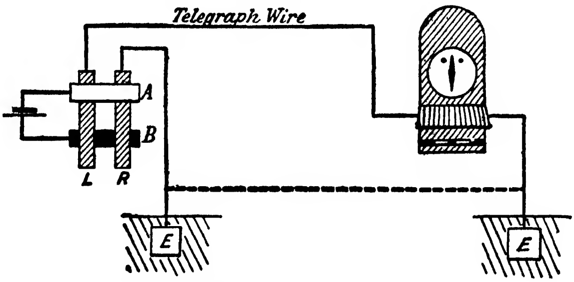


FIG. 336.—Diagram of a simple telegraph system.

potential (zero), and since the earth is a conductor it serves the same purpose as a very thick copper wire (shown by dotted line), the expense of which is saved thereby. By making use of the earth's conductivity in this manner a single wire only is needed in order to connect two offices together telegraphically.

A special form of commutator is used which consists of two strips of metal (L and R) which are capable of moving up and down like the notes of a piano.[1] In their upper position they are both in contact with a cross-piece of metal (A) connected to the −ve terminal of the battery. If L be pressed down its contact with A is broken and contact with B is made, thus the −ve terminal of the battery is connected to earth, while B (and therefore L) are connected to the +ve terminal, and a current will flow along the telegraph wire towards the distant end causing the needle in the instrument to deflect in a certain

[1] A working model of this commutator may readily be made from four strips of sheet brass. The fixed ends of L and R are raised above the base-board by screwing them to a narrow strip of wood about 1·5 cm. high. The strip A is fixed at both ends to raised blocks of wood of such a height that both L and R touch its under surface when at rest. The strip B is fixed to the base-board. In order to facilitate making connections, binding-screws should be used instead of ordinary screws.

direction. If L be released and R pressed down into contact with B a current in the reverse direction is obtained, and the needle will be deflected in the opposite direction. A recognised code of signals is adopted whereby the letters of the alphabet are represented by various combinations of left and right motions of the needle; thus a single swing to the left represents the letter *e*, a single swing to the right represents *t*, a swing to right followed by a swing to the left represents the letter *n*.

In order that the telegraph operator may be able to interpret messages by ear as well as by sight, two small pieces of tinplate are fixed on either side of one end of the pointer, the movement of which will cause the familiar tinkling sound when the instrument is working. The pieces of metal are cut to slightly different sizes in order that the deflections in opposite directions may be readily distinguished by the sound emitted.

The Morse system.—The **Morse sounder,** shown on the right of Fig. 337, consists of an electro-magnet *m* with a soft

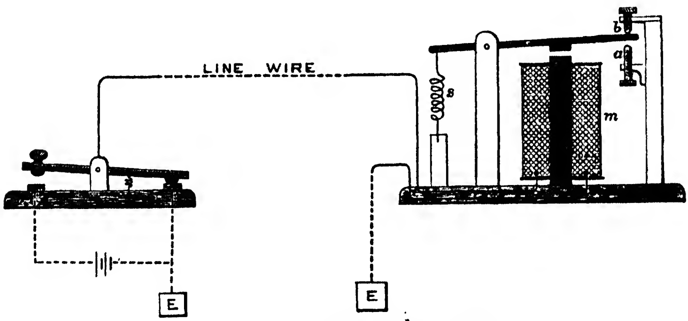


FIG. 337.—The Morse sounder and key.

iron armature fixed on a pivoted lever which has freedom of movement between two adjustable stops *a* and *b*. When no current is passing through the instrument a spring *s* keeps the lever in contact with the upper stop *b*. When a current passes, the lever is pulled down into contact with the stop *a*. The signals are based upon the duration of the interval which elapses between the striking of the stop *a* and of the stop *b*; and this interval depends solely upon the duration of the

current. The two signals, short and long, are usually termed 'dot' and 'dash' respectively; and the recognised relationship between these intervals is that the latter is three times as long as the former.

The dot-and-dash signals of the Morse alphabet correspond to a movement to the left and to the right respectively of the needle instrument.

A single dot represents the letter E, and a single dash represents the letter T. A dot before each of these represents the letter I and A; while a dash before each represents the letters N and M. A dot before each of these represents, the letters S, U, R and W; and a dash before each represents D, K, G and O. Thus:

• E	• • I	— • N
— T	• — A	— — M

• • • S	• — • R	— • • D	— — • G
• • — U	• — — W	— • — K	— — — O

By placing either a dot or a dash before each of the last eight letters a distinctive signal is given to all other necessary letters. Numerals are represented by combinations or groupings of five signals each.

In this system the operator receives the message *by ear*. More rapid transmission is obtained by a mechanical method in which a small disc, revolving in an inking fluid, is attached to the left-hand end of the lever. The depression of the lever brings the wheel into contact with a strip of paper which is moved by clockwork at a constant speed. In this manner the dots and dashes are permanently recorded on the paper.

The signals are transmitted by a **key**, represented on the left of Fig. 337. This consists of a metal lever mounted on a wooden stand. The line wire is connected to the middle of the lever. When not in action, a spring keeps the line wire in communication with the earth; when the front end of the lever is depressed a battery circuit is closed, and a current passes along the line wire to the sounder at the receiving end.

When the line wire is extremely long the current may not

be strong enough to operate the sounder, unless excessive battery power is used. In such a case a **relay** (R, Fig. 338) is

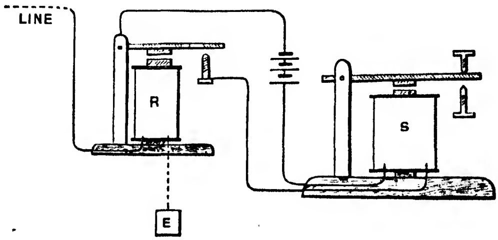


Fig. 338.—A telegraph relay.

inserted in the circuit near to the sounder S. The weak current passing along the line wire is passed through the relay, which is simply an electro-magnet with an armature attached to a lever. The depression of this lever puts into circuit a local battery which is strong enough to operate the sounder efficiently.

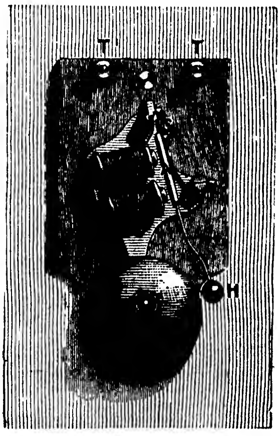


Fig. 339.—An electric bell. When a current passes through the coils of the electro-magnet M, the piece of soft iron A is attracted, the hammer H strikes the bell, the circuit is broken at C, and the spring S carries A back to its original position.

The electric bell.—The electric bell (Fig. 339) is also an application of the action of an electro-magnet. A flexible steel spring S carries a strip A of soft iron arranged across and near the poles of a horse-shoe electro-magnet M, and a side screw C is adjusted so as just to touch the side of the spring. The wires from the electric cell are joined to the terminals T and T′. A wire under the base-board joins T to the screw C; T′ is joined to one end of the coil of the electro-magnet, the other end of which is joined to the spring S. When a current passes through the circuit the electro-magnet attracts the strip of soft iron, and the circuit is broken at C; the

electro-magnet then ceases to attract the iron, and the spring returns to its former position, thus re-making the circuit. Each time the soft iron is attracted the hammer H strikes the bell.

In houses where each of several rooms and outer doors is provided with a ' bell-push ' for completing the circuit through the bell, an **indicator-board** is erected near to the bell for the purpose of indicating from which room the bell has been rung. The circuit from each room includes a small electro-magnet, fixed inside the indicator-board. Each electro-magnet has a soft iron armature *suspended* near to its poles (but not touching) ; and a numbered flag is attached to the armature. When a circuit is closed momentarily by the pressing of a bell-push, the corresponding armature is made to swing to-and-fro ; and this indicates in which room the bell has been rung. Fig. 340 represents the circuits through an indicator-board fitted for two rooms.

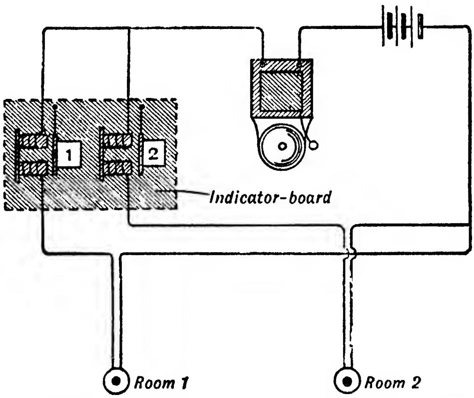
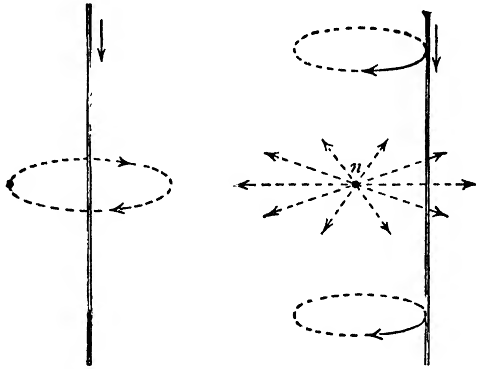


FIG. 340.—THE CIRCUIT OF AN ELECTRIC HOUSE-BELL. With its indicator-board.

Motion of a linear current in a magnetic field.—It has been explained previously that a magnet-pole tends to rotate in a circular path round a straight wire conveying a current (Fig. 341, i) ; here, the wire is fixed and the pole is free to move. When the pole is fixed, and the wire is free to

move, the wire tends to rotate in a circular path round the pole, and *in the same direction* (Fig. 341, ii).



(i) Fixed current, and moving pole **(ii) Fixed pole, and moving current**

FIG 341.—THE MOTION OF A CURRENT WHEN IN A MAGNETIC FIELD. (i) A single N-seeking pole, when near to a current passing down a vertical wire, tends to move in a circular path, and in the direction shown (compare Fig 331, p. 483). (ii) When the pole is fixed and the current is free to move, the latter tends to rotate round the pole, and *in the same direction*. At any instant, the direction of motion is perpendicular both to that of the current and to that of the magnetic force

The reason for this may be understood by a consideration of the resultant magnetic field due to a magnet and to a straight conductor conveying a current. Fig. 342 represents the field

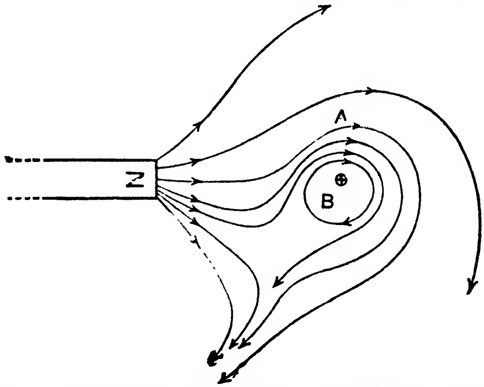


FIG. 342 —The resultant magnetic field due to a magnet and to a current traversing a straight wire fixed perpendicularly to the paper.

due to a current passing vertically downwards through the paper and near to the N-seeking pole of a bar-magnet. In the region marked A the directions of the two fields practically coincide—both are from left to right—and the resultant field is stronger than either of the components; the lines of force here, therefore, are relatively crowded together. In the region marked B, the directions of the two fields are practically opposite, the resultant field is relatively weak, and the lines of force are far apart: there must be a 'neutral point' not far from this region. These conditions give to the resultant lines of force the contour shown in the diagram.

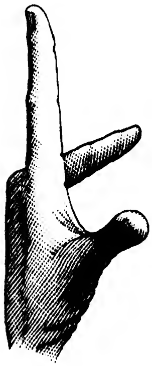


FIG. 343.—Fleming's left-hand rule.

If to these lines of force we attribute their property of *tending to shorten*, it is readily seen that the conductor conveying the current will be urged to move from the region A towards the region B. When the magnet is reversed, or when the direction of the current is reversed, the wire will tend to move in the opposite direction.

Fleming's Left-hand Rule is a useful means of remembering the direction in which the conductor tends to move:—Hold the thumb and forefinger of the left hand as fully extended as possible and bend the middle finger at right-angles to the palm (Fig. 343). If the **F***orefinger represents the direction of the magnetic* **F***ield, and the* **M***iddle finger represents the direction of the current* **I** *then the* **T***humb will indicate the direction of* **M***otion.*

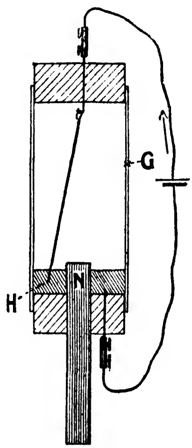


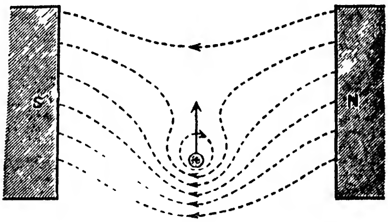
FIG. 344.—Apparatus for showing rotation of a linear current round a magnet pole.

This rotation of a current round a magnetic pole can be demonstrated by the apparatus shown in Fig. 344. G is a glass tube (30 cm. × 4 cm.) closed at both ends with corks. A cylindrical bar-magnet is fixed through the centre of the lower cork with its north-seeking pole uppermost and projecting a short distance into the tube;

a copper wire is also fixed through the same cork. A thick wire bent into the form of a hook is passed through the centre of the upper cork, and supports a thin wire the lower end of which dips into the mercury (H).

The experiment is more effective if the bar-magnet is replaced by a rod of *soft* iron which, below the cork, is wound with a number of turns of cotton-covered thick copper wire, thus forming an electro-magnet. By using a separate battery for this, the effect of reversing the polarity of the magnet can be shown. It can be demonstrated also that the rapidity of rotation is increased by increasing the intensity of the magnetic field, and by increasing the strength of the linear current.

The motor principle.—The principle of an electric motor depends entirely upon the force which acts upon a conductor, conveying a current, when situated in a magnetic field. The one essential condition is that the direction of the magnetic lines of force must *not* be parallel to the conductor: if these are parallel, the force is zero; and it is a maximum when the



(iii)

FIG. 345.—The motor principle.

directions are at right angles. Fig. 345 represents a conductor conveying a current *downwards*, and supported in the magnetic field between two opposite magnet-poles. In the absence of a current, the magnetic lines of force will be practically straight lines; but, when a current passes down the conductor, the resultant magnetic field will have the character shown in the diagram, and the conductor will be urged in the direction of the arrow. Verify this direction by applying the *left-hand rule.*

A simple experiment by which this can be demonstrated is shown in Fig. 346. Each of two corks is pierced with a piece of copper wire, and a length of thin copper *tinsel* is soldered to the wires. The corks are clamped so that the tinsel hangs vertically and *quite loosely*. Two large bar-magnets are supported horizontally, in the position shown. The tinsel forms part of a circuit which includes an accumulator, a rheostat and a commutator. When a current passes *down* the tinsel, it will be bulged outwards in the direction shown; and, when the current is reversed, it is bulged in the opposite direction.

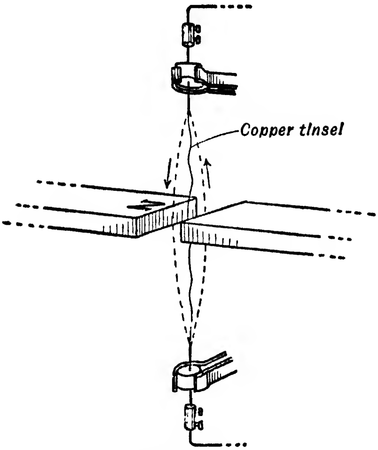


FIG. 346.—A method of demonstrating the motor-principle.

The same principle may be applied to a rectangular coil of wire (Fig. 347) which can be rotated round a horizontal axis O. The two ends of the coil are joined to the two halves

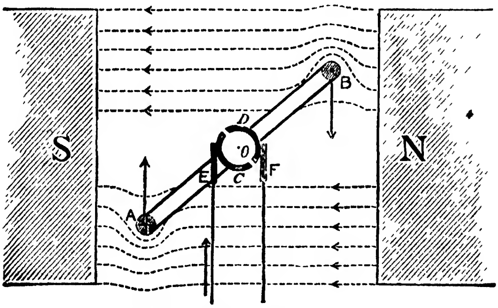


FIG. 347.—The principle of the motor.

(C and D) of a split ring, called the *commutator*, which is fixed to the coil and in front of it. Two fixed metal brushes, E and F, rub the surface of the commutator as it rotates, and the current is conveyed to the coil through the brushes. Current entering by the brush E passes to C, then *down* the horizontal wire A; it returns by the wire B, to D, and leaves through

the brush F. The character of the resultant magnetic field is indicated, and the forces acting on the current tend to move A upwards, and B downwards. These forces continue to be effective until the coil is vertical; but, in that position, the current will become reversed, since D will be in contact with brush E, and C with brush F; the forces therefore will be reversed, and the coil will continue to rotate.

The *torque*, or turning moment of the forces, is a maximum when the coil is horizontal, and it is zero when the coil is vertical. This variation in the torque during each half revolution would give a very irregular motive power; and, of course, feeble too, unless the field is very intense and the current strong. In practice, therefore, a large number of coils are wound on the same axle, and uniformly distributed so that a uniform torque is obtained. The coils are wound on a cylindrical *drum* consisting of thin sheets of soft iron fixed together. This drum serves both to support the coils and to increase the magnetic field by substituting for air a continuous iron path—except for the narrow air gaps required for free rotation. Just as the number of coils is increased, so must the number of parts into which the commutator ring is split increase. The drum, and the coils on it, is termed the **armature**.

The absolute unit of current, the ampere, and the coulomb.—The force which acts upon a conductor conveying a current, when supported in a magnetic field and at right angles to the direction of the lines of force, depends upon (i) the intensity of the magnetic field and (ii) the strength of the current. This fact makes it possible to obtain a definition of the unit of current which is based upon the fundamental units of the centimetre and the dyne. Suppose that, in Fig. 348, a uniform magnetic field has an intensity equal to 1, and that a straight conductor, supported in this field with its length perpendicular to the lines of force, is conveying a steady current. *When the current*

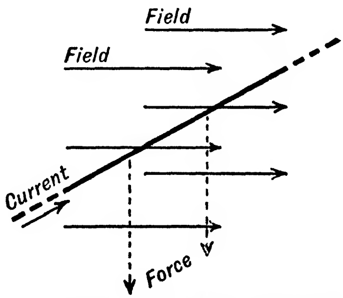


FIG. 348.—A linear current, at right angles to the lines of force of a magnetic field, is acted upon by a force.

is adjusted so that each centimetre length of the conductor is acted upon by a force of 1 *dyne, the strength of the current is said to be equal to one absolute unit.* In general, when the intensity of the field is equal to B units, and the strength of the current is I absolute units, the force acting on each centimetre length is BI dynes. For reasons which will be explained in a subsequent paragraph, the *practical unit* (which is called the **ampere** *) is equal to one-tenth of this unit.

The *quantity* of electricity, transmitted in 1 second past any point in a conductor which is traversed by a current of 1 ampere, is called the **coulomb**. Hence

$$\text{Quantity (in coulombs)} = \text{Current (in amperes)} \times \text{Time (in seconds)}.$$

Several instruments used for measuring the strength of a current depend upon the principle of Fig. 348. But, in at least one important example (viz. the 'tangent galvanometer') the conductor conveying the current is *circular* instead of linear: it will be well, therefore, at this stage, to apply the above principle to a conductor bent into the form of a circle.

Imagine a single N-seeking pole, of strength m units, to be placed at the centre of a wire circle, of radius r cm. (Fig. 340).

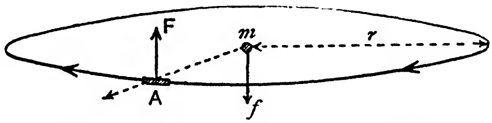


FIG. 349.—The force acting on a magnet-pole supported at the centre of a circular conductor conveying a current.

The intensity of the magnetic field, due to the magnet-pole alone, at any point A of the conductor is m/r^2; and if the strength of the current is I absolute units, the force F acting on each centimetre length is $m\mathrm{I}/r^2$ dynes. But the total length of the wire conductor is $2\pi r$ cm.; hence the total force acting on the conductor is $(2\pi r \times m\mathrm{I}/r^2)$, or

$$2\pi m\mathrm{I}/r \text{ dynes.}$$

* Sometimes the strengths of small currents are expressed in terms of either the **milliampere**, which is the one-thousandth part of the ampere, or the **microampere**, which is the one-millionth part of the ampere.

The direction of the force may be deduced by the *left-hand rule*; and it might be actually measured by suspending the coil from the arm of a balance.

Suppose, now, that the coil be fixed, and that the magnet-pole be free to move, the force f acting on the pole will be equal to the above, but in the opposite direction. If the magnet-pole has unit strength (*i.e.* if $m=1$) then

$$f=2\pi I/r \text{ dynes.}$$

This is, by definition, **the intensity of the magnetic field at the centre of the coil.**

When the conductor consists of a long length of wire wound into n circular turns, of *average* radius r cm., the intensity of the field at the centre is n times as great, and

$$f=2\pi n I/r \text{ dynes.}$$

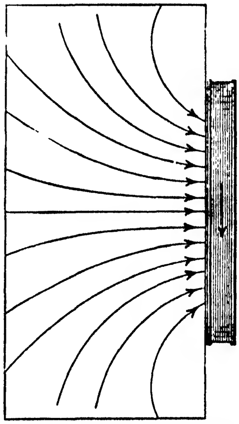


FIG. 350.—The magnetic field due to a current traversing a circular coil of wire.

Fig. 350 represents the character of the magnetic field due to a circular coil of wire conveying a current. A map of the field may be obtained, by the compass-needle method, by supporting a small drawing-board and sheet of paper horizontally and along a diameter of the circular coil. (When plotting the field, the board and coil must be adjusted frequently so that the compass-needle is always in the magnetic meridian; otherwise, the lines of force drawn will represent the *resultant* field due to the earth and to the current.)

EXERCISES ON CHAPTER XXXVII.

1. Why do we believe that the electricity obtained from an electrical machine is the same as that obtained from a Daniell cell? Explain the differences observed in the two cases.

(Lond. Univ. Matric.)

2. A plate of pure zinc, and a plate of copper, are dipped into dilute sulphuric acid, and then connected by copper wire. What changes take place in the plates, wire, and acid, when the circuit is closed?

3. Explain why the original simple cell discovered by Volta is no longer used as a source of current, and describe the method used in some other form of cell to overcome the defects of the original type.

For what purposes are primary batteries used at the present time ? (Lond. Univ. Matric.)

4. Describe the simple voltaic cell, and point out its defects. Explain the action of a cell suitable for use with an electric bell. (Cen. Welsh Bd.)

5. What is meant by polarisation in electric cells ? What means are taken to prevent this in Daniell and Leclanché cells ? Describe in detail the chemical changes going on in a Daniell cell when the current is flowing. (Camb. S.C.)

6. Describe and explain two methods for finding which is the positive pole of a voltaic cell supposing the cell to be boxed in so that only the terminals are accessible. (Bristol, 1st S.C.)

7. A long straight wire is stretched on a table in the direction of the magnetic meridian, and a dip circle, with its plane parallel to the magnetic meridian, is placed on the table near to the wire and on the west side of it. Will the dip of the needle be altered when an electric current is passed along the wire from south to north, and, if so, how ? Give reasons.

8 Two long wires are placed parallel to each other in the same horizontal plane and in the magnetic meridian. A magnetic needle capable of turning in any direction about its point of suspension is placed exactly half-way between them. How will it behave if the same electric current flows through the easterly wire from south to north, and through the westerly wire from north to south ? (The action of the earth on the magnetic needle may be neglected.)

9. Describe two experiments which indicate that a coil through which a current is passing behaves like a magnet. (Lond. Univ. Matric.)

10. A wire lies east and west (magnetic) immediately over a compass-needle. How is the direction in which the needle points affected when a *strong* current flows through the wire (1) from west to east, (2) from east to west ?

11. Draw a plan showing how the current must circulate in the coils of a horse-shoe electro-magnet, to make the poles (*a*) both north, (*b*) one north and the other south.

12. A current flows down a vertical wire, and is of such strength that at a distance of one foot from it its magnetic field is equal to the horizontal field of the earth. Indicate in a diagram the directions in which a freely suspended compass-needle would set if carried round the wire at a distance of one foot from it, when the needle is N., N.E., E., S.E., S., S.W., W., and N.W. of the wire.

13. A magnet is placed at the centre of a circular coil of wire through which a current is passed. What is the direction of the force acting on the north pole of the magnet, and how does the force depend on the direction of the current ?

14. A small compass needle is suspended at the centre of a vertical copper ring through which a current is passed. How is the needle affected by the current (i) when the ring is in the magnetic meridian, and (ii) when it is at right angles to the magnetic meridian?

What are the forces acting on the needle in each case?

15. A wire is stretched from east to west (magnetic). How, without breaking it, can you test whether, and in what direction, an electric current is passing through it?

16. In order to find, experimentally, how the intensity of the magnetic field due to a current traversing a *straight* wire varies with the distance (*d*) from the wire, the following observations were taken with a vibration-magnetometer :—In the earth's field alone it described 30 vibrations in 115·2 seconds. When placed 9·8 cm. to the west of a vertical wire conveying a current *downwards*, it described 30 vibrations in 86·9 seconds; and when 19·2 cm. to the west of the same wire it described 30 vibrations in 98·1 seconds. Prove from these observations that the intensity of the field, due to the current alone, is inversely proportional to the distance from the conductor.

17. An electric current is flowing along a wire. You are given a pivoted compass needle, and are required to find out by its aid which way the current is flowing. How would you proceed (*a*) if the wire in question lies horizontally; (*b*) if the wire runs vertically; (*c*) if the wire is coiled up in a circular coil or open hank?

18. The lines of force of a uniform magnetic field are horizontal, and their direction is from East to West. In this field is suspended a vertical wire traversed by a steady current downwards. By means of a diagram, show the nature of the resultant field, mark the conductor, and deduce the direction in which the conductor will tend to move.

19. A fixed vertical wire conveys a current downwards. Near to it is suspended another vertical wire, also conveying a current downwards. Make a diagram to show the nature of the resultant field, and deduce whether the suspended wire will tend to move; and, if it does so, in what direction will it move? What will be the effect of reversing the direction of the current in the suspended wire?

CHAPTER XXXVIII.

CHEMICAL AND HEATING EFFECTS OF A CURRENT.

CHEMICAL EFFECTS OF AN ELECTRIC CURRENT.

Electrolysis.—All conductors of electricity may be divided into two groups,

(i) *metals (solid or molten), mercury, and liquids which are not decomposed when a current passes through them, and* (ii) *those compounds, whether fused or in solution, which undergo decomposition by the current.*

The latter are termed **electrolytes**; and when traversed by an electric current, they are said to undergo **electrolysis**. Dilute sulphuric acid, hydrochloric acid, and chemical salts (*e.g.* copper sulphate, sodium chloride, sodium sulphate, etc.) are typical electrolytes. Perfectly pure liquids, *e.g.* water, sulphuric acid, and alcohol, are not capable of electrolytic decomposition.

A current is conveyed to and from an electrolyte by immersing in it rods or plates of a metal; these are termed the **electrodes**. That by which the current enters is termed the **anode**, and that by which it leaves is termed the **cathode**. The elements (or groups of elements) liberated are termed **ions**; the ion liberated at the anode is termed the **anion**, and that liberated at the cathode is termed the **cation**.

Fig. 351 represents a simple form of apparatus for demonstrating the principles of electrolysis. Two strips (about 8 cm. × 1 cm.) of thick sheet lead are cut, and a short thick copper wire is soldered to each strip. The free end of each is bent upwards, like the letter J. With a small dish of melted sealing-wax, the blade of an old pocket-knife, and a Bunsen

flame, the strips (except about 2 cm. of the extreme end) are coated with a thin layer of the wax. The wax is an 'insulator,'

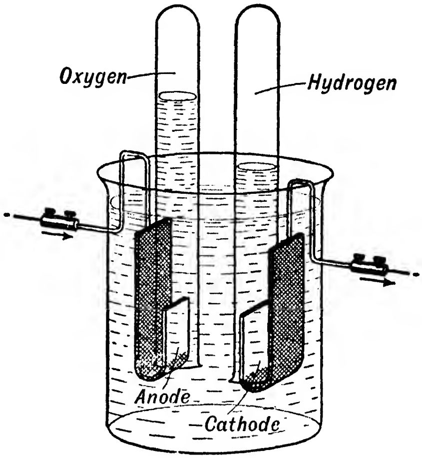


FIG. 351.—Apparatus for demonstrating the principles of electrolysis.

and the coating thus applied prevents the current from entering or leaving the liquid at any point of the metal which is thus covered. The copper wires, when bent, serve to support the strips inside a wide beaker. In other forms of apparatus, strips of platinum foil are used instead of lead; but this metal is expensive, and therefore not always available.

Tap water is not a good conductor, but its electrolysis is possible when a small quantity of sulphuric acid is added to it: 5 c.c. of strong acid added to each litre of water is suitable. Three dry cells joined in series, a rheostat, and the electrolytic apparatus are joined together by means of cotton-covered copper wire. The gases liberated from the anode and cathode can be collected in two test-tubes previously filled with the dilute acid and inverted over the ends of the electrodes. After the current has passed for several minutes, it is noticed that the volume of gas given off from the anode is only about one-half of that given off from the cathode; and, on testing the gases, it is found that the former gas is *oxygen*, and that the latter is *hydrogen*.

The student will remember that the accumulation of hydrogen on the copper plate of a simple voltaic cell causes *polarisation* (p. 473), and an opposing E.M.F. (or *back* E.M.F.) is

thereby set up, since the hydrogen is a readily-oxidizable element, and behaves in a similar manner to the zinc plate of a simple voltaic cell.

In the water voltameter this back E.M.F. is set up. If E = total E.M.F. of battery, and E' = back E.M.F. in the voltameter, the resultant E.M.F. for the complete circuit is $E - E'$, and the magnitude of the current obtained depends directly upon the magnitude of this resultant E.M.F. If $E' = E$ then no current will be obtained. In the case of a water voltameter $E' = 1{\cdot}47$ volts, so that the E.M.F. of the battery must be greater than this in order to electrolyse water. This explains why a single accumulator (E.M.F. $= 2{\cdot}0$ volts) will electrolyse water, and why it is necessary to use at least two Daniell cells (E.M.F. $= 1{\cdot}07$).

Many chemical compounds, when dissolved in water, also are decomposed by an electric current. *The metal contained in the compound is always deposited on the cathode* ; and, in this sense, resembles hydrogen in the electrolysis of water. Upon this principle depend the processes of gold-plating, silver-plating and nickel-plating. Copper-plating is perhaps the simplest of these processes ; and it may be demonstrated with the apparatus of Fig. 351, using a 10 per cent. solution of copper sulphate (' blue vitriol ') with 5 c.c. of strong sulphuric acid added to each litre of the solution. One dry cell is sufficient as a source of current. After the current has passed for 15-20 minutes, the deposit of copper on the cathode is evident. When lead electrodes are used, nothing appears to happen at the anode if the current is weak ; but, with a fairly strong current, oxygen is liberated, and in sufficient quantity to be identified by a chemical test. Supposing, now, that the direction of the current is reversed, the anode and cathode are reversed, and the deposit of copper disappears from the one electrode and appears on the other.

All processes of electrolysis are examples of the *transformation of energy* from one kind to another kind. It has been stated previously (pp. 117-8) that *chemical action* and an *electric current* are two forms of energy : in these experiments the energy of an electric current is transformed into chemical action. Further, the same experiments have involved the

converse energy changes, for the current itself, derived from the battery, is due to the chemical action taking place within the cells. According to the principle of *Conservation of Energy* (p. 118) a given quantity of energy in the form of an electric current, of definite strength and passing for a definite time, should be convertible into a perfectly definite quantity of chemical action, and *vice versâ*, for no new energy can be created. Although stated in different terms, the truth of this was demonstrated by Michael Faraday.

Faraday's laws of electrolysis.—Faraday, in 1833, fully investigated the phenomena of electrolysis, and deduced the following laws :

(i) **The mass of an ion set free by a current is proportional to the quantity of electricity which has passed.**

Thus, the amount of chemical action due to a current depends directly upon both the strength of the current and also upon the time ; a weak current flowing for a given time is equivalent to a strong current flowing for a relatively shorter time.

(ii) **If several different electrolytes are included in the same circuit, the relative masses of the liberated ions are proportional to their chemical equivalents.**

The chemical equivalent of an element is the weight of it which will combine with, or replace, 1 part by weight of hydrogen. It is numerically equal to the atomic weight of the element, compared to that of hydrogen, divided by the valency ; the valency being the number of hydrogen atoms which will combine with, or are replaced by, one atom of the element.

Electro-chemical equivalents.—The electro-chemical equivalent (E.C.E.) of an element is the weight in grams deposited by the unit quantity of electricity (1 coulomb). The accurate determination of the E.C.E. of at least one element is important, since by the second law of Faraday this determination may be used for the purpose of calculating the E.C.E. of all other elements. Lord Rayleigh has found that one coulomb of electricity deposits 0·001118 gm. of silver. A current of one ampere flowing for one second will deposit this weight of silver : this

quantity has been found to be so accurate and trustworthy that it is used as a recognised definition of the **International Ampere**.

Since the chemical equivalent of silver is 107·07, the E.C.E. of hydrogen is 0·001118/107·07 = 0·00001046. In a similar manner the E.C.E. of any other element may be calculated.

ELECTRO-CHEMICAL EQUIVALENTS.

Element.	Chemical Equivalent.	E.C.E. (grams per coulomb).
Silver	107·07	0·001118
Hydrogen	1·00	0·00001046
Oxygen	7·935	0·00008291
Copper	31·54	0·0003294
Nickel	29·12	0·0003041
Gold	65·21	0·0006812

EXAMPLE.—If the E.C.E. of nickel be 0·000304, calculate how much electricity is required to give a coating of nickel 0·1 mm. thick to a surface of 1000 sq. cm. (density of nickel = 8·8 gm. per c.c.).

Volume of nickel = $1000 \times 0 \cdot 01 = 10$ c.c.

Mass ,, ,, = $10 \times 8 \cdot 8 = 88$ grams.

Quantity of electricity required = 88/0·000304 = 289,600 coulombs.

Voltameters.—The accuracy with which the E.C.E. of several elements is known enables the process of electrolysis to be used for the measurement of current: the method is particularly useful in the case of very weak currents. Any appliance devised for this purpose is termed a voltameter.

For the most accurate measurements of current, a **silver voltameter** is used: its usual form is a platinum dish, which serves as the cathode, containing an aqueous solution of silver nitrate. In this solution a silver plate is suspended horizontally by means of platinum wires; the silver plate is the anode.

Fig. 352 represents a convenient form of the **copper voltameter**; the two outer plates of copper form the anode, and the central plate is the cathode, which should be much smaller

than the anode plates. The plates hang from copper wires C which are supported by two vulcanite rods, V and V′. The solution used is a 15% solution of copper sulphate, to each litre of which 5 c.c. of concentrated sulphuric acid has been added. The cathode should be sufficiently large to allow 50 sq. cm. of surface for each ampere of current.

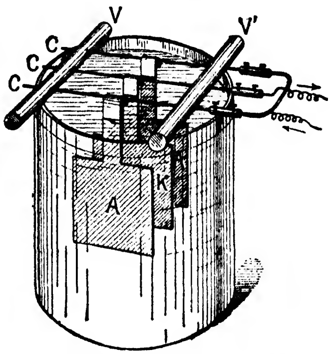


FIG. 352.—A copper voltameter.

The first of Faraday's laws may be demonstrated by means of a copper voltameter (i) by proving that the weight of copper deposited by a steady current is proportional to the *time* during which the current continues, and (ii) by proving that the weight of copper deposited in a given time is proportional to the strength of the current, as measured by a trustworthy ammeter. Or, these two tests may be combined, by proving that the weight of metal deposited is proportional to the product (*time* × *current*). The manipulation of such experiments is described in Expt. 299.

The second of Faraday's laws may be demonstrated by passing the same current through several different voltameters, connected in series, and arranged for the deposition of such metals as copper, silver, nickel, tin and zinc.

In a laboratory provided with *ammeters* or suitable galvanometers, a voltameter would not be used for measuring the strength of a current, owing to the considerable time required for a single observation ; but it serves as a most useful method of testing an ammeter, the accuracy of which may be doubtful. In the same sense it serves as a means of determining how the readings of a *tangent galvanometer* (p. 528) may be used for measuring the actual strength of the current which is passing through the instrument at the moment.

EXPT. 299.—**The correctness of an ammeter, verified by a voltameter.**—An ammeter with a scale graduated from 0-3 amperes is suitable for a laboratory test. Fit up the circuit as shown

in Fig. 353. Adjust the rheostat so that an appropriate strength of current is obtained. Break the circuit at the *key*. Remove the copper cathode, rinse it in water, dry it, clean the surface on both sides with fine sand-paper, dust it with a *clean*, *dry* duster, and weigh it accurately. Replace it in the voltameter. Have a watch (with a seconds' hand), or a stop-watch, near to the key, and note the time at the instant when the circuit is closed at the *key*. At once, note the reading of the ammeter as accurately as possible. Continue to observe the ammeter: if any change of current is noticed, adjust the rheostat so that the original strength of current is maintained.

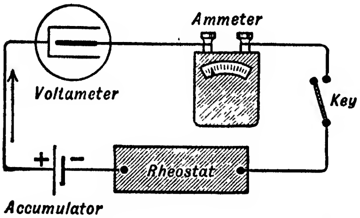


FIG. 353.—Verification of the accuracy of an ammeter

Continue this for at least 30 minutes. Have the watch ready, and note the exact time when the circuit is broken at the key. Remove the cathode (do not touch the deposited metal), *quickly* rinse it in water, then plunge it for a moment into methylated spirit contained in a beaker; dry it quickly by holding it *well above* a small Bunsen flame, put it aside to cool, then weigh it. From the data, calculate the strength of the current. (The methylated spirit is used in order to remove the water quickly. The deposited copper is granular, and not polished; and finely divided copper, *when wet*, rapidly oxidises in the air; and, therefore, rapidly increases slightly in weight. Hence the importance of removing the wetness quickly.)

EXAMPLE. **Verification of an ammeter** (reading 0-1·5 amp.).

Ammeter reading - - - -	**1·142** amp.
Weight of cathode (final) - - -	20·080 gm.
,, ,, (initial) - - -	18·950 ,,
Weight of copper deposited - -	1·130 ,,

Time, of deposition, =50 min.=3000 sec.

From Tables, the weight of copper deposited by 1 amp. in 1 sec. is 0·0003294 gm. (*i.e.* the E.C.E. of copper). Hence

$$\text{Current}=\frac{1\cdot130}{3000\times\cdot0003294}=\mathbf{1\cdot144}\text{ amp.}$$

The reading of the ammeter differs from this only by about 1 in 500; and the errors of experiment are probably greater than this. So, for practical purposes, the above ammeter may be regarded as correct.

Accumulators (or **secondary cells**).—The accumulator must be familiar to most students who have worked in a Physics Laboratory, or who have a wireless receiving outfit, with its so-called 'low-tension (L.T.) battery.' As this type of cell is an application of the principles of electrolysis, it is appropriate that the theory of its action should be described here.

The fundamental principle of the accumulator may be demonstrated by means of two lead plates (about 8 cm. × 6 cm.) immersed in dilute sulphuric acid (specific gravity = 1.20 approximately), and fitted up like the simple voltaic cell (Fig. 320, p. 470). It will be remembered that, in the electrolysis of water, using lead plates as electrodes, hydrogen is liberated at the cathode and oxygen at the anode : but all of the oxygen formed does not appear as gas, since a small portion of it combines with the lead on the surface of the anode, forming a dark grey *peroxide of lead* (PbO_2). Suppose that the current is allowed to pass for some time, so as to obtain an appreciable quantity of this peroxide—the apparatus being arranged as in Fig. 354, i ; and, then, that the battery is removed from the circuit, the voltameter reversed in position,

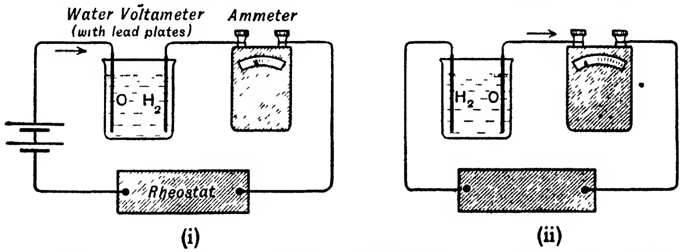


FIG. 354.—THE PRINCIPLE OF THE LEAD ACCUMULATOR: (i) charging the cell, and (ii) discharging the cell.

and the circuit closed (Fig. 354, ii), the ammeter will show that a small current is obtained from the voltameter. Evidently, this current passes through the voltameter in the *opposite* direction to that of the charging current : the plate which was the anode when being charged becomes the *positive* plate of the charged voltameter. During the discharge, an electrolytic process is going on, and, as usual, ***hydrogen travels with the current***, and migrates towards the lead-peroxide ; on this

it acts chemically, removes oxygen from it, and soon restores the original lead. As soon as this process is completed the current ceases. The lead plates in this experiment become more efficient with repeated use.

In the process of charging, the *energy* of the electric current is converted into *chemical action*; and the reverse operation takes place in the process of discharging. The original type of accumulator, devised by M. Planté, consisted of two sheets of lead rolled up together and separated by felt or similar material. Frequent charging and discharging results in the formation of porous or spongy lead on the surface of the plates, thus increasing the amount of available surface. With a view to accelerate this process of *forming*, the plates now used consist of lead *grids* (Fig. 355), into the spaces of which is firmly pressed a paste made from oxides of lead and sulphuric acid; in both cases lead sulphate is formed.

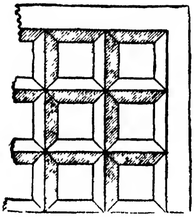


FIG. 355.—Grid of an accumulator.

The reactions which take place during the *forming* of the plates is as follows:

At anode: $PbSO_4 + O + H_2O = PbO_2 + H_2SO_4$.

At cathode: $PbSO_4 + H_2 = Pb + H_2SO_4$.

During the discharge of the cell the following reactions take place:

$$\text{At the positive plate: } \left.\begin{aligned} PbO_2 + H_2 &= PbO + H_2O, \\ PbO + H_2SO_4 &= PbSO_4 + H_2O. \end{aligned}\right\}$$

$$\text{At the negative plate: } \left.\begin{aligned} Pb + O &= PbO, \\ PbO + H_2SO_4 &= PbSO_4 + H_2O. \end{aligned}\right\}$$

Evidently, during the passage of the current through the cell from the negative to the positive plate, sulphuric acid is electrolysed. The hydrogen travels with the current and is liberated at the positive plate.

It will be noticed that, during charging, the quantity of sulphuric acid in the liquid of the cell is increased: the specific gravity consequently increases. During discharge, acid is taken from the liquid, and is used up in forming solid lead-sulphate on the plates: hence the specific gravity of the liquid diminishes. In fact, the specific gravity of the liquid

(as determined by means of a hydrometer) is often used as a means of deciding whether a cell is fully charged or in need of further charging.

Cells are made usually with several positive and negative plates arranged alternately and close together, the two outer plates always being negative. Fig. 356 represents a typical modern accumulator.

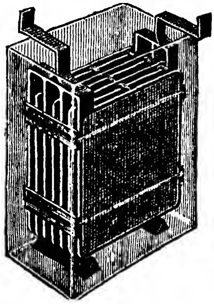


FIG. 356.—An accumulator.

Theory of electrolysis.—Recent discoveries as to the constitution of the atoms and molecules of matter provide a possible explanation of the phenomena of electrolysis. It has been stated previously (p. 429) that the atom of matter consists of a +ve *nucleus*, round which rotate one or more −ve electrons, each element having a characteristic number of such electrons, and that the different elements can be arranged in a Table according to their **Atomic Number**—this Number, in each case, representing the number of electrons associated together outside the nucleus.

These electrons are not all contained in one orbit; but, as the number increases, they tend to arrange themselves in groups, each group occupying an orbit at a different distance from the nucleus; and it would seem that one group never contains more than eight electrons. The single electron of the hydrogen atom, and the two electrons of the helium atom, occupy an orbit quite near to the nucleus. The additional electron of the next element in the Table takes its place in a wider orbit; and the same orbit is occupied by each additional electron until the number of them is eight. The further electrons of succeeding elements occupy a still wider orbit.

Consider, now, the case of *sodium chloride*—the compound formed by the union of the metal sodium and the gas chlorine. The atomic number of chlorine is 17; the inner orbit has two electrons, the next orbit eight electrons, and the outer orbit only *seven* electrons. The chemical activity of this element may be attributed to its eagerness to complete this outer ring. The atomic number of sodium is 11; and, therefore, its outer

ring has only one electron ; and this, probably, is but loosely held (Fig. 357, i). When chlorine and sodium come into contact, they combine vigorously : the chlorine atom draws the outer single electron of the sodium atom into its own outer ring, thus completing it, and the two atoms are firmly combined together (Fig. 357, ii).

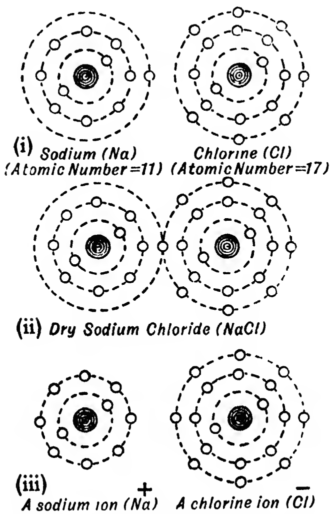


FIG. 357.—THE NORMAL ATOM AND THE ION. (i) Normal atoms of sodium and chlorine; (ii) the molecule of sodium chloride; and (iii) the molecule dissociated into ions.

The conditions, however, become modified when the solid is dissolved in water. A considerable fraction of the molecules become broken up : the chlorine atom separates from the sodium atom, *but it still retains the electron which previously was jointly held by the two atoms* (Fig. 356, iii). So, the chlorine atom becomes a −ly charged **chlorine ion** ; and the sodium atom (deprived of its outer electron) becomes a +ly charged **sodium ion**. This phenomenon of **dissociation**, if correctly stated, is perhaps due to the fact that water has a high 'specific inductive capacity,' and the force of electric attraction between the atoms is thereby largely reduced (p. 434). In the solution, during any short time interval, a number of these oppositely charged ions may re-combine ; but, during the same interval, an equal number of molecules will *dissociate* and thus maintain the number of ions.

The student of chemistry, knowing how readily sodium attacks water, may doubt the possibility of these sodium ions remaining, as such, in the water ; but, the sodium ion $\overset{+}{\textbf{Na}}$ may have chemical properties quite distinct from those of the normal sodium atom, Na.

Imagine, then, that two strips of platinum, connected to the poles of a cell or battery, are dipped into the solution.

The $\overset{+}{Na}$ ions will migrate down the potential-gradient towards the cathode, while the $\overset{-}{Cl}$ ions will migrate in the opposite direction towards the anode. At the cathode, the $\overset{+}{Na}$ ion picks up another electron, and becomes an ordinary sodium atom; this at once attacks a water-molecule, displaces a hydrogen atom, and itself forms *caustic soda*; the hydrogen gradually accumulates in bubbles on the cathode, and is finally liberated. At the anode, the $\overset{-}{Cl}$ ion gives up its extra electron, and becomes an ordinary chlorine atom. This gas dissolves in the water; but, in due time, the water becomes 'saturated' with it, and the gas is then visibly liberated from the surface of the anode.

In the electrolysis of water, in which a dilute solution of sulphuric acid (H_2SO_4) is used, the acid becomes dissociated into $\overset{+}{H}_2$ ions and $\overset{-}{SO}_4$ ions. The former migrate to the cathode, acquire electrons, and are liberated as hydrogen gas. The latter migrate to the anode and give up their electrons; each SO_4 group then attacks a water molecule, appropriates the hydrogen, and liberates the oxygen: thus,

$$SO_4 + H_2O = H_2SO_4 + O.$$

Similarly, in the electrolysis of copper sulphate, the solution contains $\overset{+}{Cu}$ ions and $\overset{-}{SO}_4$ ions. When platinum electrodes are used, the copper is deposited on the cathode as a metallic film, while the $\overset{-}{SO}_4$ ions arriving at the anode give up their charge, attack the water, and liberate oxygen. But, when copper electrodes are used (as in the copper voltameter), the $\overset{-}{SO}_4$ ions, after giving up their charge, attack the copper anode, each one forming another molecule of copper sulphate ($CuSO_4$), which dissolves in the water—thus maintaining the strength of the solution.

Generation of Heat by an Electric Current.

Heating effects.—A steady electric current traversing a fixed metal wire loses energy. As this energy cannot be lost it must assume some other form, and in equivalent quantity. The wire is fixed, and therefore cannot do 'mechanical' work (as in the electric motor); nor, for the same reason, does it acquire 'kinetic energy'; nor does the wire undergo any chemical change. As in most other cases of transformation of energy, the final state is **heat**, and this is the form in which

the energy of the current appears : *the wire becomes perceptibly warmer.* In order to understand this, we may imagine that the current consists of a vast number of **electrons** travelling at high speed along the metal wire ; even with a current so small as one ampere the number of electrons passing any point of the wire in each second is many millions of millions. In their passage through the metal they are constantly colliding with the atoms, and bombard them like rifle bullets bombarding a target. In the latter case, the bullets and target become warmer ; and the same effect is obtained when the atoms of a metal conductor are bombarded by a stream of electrons.

This principle is applied in many practical ways : *e.g.* in the incandescent **electric lamp**, where a current raises to white-heat the temperature of a very thin metal wire inside the lamp ; the electric **radiating-stove**, too, consists of long thin wires (of *nichrome*, an alloy of nickel and chromium) heated to dull redness by a current ; the safety-fuses (or ' cut-outs ') inserted in the lighting circuits in buildings, the electric cooking-stove and the electric iron used for domestic purposes, all depend upon the same principle.

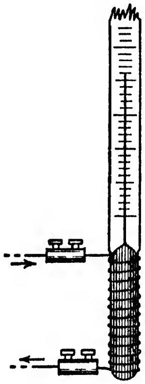


FIG. 358.—The heating effect of a weak current.

The effect can be shown by including a short piece of thin platinum wire in a circuit consisting of an accumulator and a rheostat : the platinum will glow brightly when the current is sufficiently strong. When the available current is very small, it can be demonstrated by the simple device shown in Fig. 358. About 12 turns of thin soft-iron wire (such as is used by florists for bouquets) is wrapped tightly round the bulb of an ordinary thermometer. This is clamped in a vertical position, and the wire is included in a circuit containing an accumulator (or a dry cell), a rheostat and an ammeter. As soon as the circuit is closed and adjusted so that the current is about 0·3 ampere, it will be noticed that the reading of the thermometer gradually increases, and continues to do so for several minutes ; finally the temperature becomes constant, when the rate at which the wire loses heat is equal to the rate at which heat is being generated by

the current. With a stronger current, the final reading of the thermometer will be higher.

According to Joule's Mechanical Equivalent of Heat (p. 225), 1 *calorie* of heat is equivalent to $4{\cdot}2 \times 10^7$ *ergs* of mechanical work. So, if the heat generated by an electric current is measured, a simple calculation will convert this into its equivalent, in *ergs* of work.

The *mechanical work* done in lifting a weight can be calculated, and expressed in ergs: this represents the *potential energy* acquired by the weight. If allowed to fall, it passes from a point of higher (gravitational) potential to a point of lower (gravitational) potential; and, when it reaches the ground, all of its original potential energy re-appears as *heat*. In fact, the ergs of work done in lifting the weight have been converted into an equivalent number of calories of heat. This transformation is analogous to the passage of an electric current along a conductor.

An electric current traverses a wire because one end of the wire is at a higher (electric) potential than the other end; and the quantity of energy expended depends upon (i) the quantity of electricity transmitted, and (ii) the difference of potential between the ends of the wire.

The absolute unit of potential-difference (electro-magnetic).—When discussing the electric field of force near to a charged body (p. 434), it was stated that the potential-difference between two points in the field is equal to unity when one erg of work is done by the electric force in transmitting unit quantity from the point of higher potential to the point of lower potential.

The same definition applies in the case of a current traversing a wire; but, the unit of quantity is not the same as before. The electrostatic unit (p. 433) is based upon the mutual repulsion of two similar charges; but, in the case of an electric current, the unit of quantity is based upon the magnetic effect of the current. The definition of this unit—called the *electromagnetic*, as distinct from the *electrostatic*—may be defined thus:—**Unit difference of potential exists between two points of a conductor when 1 erg of work is done by the electric force in transmitting unit quantity between the two points.**

The volt (or practical unit of potential-difference).—Until about the middle of the nineteenth century all electrical measurements were primitive, and the units in terms of

which the measurements were expressed were equally primitive. Thus, potential-differences (electromagnetic) were expressed in terms of that which is obtained between the terminals of a Daniell cell, which, at that time, was one of the most trustworthy standards available. But when Gauss introduced the *absolute* system of units, electrical measurements were placed on a more satisfactory footing. It was found that the absolute unit of potential-difference, as defined in the previous paragraph, was inconveniently small; and, so as not to disturb more than necessary the magnitude of the units previously used, it was decided to take, as a *practical* unit, one which is 10^8 times as great as the absolute unit, because this multiple gives a potential-difference nearly equal to that of the Daniell cell. To this practical unit the name **volt** was given. In terms of this unit, the potential-difference between the terminals of a Daniell cell is 1·07 volt (or $1{\cdot}07 \times 10^8$ absolute units).

Ohm's Law.—The units of current-strength and of potential-difference having been explained, it is appropriate to refer now to a most important relationship between these quantities. G. S. Ohm, in 1826, conducted experiments which resulted in a statement of the following simple relationship: **In any wire, at a uniform temperature, the current is directly proportional to the potential-difference between its ends**; or E/I is a constant ratio (where E and I represent the potential-difference and the current respectively).

The numerical magnitude of this ratio is termed the **resistance** of the wire (or other conductor). For example, in the case of a short thick copper wire, a small potential-difference will cause a large current; the ratio E/I therefore is small, and we say that 'the resistance of the wire is small.' When the wire is long and thin, the same potential-difference will cause a much smaller current, the value of E/I is larger, and we say that 'its resistance is higher.' If the units in which the three quantities are measured have the requisite relationship, then

$$\text{Resistance (R)} = \frac{\text{Potential-difference (E)}}{\text{Current (I)}},$$

or

$$I = \frac{E}{R}.$$

EXPT. 300.—**Demonstration of Ohm's Law.**—In Fig. 359 AB is a long uncovered wire, preferably of an alloy such as manganin, the resistance of which is not affected appreciably by change of temperature. This wire forms part of a circuit which includes two or three accumulators, a rheostat and an ammeter. If a low-reading ammeter is not available, a tangent galvanometer and commutator may be used for measuring the current (instructions for using a tangent galvanometer are given on p. 530). The potential-difference between the ends of AB is observed by means of a voltmeter.

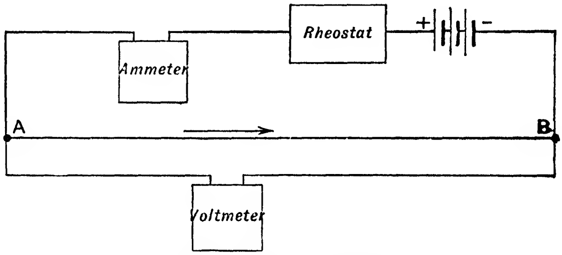


FIG. 359.—Demonstration of Ohm's Law.

Adjust the rheostat so that the circuit is traversed by the smallest current which can be read accurately on the ammeter. Note the readings of both ammeter and voltmeter. Increase the current and repeat both readings. Continue this process until several independent pairs of readings are obtained. Tabulate your observations thus :

Potential Difference (P.D.)	Current (I)	$\frac{P.D.}{I.}$

The absolute unit of resistance, and the ohm.—Previously to the introduction of the absolute system of electrical units, the unit of resistance was quite arbitrary. Measurements were limited to the small-scale experiments of the laboratory, and one unit used was 'one foot of No. 11 copper wire.' At a later date, when telegraph systems and other large scale work was becoming general, the unit was increased to '1 mile of No. 16 copper wire'; while, in France,

a unit frequently used was ' 1 kilometre of iron wire of 4 mm. diameter.' As it was so difficult to ensure uniform purity of these metals, and as their resistance varies according to the degree of purity, these units necessarily were very uncertain. Since, by distillation, the metal mercury can be obtained in a condition of uniform purity, Werner Siemens, in 1860, introduced as a unit of resistance ' a column of mercury, 1 metre long and 1 sq. millimetre cross-section, at a temperature of 0° C.' This was known as the ' Siemens mercury unit.'

Soon afterwards, and following the introduction of Gauss's absolute system, an experimental method was devised to determine the magnitude of the *absolute unit* of resistance. This was found to be far too small for convenient use, and that if it were 10^9 times as great it would correspond closely to the ' Siemens mercury unit.' So it was generally agreed that the new practical unit of resistance should be equal to 10^9 absolute units ; and to this the name **ohm** was given.

The **International Ohm,** now universally recognised, is defined as **the resistance of a column of mercury at 0° C, 106·3 cm. long, of uniform cross-section, and weighing 14·452 gram.**

It will be remembered that the *volt* is equal to 10^8 absolute units of potential-difference ; and the student will now understand the reason for the *ampere* being equal to *one-tenth* of the absolute unit of current. For, by Ohm's law,

$$\text{I (amperes)} = \frac{\text{E (volts)}}{\text{R (ohms)}} = \frac{(\text{E} \times 10^8) \text{ absolute units}}{(\text{R} \times 10^9) \text{ absolute units}}$$

$$= \frac{\text{E}}{\text{R} \times 10} = \frac{1}{10} \text{ absolute unit.}$$

Consequently, the *coulomb* must be equal to one-tenth of the *absolute unit* of quantity.

Work done by an electric current.—It has been stated (p. 513) that 1 erg of work is done when the absolute unit of quantity passes between two points of a conductor which have a potential-difference of one absolute unit. Hence, when Q coulombs pass between two points which differ in potential by E volts,

$$\text{work done} = (\text{Q} \times 10^{-1}) \times (\text{E} \times 10^8) = \text{QE} \times 10^7 \text{ ergs.}$$

As the erg represents an extremely small quantity of work, a unit 10^7 times as great, and called the **Joule**, is generally used ; hence

$$\text{work done} = QE \text{ joules.}$$

It is much easier to measure the strength of a current and a period of time than it is to measure the quantity of electricity conveyed by the current : it is more convenient therefore to express the work done in terms of the current-strength ; and, as $Q = It$,

$$\text{work done} = EIt \text{ joules.}$$

Since, by Ohm's law, $E = IR$, then

$$\text{work done} = I^2Rt \text{ joules.}$$

It has been stated previously that the work done by an electric current traversing a fixed wire reappears as heat, in equivalent quantity. According to the 'mechanical equivalent of heat,'

$$1 \text{ calorie} = 4{\cdot}2 \times 10^7 \text{ ergs} = 4{\cdot}2 \text{ joules.}$$

Hence, in the electric circuit,

$$\text{heat generated} = I^2Rt \text{ joules} = \frac{I^2Rt}{4{\cdot}2} \text{ calories.}$$

This constitutes what is known as **Joule's law,** which may be stated in the following terms: **The heat generated in a simple circuit is proportional (i) to the square of the current strength, (ii) to the resistance, and (iii) to the time during which the current continues.**

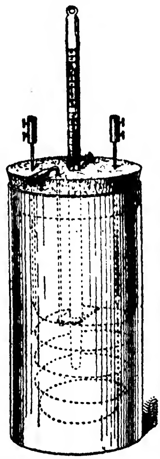


FIG. 360.—Apparatus for proving Joule's law.

A form of apparatus suitable for the approximate verification of Joule's law is shown in Fig. 360. The ends of a coil of thin manganin wire are connected to thick copper wires passing through a cork which fits a copper calorimeter. The coil is more convenient if wound as a long narrow spiral which bends round near to the bottom of the calorimeter—like a letter U. The cork also carries a thermometer (graduated to 0°·2 C.) and a stirrer.

EXPT. 301.—**Joule's Law.**—(i) Connect up the apparatus in series with an accumulator, a low-reading ammeter, a rheostat and a key : a stop-watch also is necessary. Adjust the rheostat

so as to obtain a current of suitable strength. Break the circuit. Pour into the calorimeter a carefully measured volume of cold water, and sufficient to immerse the *whole* of the spiral. Wrap a layer of cotton-wool round the calorimeter, to minimise the loss of heat by radiation. Stir the water, and note its temperature. Close the circuit and start the stop-watch simultaneously : read the ammeter and note the strength of the current. Maintain a constant current by means of the rheostat, and frequently stir the water. When the temperature has risen about 3° C., note the temperature and the time; but do not stop the experiment or the watch—let it continue until the temperature has risen another 3° C. or more ; then note the temperature and the time ; and stop the experiment. From these observations, verify that *the rise in temperature is proportional to the time.*

(ii) Adjust the rheostat so that the strength of the current is nearly twice as great as in the previous experiment. Break the circuit, empty the calorimeter, dry the inside of it, and pour into it *exactly* the same volume of cold water as used in the previous experiment. Proceed exactly as before, and allow the current to continue for *the same period of time* as in the previous experiment. Using the data of both experiments verify that *the rise in temperature is proportional to the square of the current.*

(iii) In order to prove that the heat generated is proportional to the *resistance*, it is necessary to use two appliances similar to Fig. 360 and fitted with coils which differ considerably in length. The resistance of the coils must be measured beforehand, by one of the methods described in a later chapter (p. 544). Weigh each calorimeter, and calculate its 'water-equivalent' (p. 179). Pour into each of them a carefully measured volume of cold water: or, if the calorimeters are not too large, the water may be weighed.

Connect the two coils *in series*, and in series with an ammeter, rheostat and accumulator. Pass a steady current through the circuit for a period sufficiently long to produce a considerable rise in temperature. Note the readings of each thermometer before and after the experiment. From the data obtained, verify that *the heat generated is proportional to the resistance.*

Electric power.—**Power** is defined as **the rate at which work is done.** The 'horse-power' is defined as 33,000 foot-pounds *per minute*, or 550 foot-pounds *per second.* In the case of a steady electric current traversing a metal conductor, the work done is equal to the product E*It* joules, where E is

the potential-difference (in volts) between the ends of the conductor, I is the current-strength (in amperes) and t is the time occupied (in seconds). The *power*, or rate at which the work is done, is calculated by dividing the quantity of work done by the time occupied in doing it ; hence,

$$\textbf{power} = \mathrm{EI}t/t = \mathrm{EI} \text{ joules per sec.}$$

The unit rate of work is one joule per second ; and this is called the **watt.**

Hence,

$$\textbf{power} \text{ (in watts)} = \text{volts} \times \text{amperes.}$$

The relationship between the Horse-power and the Watt may be derived in the following manner : Since 1 lb. weight $= 453 \cdot 6$ grams $= (453 \cdot 6 \times 981)$ dynes, and 1 ft. $= 30 \cdot 49$ cm., then

$$1 \textit{ ft. lb. per sec.} = (453 \cdot 6 \times 981 \times 30 \cdot 49)/10^7 = 1 \cdot 357 \textit{ joules per sec.} \text{ (or } \textit{watts}\text{).}$$

$$1 \text{ Horse-power} = (550 \times 1 \cdot 357) = 746 \textit{ watts} = 0 \cdot 746 \textit{ kilowatt.}$$

Hence, 1 *kilowatt* is approximately equal to $1\frac{1}{3}$ *horse-power.*

The power supplied for lighting houses, driving machinery and other public purposes is measured in terms of a unit called the **Board of Trade Unit,** which is a rate of working equal to 1000 watts, continued for one hour: this is briefly called the **kilowatt-hour.** It is the number of these units which is recorded on the dials of an electric meter.

Metal filament lamps.—At the present time the most efficient incandescent lamp contains a long thin filament of the metal *tungsten,* which is raised to white-heat by the passage through it of an electric current ; the filament is enclosed in a glass bulb from which practically all the air has been removed. In a 25 candle-power (c.p.) lamp, made for use on a circuit where the electric pressure is 230 volts, the filament is about 85 cm. long and 0·02 mm. diameter. Fig. 361 represents how this long filament is mounted in a zig-zag manner from the ends of wires which radiate outwards from the top and bottom of a glass rod.

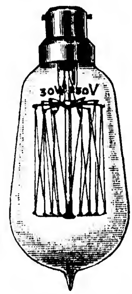


FIG. 361.—A metal filament electric lamp.

The metal tungsten is obtained from a mineral called *wolfram*, which is really a tungstate of iron. It is used in preference to other metals because of its high melting point (3500° C.) ; so it may be heated to an extremely high temperature without risk of being fused.

It is usual to have etched on the outside of the glass bulb numbers like '230-30W' ; this means that the lamp is made for use on a 230-volt circuit, and that its rate of consumption of electrical energy is 30 watts. The intensity of the light given by the lamp would be about 25 c.p. A lamp marked '230-40W' would be a 32 c.p. lamp for use on a 230-volt circuit ; the mark '110-60W' would mean that the lamp is intended for a 110-volt circuit, and that the light given by it would be 50 c.p.

The **efficiency** of a lamp is *the power consumed for each candle-power of light given* ; hence, if a 30-watt lamp gives 25 c.p. the efficiency is $30/25 = 1{\cdot}2$ watts per candle-power. Similarly, if a 60-watt lamp gives 50 c.p. its efficiency is $60/50 = 1{\cdot}2$ watts per c.p. The metal filament lamps are more economical than the old carbon filament lamp, in which the efficiency is 3-4 watts per candle-power. The life of a well-made lamp is about 1000 hours.

In recent years a filament lamp has been devised which requires only 0·5-0·8 watt of electric power for one candle-power of light ; it is sometimes called the **half-watt lamp**, but as its efficiency is seldom so low as 0·5 watt the term is misleading, and it is more correct to use the term **gas-filled lamp**. It resembles closely the ordinary lamp, but the bulb is filled with nitrogen or with argon instead of being a vacuum.

The strength of the current traversing the filament of an incandescent lamp may be calculated by dividing the number of *watts* (etched on the glass bulb) by the voltage of the supply. For example, a lamp marked '110-40W' will require a current equal to $40/110 = 0{\cdot}36$ ampere. Or, it may be determined experimentally by measuring the *heat* evolved in a given time, converting this by calculation into its equivalent in *joules*, and equating it to the electrical energy developed in the lamp, as expressed by the product EI*t*—which, of course, is also in joules. Here it is assumed that the voltage of the supply is known accurately.

Fig. 362 suggests a convenient method for this experiment. A wooden board AB (18 in. × 6 in.) has a wire (insulated) sunk in a shallow groove along its centre line. At different points of this wire are a 'lamp-holder' with wooden plug connector,

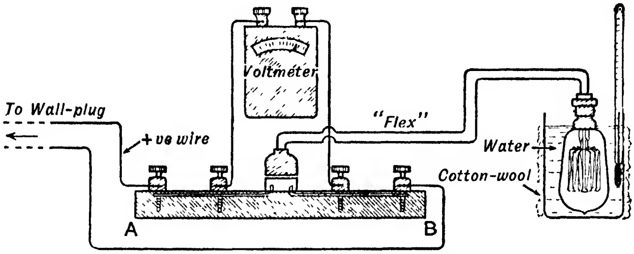


FIG. 362.—Indirect method of measuring the current traversing a filament-lamp.

and four binding-screws. The outer binding-screws are joined by 'flex' to a wall-plug and switch (or to an electric light pendant). The inner binding-screws are convenient for connecting to a voltmeter (if available), for verifying the voltage of the supply. The lamp to be tested is connected by 'flex' to the lamp-holder: it is clamped vertically and immersed in a known weight of water contained in a metal calorimeter of known weight. To minimise loss of heat by radiation, the calorimeter is protected by a cotton-wool jacket. The thermometer can be used as a stirrer.

EXPT. 302.—**Indirect measurement of the current traversing a filament lamp.**—Weigh the empty calorimeter. Place the lamp in position, and find roughly the volume of water required to immerse the *whole* of the glass bulb (but not more than this). Remove the lamp and dry it. Weigh the calorimeter with the water in it. Fit up the experiment as shown in Fig. 362. Stir the water, and note its initial temperature. Have a stop-watch ready, and start it at the instant when the current is switched on. Allow the current to continue until the temperature has risen about 10° C. Stir the water thoroughly, switch off the current and note the time; then quickly read the final temperature. The following data were obtained in a similar experiment:

Data, etched on the lamp, **230-40W.**

(i) Weight of calorimeter + water	- -	418·05 gm.
,, calorimeter	- - - -	114·80 ,,
,, water	- - - - -	303·25 ,,

Initial temperature $=21^\circ{\cdot}2$ C.
Final ,, $=33^\circ{\cdot}0$ C.
Rise in ,, $=11^\circ{\cdot}8$ C.

Time $=7$ min. $=420$ sec.
(Specific heat of copper, 0·093.)
Water-equivalent of calorimeter $=114{\cdot}8\times0{\cdot}093=10{\cdot}7$
Total water $=303{\cdot}25+10{\cdot}7=314$ gm. approx.
Heat generated $=314\times11{\cdot}8=3705$ calories
$=(3705\times4{\cdot}2)=$ **15,560 joules.**

Voltage $=230$ volts.
Electric work done $=EIt=230\times I\times420=I\times$ **96600 joules.**
Hence, $I\times96600=15{,}560$,
or $I=$ **0·161 ampere.**

(ii) **By calculation**: $230\times I=40$ watts,
or $I=40/230=0{\cdot}174$ *ampere.*

[It would be anticipated that the experimental result would be too low, (i) because of loss of heat by radiation and by evaporation of the water, and (ii) because no allowance has been made for heat absorbed by the glass bulb.]

The arc lamp.—Although the carbon-filament incandescent lamp is now practically obsolete, carbon is still an important element in the **arc lamp**, which is frequently used for street lighting, lighthouses, projection lanterns, etc. On allowing the ends of two sticks of compressed carbon to touch each other, when connected to the terminals of a battery giving a voltage of at least 30 volts, the current which passes across the point of contact generates considerable heat just where the ends of the rods touch, because the resistance at the point of contact is great. When the rods are now separated slightly, the current continues to pass as a highly luminous flame between the points. The maintenance of the arc is due to the fact

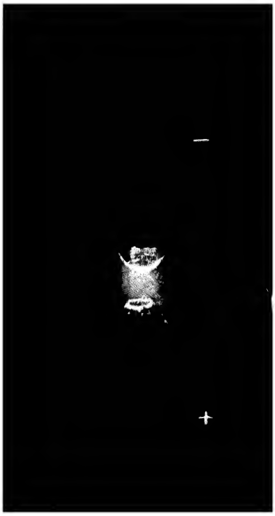


FIG. 363.—The electric arc.

that carbon volatilises at a very high temperature, and the vapour thus formed continues to conduct the current ; but the local resistance is still high, and the heat generated is therefore great.

The arc really consists of a stream of carbon particles passing from the positive terminal to the negative terminal, and for this reason the former becomes hollow or crater-shaped, and the latter pointed (Fig. 363). In contact with the air this carbon vapour slowly burns, and the rods become shorter ; if the arc becomes too long it is extinguished, and it is necessary therefore, to have some method of adjusting the distance separating them. In all arc lamps there is a complicated mechanism, operated by means of the current itself passing through coils with iron cores, for automatically creating the arc and controlling its length. The temperature is highest in the crater of the positive carbon, where it is nearly 4000° C. ; a fragment of the most refractory metal is quickly vaporised when placed in the crater.

EXERCISES ON CHAPTER XXXVIII.

1. Plates of copper and platinum are dipped into a solution of copper sulphate, and a current is passed through the cell from the copper to the platinum. Describe the effects produced ; also what happens when the current is reversed.

2. What is meant by the statement that the electrochemical equivalent of copper is 0·000328 gm. per coulomb ? Give any practical use to which this knowledge might be put.
(Joint Matric. Bd., S.C.)

3. State Faraday's laws of electrolysis, and explain how you could verify them experimentally. A current passing for 20 minutes through a copper sulphate cell deposits 0·472 gm. of copper on the cathode. Calculate the average strength of the current. [Electrochemical equivalent of copper=0·000328 gm. per coulomb.]
(Camb. S.C.)

4. A piece of metal weighing 200 grams is to be plated with $2\frac{1}{2}\%$ of its weight in gold. If the current strength is 1 ampere, how long will it take to deposit the required weight of gold ? [E.C.E. of gold=0·000681.]

5. A metal plate, having a surface of 200 sq. cm. is to be silver-plated. If a current of 0·5 ampere is used for a period of 1 hour, what thickness of silver will be deposited on the plate (density of silver=10·6 gm. per c.c.) ?

6. How long must a constant current of 500 amperes pass through a bath for the electrolytic deposition of copper in order to deposit sufficient copper to make 1 kilometre of No. 16 S.W.G. wire (diam.$=0 \cdot 163$ cm.)? The density of copper is $8 \cdot 95$ gm. per c.c.

7. An electric current (which is the same in all parts of the trough) flows horizontally in a trough filled with copper sulphate. A rod of copper is then supported horizontally in the trough, with its length parallel to the direction in which the current is flowing. How will the rod be affected by the current?

8. Describe an experiment to illustrate the principle of the secondary cell. What are the advantages of secondary cells as compared with primary cells? Are secondary cells suitable for use in electric bell circuits? (Bristol 1st S.C.)

9. State Faraday's Laws of Electrolysis. Give a brief account of processes by which electricity is supposed to be conducted through an electrolytic solution, and indicate how the theory affords an explanation of Faraday's laws.

(Bristol 1st S.C.)

10. The current from a voltaic battery is passed at the same time through a thin wire and through dilute sulphuric acid, connected in series. What will happen to the wire and to the dilute acid; and what change (if any) will be produced in each case by reversing the battery connections, so as to alter the direction of the current through the wire and liquid?

11. An electric current is passed through a platinum wire and a copper wire of the same size, arranged in series. If the strength of the current is sufficiently increased, the platinum becomes red-hot while the copper remains dark. Explain this.

12. A current flows through a copper wire, which is thicker at one end than at the other. If there is any difference either (1) in the strength of the current at, or (2) in the temperature of, the two ends of the wire, state how they differ from each other, and why.

13. Assuming that the rate of production of heat by a current in a wire varies as the product of the resistance and the square of the current, compare the amount of the heat developed by a current of 2 amperes in 3 minutes in a wire 3 feet long, with that produced by a current of 3 amperes in 2 minutes in 2 feet of the same wire.

14. Two wires of the same size and length, one of copper and the other of iron, are joined in series and connected to the poles of a battery. In this case the iron wire becomes hotter than the copper. The two wires are then connected in parallel to the same battery, and the copper is observed to become hotter than the iron. Explain these observations.

15. When a current of 1 ampere is passed through a certain heating coil it raises the temperature of 100 gm. of water 10° C. in 2 minutes. What is the resistance of the coil? (1 calorie = 42 million ergs.) (Part of question) (Lond. Univ., G.S.)

16. A wire of 5·23 ohms resistance was placed in a calorimeter containing 1000 grams of water, and a current of 5 amperes was sent through the coil for 10 minutes. If the initial temperature of the water is 10° C., what will be its final temperature?

17. A current is passed through a wire of 5 ohms resistance placed in a calorimeter. A steady stream of water is kept flowing through the calorimeter at the rate of 15 c.c. per minute, and the heating effect was such that the water was 4° C. warmer on leaving the calorimeter than it was on entering. Find the strength of the current.

18. During the course of an hour 0·62 gram of copper is deposited on the cathode of a copper sulphate voltameter. What is the current through the voltameter? In the same time 20 calories of heat are developed in one of the leads from the battery to the voltameter. What is the resistance of this lead? [Electrochemical equivalent of copper = 0·32 milligram per coulomb. 1 calorie = 4·2 joules.] (Joint Matric. Bd., S.C.)

19. A wire carrying an electric current of 0·4 ampere has a resistance of 2 ohms per metre. If 75 cm. of this wire are placed in 500 c.c. of water, find the rise of temperature in 15 minutes. [1 joule = 0·24 calorie.] (Part of question) (Oxf. and Camb., S.C.)

20. Define the *calorie*, the *joule*, and the *watt*.

An electric kettle, used on a 220 volt supply, takes a current of 2 amperes and raises the temperature of 1 kilogram of water from 15° C. to 65° C. in 10 minutes. What percentage of the energy supplied is used in raising the temperature of the water? The mechanical equivalent of heat may be taken as 4·2 joules per calorie. (Oxf. and Camb., S.C.)

21. State the law relating to the development of heat in a conductor by an electric current.

An electric kettle contains a coil of 8 ohms resistance and a current of 10 amperes is sent through it. Calculate how long it will take to warm two litres of water from 15° C. to boiling-point, neglecting the heat lost by the water. (Oxf. and Camb., S.C.)

22. On what does the heat given out by an electric radiator depend? Assuming that the resistance of the radiator remains constant, but that the voltage from the mains fluctuates between 95 and 105 volts, find the ratio between the maximum and minimum heating effects.

Compare the heating effects in two coils of resistance 6 and 8 ohms, the currents through them being 2·5 amperes and 1·75 amperes respectively. (Cen. Welsh Board.)

23. Explain the terms *joule, watt, kilowatt-hour.* No. 20 gauge nickel-chrome wire has a resistance of 1360 ohms per 1000 yards. What length would be required for an electric heater taking 200 watts at 110 volts ? (Lond. Univ., G.S.)

24. Define calorie, joule, kilowatt-hour.

If 1 calorie is equivalent to 4·2 joules, how much heat would be developed per hour by an electric heater taking 2·5 amp. at 220 volts ? (Lond. Matric.)

25. What is meant by the expression '*a* 40 *watt lamp*' ?

A 40 watt lamp is connected to a 220 volt supply circuit. How much current will it take and how much will it cost to run for 20 hours at 8d. per unit ? (Lond. Univ., G.S.)

26. What is the resistance of a 60-watt lamp for use on (*a*) a 110-volt supply, (*b*) a 240-volt supply ?

What would be the cost of running three such lamps connected in parallel for 10 hours if the cost of electricity is 8 pence per kilowatt-hour ? (Lond. Univ., G.S.)

27. Explain the statement that *the electrochemical equivalent of gold is* 0·000676 *gm. per coulomb.*

It is required to deposit 0·025 gm. of gold on an article in a plating bath. The E.M.F. applied is 20 volts, and the resistance of the bath is 250 ohms. How many minutes must the article remain in the bath ? (Lond. Univ., G.S.)

28. A copper voltameter is connected in series with an electric battery and a standard 2 ohm coil. The current is passed for 30 min. and the increase in weight of the cathode is 1·098 gm. The mean reading of a voltmeter connected across the 2 ohm coil is 3·70 volts. Determine the electrochemical equivalent of copper. (Lond. Univ., G.S.)

CHAPTER XXXIX.

MEASURING INSTRUMENTS.

Galvanometers.—Instruments for detecting and measuring weak currents are called **galvanometers**. All types depend upon the fact that a current traversing a coil of wire sets up a magnetic field in the region within and around the coil. In one type of instrument the coil is *fixed*, and a small magnet is *suspended* near to it (usually at its centre) and with its axis in such a direction as to be perceptibly moved when a weak current traverses the coil; in another type, the coil is *suspended* and the magnet is fixed.

The use of the instrument consists in observing the extent of the deflection, if any, of the suspended element when the coil is traversed by the current under observation. The **sensibility** of any galvanometer may be defined as the amount of deflection obtained with a given strength of current: this will be referred to in more detail in subsequent paragraphs.

The astatic galvanometer (Fig. 364).—In this instrument, the coil is rectangular in form, and it is fixed in position. The suspended element consists of an *astatic pair* (p. 402) of magnets, of approximately equal dimensions. If the two magnets are identical in their 'magnetic moments,' the force tending to make one magnet set in the magnetic meridian is exactly neutralised by the force acting on the other magnet, and the astatic pair comes to rest in a position which is determined

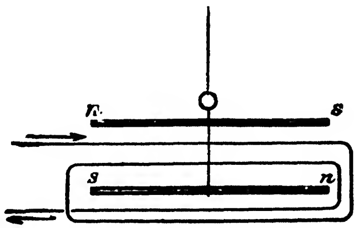


FIG. 364.—Principle of an astatic galvanometer.

entirely by the silk fibre suspension. In practice, it is impossible to obtain two magnets so identically alike, and the pair comes to rest in the meridian in obedience to the stronger magnet. In this case the pair is equivalent to a single magnet, the pole strength of which is equal to the *difference* in pole-strength of the component magnets.

If m_1 is the pole-strength of the stronger magnet, the force due to the earth's field and acting on its N-seeking pole is m_1H (where H is the horizontal intensity of the earth's field); similarly, if m_2 is the pole-strength of the other magnet, the force acting on its S-seeking pole is m_2H, and the direction of this is opposite to that of the force m_1H. Hence the resultant force due to the earth's field is $(m_1 - m_2)$H; and this is called the *controlling force*. Nevertheless, this force usually is so small that the fibre suspension largely determines the position of rest.

The instrument is adjusted so that the turns of wire in the coil are parallel to the suspended magnets; and any current traversing the coil sets up a magnetic field which has a direction perpendicular to the axes of the magnets. Moreover, by adjusting the suspension so that the upper layer of the coil lies between the needles, the presence of the upper needle has the added advantage of aiding the deflection of the lower needle: this is simply an application of Ampere's Rule (p. 482).

Any deflection is observed by attaching to the astatic pair a thin pointer, either of aluminium wire or of drawn-out coloured glass. A still more effective pointer is obtained by reflecting a narrow beam of light from a small round mirror attached to the pair; the reflected beam, acting like a weightless pointer, moves through an angle which is *twice* as great as the deflection; and this has the additional advantage that the reflected beam may be indefinitely long. The use of a beam of light is shown in Fig. 367 (p. 533).

It may be added that the astatic galvanometer is used only as a *detector* of current, and not as a means of measuring the strength of a current.

The Tangent galvanometer.—In magnetic measurements, described in a previous chapter, it was shown that when a magnet is suspended in the earth's field and is deflected by another magnetic field, the direction of which is perpendicular to that of the earth, *the deflecting force is proportional*

to the tangent of the angle of deflection. This is known as the **Tangent Law.**

In order that a galvanometer may obey the tangent law, it is necessary that the controlling force should be due to a uniform magnetic field (such as that of the earth), and that the field created by the current in the coil should be uniform within the region in which the needle is capable of moving. When the coil is circular and of considerable diameter, the field at its centre due to a current passing round it will be fairly uniform (Fig. 350, p. 497). Hence if a very short magnetised needle is suspended at the centre of a circular coil, which is placed with its plane in the magnetic meridian, then all the conditions for obeying the tangent law will be fulfilled. Such an instrument is called a **tangent galvanometer.**

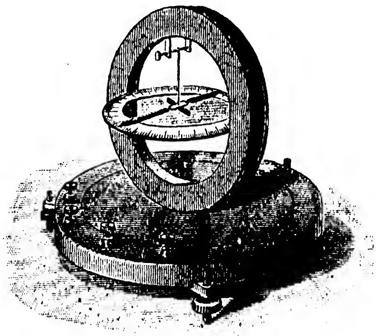


Fig. 365.—A tangent galvanometer.

Fig. 365 represents a suitable form of tangent galvanometer for simple experiments. Three separate coils may advantageously be wound on the circular wooden frame (about 20 cm. diameter), and connected to separate binding-screws fixed to the base-board of the instrument. One coil may conveniently consist of three or four turns of thick copper wire, for use with fairly strong currents; the other coils may consist of fifty and one hundred turns (respectively) of thin copper wire, for use with weaker currents. A horizontal circular scale is fixed just below the centre of the coil, and a magnetised needle (2 cm. long) is suspended by means of a single fibre of unspun silk at the centre of the coil. A long pointer is attached to the centre of the needle and at right angles to its axis; a suitable pointer consists of a strip of thin aluminium sheet, bent on each side of the centre, as shown in Fig. 365. The use of a silk fibre introduces torsion when the needle is deflected, but its controlling force is small when compared with that due to the earth's magnetic field (unless the magnet is but feebly magnetised); this error to

which the instrument is liable, may be more completely avoided by pivoting the needle on a vertical metal point fixed through the centre of the scale. The needle is protected from air currents by placing a glass shade over the instrument.

Before taking any readings with the instrument, it must be placed with its coil coinciding as nearly as possible with the magnetic meridian ; and all neighbouring magnets must be moved to a distance.

When using the instrument, error of observation may arise from three causes :

(i) **Parallax.** The eye should be vertically over the pointer when reading the deflection. This is ensured by mounting the circular paper scale, of which the central portion is removed, on a plane mirror. The eye is moved until the image of the pointer appears to be immediately below the end which is being observed.

(ii) **The suspending fibre may not coincide with the centre of the circular scale.** Errors due to this are eliminated by taking readings at each end of the pointer.

(iii) **The axis of the magnet and the plane of the coil may not coincide with the magnetic meridian.** The former possible error is probably due to torsion in the suspending fibre. The errors are eliminated by observing the deflections when the current is reversed, and taking the mean of the four readings obtained.

It has been shown (p. 496) that, at the centre of a circular coil conveying a current, the intensity (f) of the field due to the current is given by the equation

$$f = 2\pi n \mathrm{I}/r \text{ dynes,}$$

where n = number of turns of wire in the coil, r = average radius, and I = current strength (in absolute units). If I is expressed in amperes, then

$$f = 2\pi n \mathrm{I}/10r \text{ dynes.}$$

This is the 'deflecting force'; and if H is the horizontal intensity of the earth's field, then

$$f/\mathrm{H} = \tan\theta,$$

or

$$\frac{2\pi n \mathrm{I}}{10r} \Big/ \mathrm{H} = \tan\theta,$$

or

$$\mathrm{I} = \frac{10\mathrm{H}r}{2\pi n} \cdot \tan\theta \text{ amperes.}$$

If the values of H, r and n are known, the quantity $10\mathrm{H}r/2\pi n$ may be regarded as a constant quantity for the instrument ;

and the strength of the current may be calculated by multiplying $\tan\theta$ by this quantity. The expression is termed the **reduction-factor** of the galvanometer, and is usually denoted by the symbol k. Hence

$$I = k \tan\theta.$$

It is important to remember that the value of H is not the same in all localities, or even in different parts of the same room. Hence the 'reduction-factor,' whether calculated or determined by any other methods, will apply only to the exact position occupied by the galvanometer at the time. It will be noticed, from the above expression, that for a given current-strength, *$\tan\theta$ is inversely proportional to* H; consequently, when the instrument is moved into a position where the value of H is greater, then $\tan\theta$ will be proportionally less.

An accurate method of determining the 'reduction-factor' of a tangent galvanometer is to find the current-strength by means of a copper voltameter: this is connected in series with the galvanometer, to which a commutator is attached, and the current is kept constant by means of a rheostat. When one half of the time occupied by the experiment has elapsed the current (*through the galvanometer only*) is reversed, so as to obtain the four readings of the deflection. The manipulation of the voltameter has been described previously (p. 505). The following is a record of an experiment of this nature:

EXPT. 303.—**Reduction factor of a tangent galvanometer (by copper voltameter).**

Position: Physics laboratory, centre of table No. 4.

Coil used: Low resistance (4 turns of wire).

Weight of cathode: (final) - 21·585 gm.

(initial) - 20·081 ,,

Weight of copper deposited = 1·504 ,,

Duration of experiment: 60 minutes = 3600 sec.

Current = 1·504/(0·0003294 × 3600) = 1·265 amp.

Deflections:

$$\left.\begin{array}{r} 40^\circ{\cdot}1 \\ 44^\circ{\cdot}0 \\ 39^\circ{\cdot}9 \\ 37^\circ{\cdot}2 \end{array}\right\} \tan 40^\circ{\cdot}3 = 0{\cdot}836.$$

$$4\,\underline{)161^\circ{\cdot}2}$$

$$40^\circ{\cdot}3$$

$$k = \frac{1{\cdot}265}{0{\cdot}836} = \mathbf{1{\cdot}49}.$$

Suspended coil galvanometers.—The action of a magnet upon a conductor conveying a current is the basis of a type of galvanometer which possesses considerable advantages. The **d'Arsonval** (Fig. 366) is the most familiar pattern. The magnetic field is derived from a cylindrical magnet M built up of magnetised rings of hard steel. The rectangular coil C is hung on a stretched strip of phosphor-bronze, which serves to convey the current to the coil. The current is conveyed away through a very fine spiral spring, the lower end of which is connected to one of the terminal screws on the base-board of the instrument.

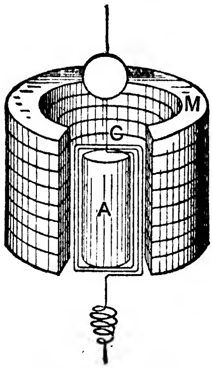


FIG. 366.—A suspended coil galvanometer.

The instrument is adjusted so that when no current is passing the plane of the coil is parallel to the magnetic lines of force. When a current passes, the vertical sides of each turn of wire in the coil are acted upon by a force; and since the two sets of forces will be acting in opposite directions, they constitute a couple, which tends to rotate the coil into a position with its plane perpendicular to the lines of force. The rotation is opposed by the torsion set up in the suspension; and the restoring force due to this torsion is proportional to the angle through which the lower end of the suspension is twisted.

Hence, if the magnetic field is uniform within the range of movement of the coil, and also radial towards the vertical axis of the coil, the moment of the couple due to the magnetic forces, and therefore the current, will also be proportional to the angle through which the coil is deflected. Uniformity and radial direction of the magnetic field are obtained by means of the soft-iron cylinder A which is fixed in position between the pole faces of the magnet; and these pole faces are usually curved, concentric with the axis of A, and made wider than represented in Fig. 366. The following are the chief advantages of this type of galvanometer:

(i) The deflections are scarcely affected by external magnetic fields.

(ii) The instrument may face in any direction, since the zero position of the coil is independent of the direction of the magnetic field in which it is suspended.

Fig. 367 shows how a reflected beam of light is used for observing deflections of the suspended coil. The beam of

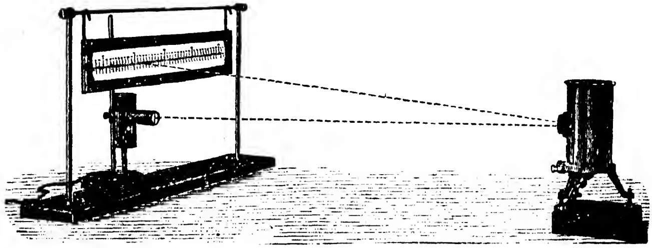


FIG. 367.—Galvanometer, with lamp and scale.

light is equivalent to a pointer of length equal to twice the distance between the mirror (attached to the coil) and the scale. The tube in front of the lamp carries a small lens, on which a fine vertical line is scratched—or a very thin wire is stretched across it. The distance of the galvanometer from the scale is adjusted so that a well-defined image of the scratch is formed on the scale.

Ammeters and voltmeters.—When approximate accuracy of the measurement of a current is sufficient, it is more convenient to use some form of **ammeter,** many of which are identical in principle with the d'Arsonval galvanometer, except that only a fixed small fraction of the current passes through the suspended coil. Fig. 368 (i) represents the external view of one form of ammeter; and Fig. 368 (ii) is a diagram of the main internal details. The permanent magnet NS is of the horse-shoe type; and a soft-iron cylinder D, fixed between the poles of the magnet, serves to concentrate the lines of force within the space occupied by the pivoted coil C. The coil is wound on a rectangular frame of thin aluminium, it rotates on a horizontal axis, and the current is conducted into and from the coil by the fine springs s_1 and s_2.

The current to be measured enters the instrument at the terminal marked +ve; at the point A the current divides;

nearly the whole current passes to the point B through the short thick wire shown, while the remainder is conveyed to

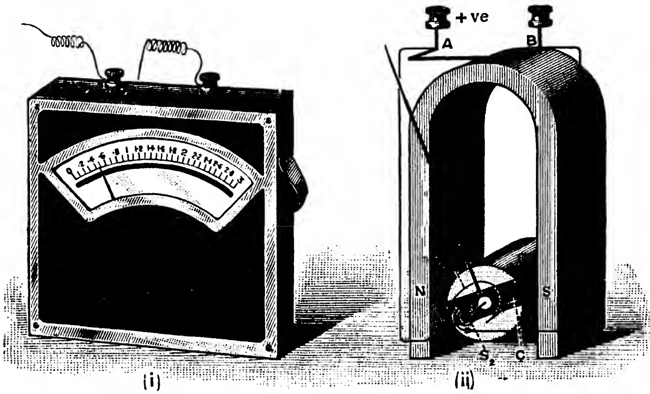


FIG. 368.—An ammeter (moving-coil type).

the coil through the spring s_1 and back through s_2 to the point B. The polarity set up in the coil causes it to rotate in a direction such that the pointer moves from left to right in front of the divided scale ; and the rotation ceases when the magnetic forces are balanced by the mechanical forces set up in the springs. The relative resistances of the thick wire between A and B, and of the coil, are adjusted so that the scale reading indicates the strength of the *total* current. This construction makes the resistance of an ammeter very small, and a very small portion of the energy of the current therefore is used up in the instrument.

A **voltmeter**, as its name indicates, is an instrument for measuring the difference of electrical pressure between the two points of a circuit to which it is connected. A frequent type of the instrument resembles closely the ammeter shown in Fig. 368, except that the thick wire between the points A and B is removed, and a high resistance is inserted between one of the terminals and the coil. This construction ensures that the current traversing the coil is very small, and not sufficient to disturb the distribution of electrical pressure along the main circuit. The coil is constructed, and the scale is

graduated, so that the instrument indicates differences of electrical pressure in terms of the *volt.*

The method of connecting an ammeter and a voltmeter in any simple circuit is shown in Fig. 369. Suppose that AB

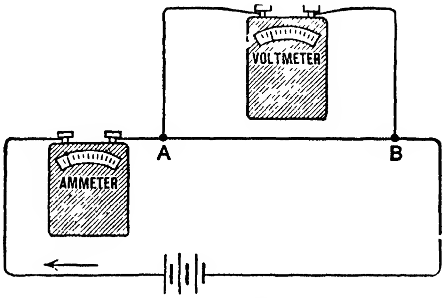


FIG. 369.—The method of using an ammeter and voltmeter, in order to find the current and the voltage used in a conductor AB.

(Fig. 369) is part of an electric circuit, and that we wish to find what current is passing along AB and the P.D. between its ends. The voltmeter forms an alternative path for the current; but as its resistance is very high the current which is diverted through it is very small, and practically all the current which passes through the ammeter passes also through the conductor AB. Since $R = E/I$, the resistance of AB can be calculated by dividing the reading of the voltmeter by that of the ammeter.

Watt-meters.—The cables which convey electric current into a house pass through instruments which measure the *electric power* consumed. As stated previously, the power is measured in **watts**, and a rate of working equal to 1000 watts, continued for one hour (briefly called the **kilowatt-hour**), is the **Board of Trade Unit.** It is the number of these units which is recorded on the dials of the meter. Suppose that a room is lighted by three lamps, each marked '30W'; the illumination, therefore, is equal to $3 \times 25 = 75$ c.p., and the rate of consumption of power will be $3 \times 30 = 90$ watts. If the lamps are kept burning for 5 hours the power consumed will be $90 \times 5 = 450$ watt-hours $= 0{\cdot}45$ kilowatt-hour (or B.O.T. unit). Assuming that the charge is 8d. per unit, the cost of the

lighting will be $8 \times 0{\cdot}45 = 3{\cdot}6$d. To this must be added a small sum towards the original cost of the lamps, so that the total cost would be approximately 4d.

There are several different types of meter for measuring the kilowatt-hours of energy supplied to an electric circuit. A type which is frequently used resembles a small electric motor (Fig. 370). The speed at which the armature of a motor rotates depends upon (i) the strength of the magnetic field and (ii) the strength of the current passing through the coils of the armature. In Fig. 370 the entire current passing through the circuit traverses the coils FF, which create the magnetic field in which the armature A revolves. The coils

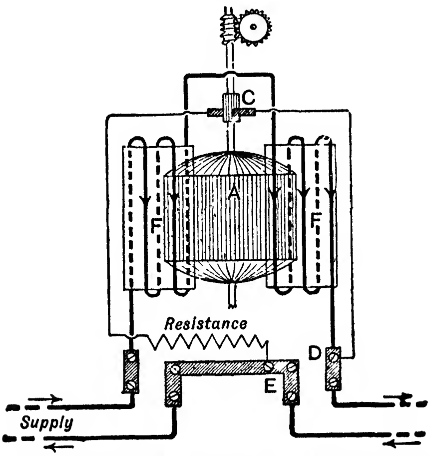


FIG. 370.—A METER FOR MEASURING ELECTRIC POWER.
It resembles an electric motor, with an armature A rotating in a magnetic field set-up by the coils FF. All the current passes through FF. The armature has a high resistance, and joins the 'flow' and 'return' wires; the current in it therefore is proportioned to *the voltage of* the supply. The rate of rotation depends upon (i) the current in the mains, and (ii) the voltage, and therefore upon the product of these (*i.e.* the watts supplied).

of the armature are connected through a high resistance to two points D and E on the flow and return wires; in fact, its circuit resembles closely that of a voltmeter, as described on p. 534, and the current through the armature is a measure of the P.D. between the two mains. Hence, the speed of rotation of the armature is proportional (i) to the current, and (ii) to

the P.D. between the mains, and it varies, therefore, with the product *current* × *voltage*, and measures the *watts* of power consumed. The axis of the armature gears into a train of wheels which record, by means of pointers and circular scales, the number of B.O.T. units which have been used; this indicator is read in just the same way as that of an ordinary gas meter. The speed of rotation of the armature is regulated by means of a special type of brake, which is not shown in the diagram; without this, the speed would gradually increase, although the current used might remain constant. Such an instrument is termed a **watt-meter.**

EXERCISES ON CHAPTER XXXIX.

1. Explain fully why the deflection of the needle of a tangent galvanometer is independent of the pole-strength of the needle.

2. Describe the tangent galvanometer, and explain how it is used to measure current.

If the galvanometer has 50 turns, each of radius 7·5 cm., and if the magnet is deflected 60°, calculate the current in amperes [H=0·18.] (Cen. Welsh Bd.)

3. A current of 0·1 ampere produces a deflection of 20° in a tangent galvanometer in a position where the horizontal intensity is 0·36 unit. What current will be required to produce the same deflection in a position where the horizontal intensity is 0·32 unit?

4. The same current is sent through two tangent galvanometers connected in series, and causes deflections of 30° and 60° respectively. Find the ratio of the reduction factors of the two instruments.

5. What is the intensity of the magnetic field at the centre of the coil of a tangent galvanometer of 20 turns of wire and 25 cm. mean radius when traversed by a current of 0·2 ampere?

6. Calculate the strength of the current, passing through a tangent galvanometer, in absolute units and also in amperes from the following data: Radius of coil, 12 cm. Number of turns in coil, 10. Deflection of needle, 45°. Value of earth's horizontal force, 0·18.

7. In a determination of the reduction-factor of the low-resistance coil of a tangent galvanometer, by electrolysis of copper sulphate, the following data were obtained:

Initial weight of cathode - -	11·337 gm.
Time, at closing of the circuit -	4 h. 48 min.
,, ,, breaking ,, ,, -	5 h. 8 min.
Final weight of cathode - -	11·563 gm.
Mean deflection of galvanometer -	31°·27

If the electrochemical equivalent of copper is 0·000329, calculate the reduction-factor of the galvanometer.

8. Describe a tangent galvanometer and explain its action. A tangent galvanometer which gives a deflection of 20° for a current of 0·01 ampere in England gives a deflection of only 12° for the same current when in Ceylon. How do you account for this? (Oxf. and Camb., S.C.)

9. A coil of six turns, each of which is 1 metre in diameter, deflects a compass needle at its centre through 45°. Find the strength of the current in amperes, having given that H=0·36 C.G.S. units.

10. A current flows through two tangent galvanometers in series, each of which consists of a single ring of copper, the radius of one ring being three times that of the other. In which of the galvanometers will the deflection of the needle be greater? If the greater deflection be 60°, what will the smaller be?

11. Of how many turns of wire must the coil of a tangent galvanometer consist, if the radius of the coil is 15 cm., and if a current of 0·01 ampere is to produce a deflection of 30°? (H=0·17).

12. A tangent galvanometer connected in series with a cell, E.M.F.=2 volts, and a coil of 5 ohms resistance, shows a deflection of 60°. Putting an additional resistance of 7 ohms in the circuit reduces the deflection to 45°. Find (*a*) the reduction-factor of the galvanometer, (*b*) the total resistance of the circuit in the first case. (Bristol 1st S.C.)

CHAPTER XL.

APPLICATIONS OF OHM'S LAW, AND GENERAL MEASUREMENTS.

Graphical representation of resistance, potential-difference, and current.—In Fig. 371 let AB represent a copper wire along which a current is flowing from A to B. If the wire be of uniform material and cross-section, the resistance of each cm. length of the wire will be the same; hence a length of 2 cm. would have twice the resistance of a length of 1 cm. In other words, the resistance will be proportional to the length, and if AB represents the length of the wire, it may also be regarded as a graphic representation of its resistance.

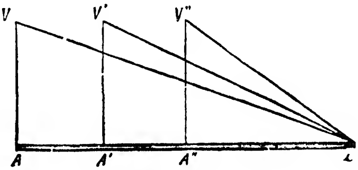


FIG. 371.

Let the potential at A be represented by AV, and that at B zero. The fall of potential along the wire will be uniform and represented by the line VB.

When the wire is shortened to A′B its resistance will be less than before, and V′B will represent the fall of potential. If the wire is still further shortened to A″B, then V″B will represent the fall of potential.

A simple experiment demonstrates that when the wire is shortened the current traversing the wire is increased: can this increase be suggested in the diagram? An increased current in the experiment is accompanied by an increase in the angle VBA in the diagram: can we regard the latter as a representation of the current?

This is evidently possible if we consider not the angle itself but rather the *tangent* of the angle, for then the tangent of VBA = VA/AB, or expressed in words—

$$\text{The strength of the current} = \frac{\text{difference of potential}}{\text{resistance}}.$$

Fig. 372 is a similar diagram, which shows how the potential-difference between *any* two points of a uniform wire may be indicated. The potentials at A and B are AV_1 and BV_2, and the potential-difference is represented by the length V_1a.

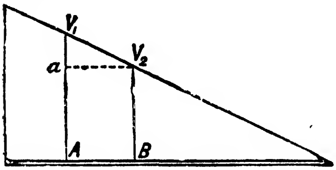


Fig. 372.

External and internal resistance.—As a general rule the equation I = E/R is applied to the entire circuit traversed by the current, including the battery as well as the external wires, all of which offer a resistance to the passage of the current. Hence the symbol R includes both the resistance of the wire (usually termed the *external resistance*) and also that of the battery (usually termed the *internal resistance*). It is better to represent these component resistances by separate symbols, and to write the equation thus—

$$I = \frac{E}{R + r},$$

where R = the external resistance, and r = the internal resistance. Since the battery has resistance, a portion of the total available P.D. will be used in driving the current through the battery, and only the remainder of the P.D. will be available for driving the current through the wire. This is rendered more evident by writing the above equation thus—

E	=	IR	+	Ir
(Total P.D.)		(P.D. used in external circuit.)		(P.D. used in internal circuit.)

This is represented diagrammatically in Fig. 273, where AB represents the internal resistance and BC the external resistance. AE is the total P.D., and Ee is the portion used up in

overcoming the resistance of the cell, while BE′ represents the difference of potential between the ends of the wire. The current is represented by the tangent of the angle ECA, hence

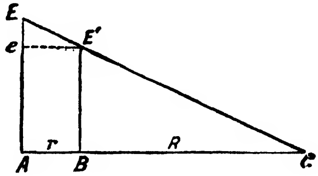


FIG. 373.

$$I = \frac{Ee}{r}, \text{ and also } I = \frac{BE'}{R};$$

or, $Ee = Ir$, and $BE' = IR$.

Therefore $AE = Ee + BE' = Ir + IR$.

Electromotive force.—The statement that 'the **electromotive force** of a dry cell is 1·43 volts' may be familiar. The term is usually abbreviated to E.M.F. There is not much difference between its meaning and that of potential-difference (or P.D.). Since the cell is the active source of the current, it is now customary to distinguish between the total P.D. which the cell is capable of setting up and the P.D. between any two selected points of the circuit along which the current is passing. *Even when the cell is standing idle there is a* P.D. *between its terminals; and this special* P.D. *is called the electromotive force.* But when the terminals are joined by conductors, the E.M.F. is used up in sending a current round the whole circuit, which includes the cell itself; and as the cell always has more or less resistance, part of its E.M.F. is used up in driving the current through itself, and only the remainder is available for driving the current through the wires (or *external circuit,* as it is called).

The E.M.F. of a cell depends only on the metals and liquids used, and not upon its size; hence, the E.M.F. of a cell no larger than a thimble will be exactly the same as that of a full-sized cell of the same construction. But the great advantage of the large size is that it offers much less resistance to the passage of a current through it. Thus, in the case of the simple cell (p. 470), the E.M.F. is not altered when the plates are separated farther apart or when the plates only just dip into the acid; but the current given under the latter conditions is much less, because the internal resistance is so much greater. One of the great advantages of the accumulator is that it has a very low internal resistance.

EXAMPLE.—The total E.M.F. of a battery is 10 volts. When the poles of the battery are connected by a wire a current of 2 amperes is obtained, and the potential-difference of the battery poles drops to 7·5 volts. Find the resistance of the battery and of the wire.

$$I=\frac{E}{R+r}, \text{ or } R+r=\frac{E}{I}=\frac{10}{2}=5 \text{ ohms.}$$

Potential-difference between the ends of wire $=IR$;

or $7{\cdot}5=2\times R.$

Hence $R=3{\cdot}75$ ohms.

But $R+r=5$ ohms, therefore $r=5-3{\cdot}75=1{\cdot}25$ ohms.

EXPT. 304.—**Internal resistance.** Pierce the axis of an ordinary cork with the supporting wire of the copper plate of a simple voltaic cell, and mount the zinc plate on a cork in a similar manner. Fix the corks in the screw clamps of two separate retort-holders, so that the metal plates are vertical and just above the level of the table. In this manner the plates may be rigidly supported within a shallow glass dish, *e.g.* a crystallising dish, and their distance apart and their depth of immersion within the dish varied. Fill the dish with very dilute sulphuric acid, and connect the plates to the thick coil of a tangent galvanometer by means of copper wires. Place the plates close together and observe the deflection. Separate them gradually and observe how the deflection diminishes, showing that the resistance of the cell is increased when the length of the liquid column between the two plates is increased.

Now raise the plates slightly, or remove some of the acid by means of a pipette, so as to reduce the cross-section of the liquid column. Notice how the deflection diminishes as the cross-section of the liquid column becomes less.

EXPT. 305.—**Measurement of internal resistance (voltmeter method).** Use for the purpose a Daniell cell, a dry cell, or a Leclanché cell. A voltmeter reading 0-3 volts is suitable for the observations. Connect the terminals of the cell to the voltmeter, and note the reading. This is the total E.M.F.: denote this by the symbol E. Without altering the connections, join the terminals together through a 10-ohm coil, using short thick copper wires for the purpose, and again read the voltmeter: denote this by the symbol *e*. This is the potential-difference required to drive the current *through the coil.* Except when a Daniell cell is used, polarisation may affect the latter reading, and it may be smaller than it should be.

It is well, therefore, to repeat the measurement of E; and if it differs from the initial reading, the *average* value should be used.

Since
$$\left.\begin{aligned} E &= I(R+r), \\ e &= IR; \end{aligned}\right\}$$
then
$$(R+r)/R = E/e,$$
or
$$r = R \times (E-e)/e.$$

Substitute the values of R, E, and e, and calculate the value of r.

Methods of Measuring Resistances.

Resistance boxes.—For the comparison and measurement of unknown resistances it is necessary that other resistances of known magnitude be available. In a **resistance box** a series of separate coils are arranged so that they may be used in any desired combination. Each coil consists of silk-covered wire wound on a cylinder; the free ends of the coil are connected to adjacent brass blocks b (Fig. 374) on the outside of the cover of the box; the space between each pair of blocks is occupied by a conical brass plug fitted with an ebonite handle p. When the plug is inserted, any current passing along the brass blocks is conveyed through the plug; but when the plug is removed, all the current has to pass through the resistance coils beneath. The resistance of each coil is marked, in *ohms*, on the lid of the box.

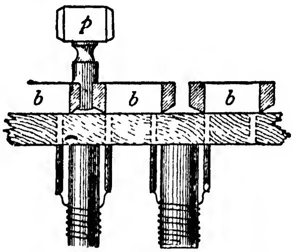


FIG. 374.—Construction of a resistance box.

In another type of resistance box, termed the 'dial pattern,' the coils are joined to metal studs arranged in a circle; one or more of the coils can be inserted into the circuit by rotating a metal arm pivoted at the centre of the circle.

It should be remembered always that it is very easy to damage a resistance box by sending too strong a current through it.

The Wheatstone net.—Let the points A and C (Fig. 375) be connected by two conductors ABC and ADC in parallel. The current entering at A divides into two portions, i_1 and i_2, which rejoin at C. The potential falls gradually along each

branch; and a point D, in ADC, may be found which has the same potential as the point B. The position of D may be determined by connecting the two points through a galvanometer G, and adjusting the point of contact at D until there is no deflection. Under this condition, since no current passes through G, the currents i_1 and i_2 are individually uniform along the conductors ABC and ADC.

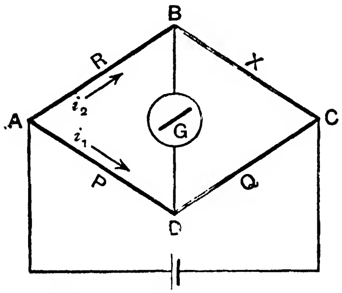


FIG. 375 —The Wheatstone net.

If V_A, V_B, V_C, and V_D are the potentials of the points A, B, C, and D respectively; and if P, Q, R, and X are the resistances of AD, DC, AB, and BC respectively; then, by Ohm's law,

$$V_A - V_B = i_2 R,$$
$$V_A - V_D = i_1 P.$$

But $$V_A - V_B = V_A - V_D;$$

therefore $$i_2 R = i_1 P; \quad \text{or} \quad i_1/i_2 = R/P.$$

Similarly $$i_2 X = i_1 Q; \quad \text{or} \quad i_1/i_2 = X/Q.$$

Hence $$X/Q = R/P,$$

or $$\frac{X}{R} = \frac{Q}{P}.$$

This result explains how it is possible, in an experiment with four separate resistances, to determine any one of them if the other three are known; or to obtain the ratio of any two of them if the ratio of the remaining two is known.

The metre bridge.—The metre bridge (Fig. 376) is the simplest application of the previous paragraph to the measurement of an unknown resistance. It consists of a uniform German-silver (or iridio-platinum) wire, one metre long, stretched alongside a metre scale, and with its ends soldered to stout copper strips E and F. There are four gaps between copper strips fixed along the other edge of the board, but in simple measurements the gaps U and V are closed by copper strips held in position by binding screws. The resistances R

and X which are to be compared are attached to binding screws as shown. The galvanometer circuit is closed by

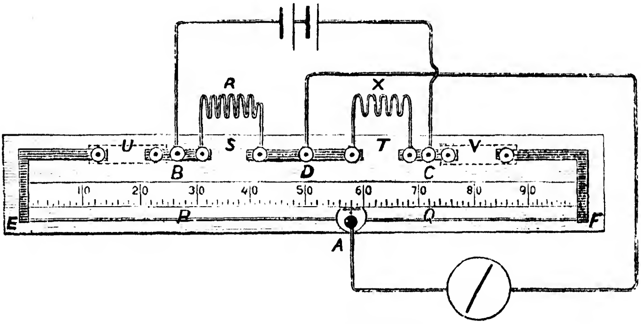


FIG. 376.—The metre bridge.

depressing the knob A, which makes contact between a knife-edge and the bridge wire. Having found by trial the position of A which gives no deflection, then, since the wire is uniform and the resistance of any portion is proportional to its length, the ratio R/X will be equal to the ratio of the lengths P and Q.

Trustworthy results are obtained only when the resistances R and X are of the same order of magnitude. Thus, if R is about 99 times as great as X, the position of A for no deflection will be near to the scale-reading 99; and the length of Q will be only about 1 cm. The liable error in reading so short a length is great, and the result of the calculation will be equally liable to error. The result is most trustworthy when the position of A for no deflection is near to the middle of the wire.

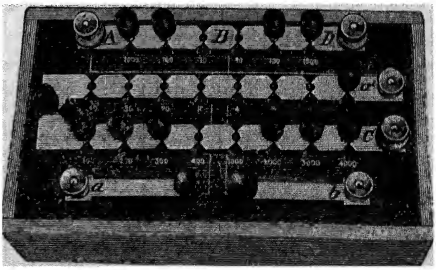


FIG. 377.—A Post-office box.

The Post-Office box.—The Post-office box (Fig. 377) is simply a convenient form of metre bridge, with which a resistance can be measured with far greater accuracy. It appears to consist simply of three rows of resistance coils, and

of two ' tapping keys ' in front ; but if examined in detail, it will be found to contain the chief features of the ' Wheatstone net.' Fig. 378 will help to explain these points: Fig. 378 (i) is a reproduction of the Wheatstone net ; and, as previously

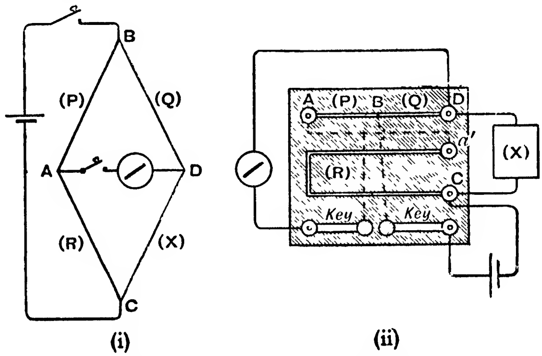


FIG. 378.—THE POST-OFFICE BOX. (i) The Wheatstone net, and (ii) its modified form as arranged in the P.O. box.

explained, there is no deflection of the galvanometer when P/Q = R/X. Fig. 378 (ii) is a skeleton diagram of a Post-office box ; and, for the sake of clearness, the same lettering is used in the two diagrams.

Imagine that the current from the cell enters at the point B : it then divides to left and right along the arms BA and BD. In the P.O. box these are known as the ' ratio-arms,' and each consists of three coils, the resistances of which are 10, 100 and 1000 ohms respectively. The points A and *a'*, in the box, are electrically the same, because they are joined by a thick wire below the cover of the box ; and the coils between *a'* and C constitute the third arm (R) of the Wheatstone net. The unknown resistance X is joined *by short thick wires* to the binding-screws at D and C. Instead of connecting the galvanometer to either A or *a'*, it is joined to the key on the left side of the box, the stud of which is permanently connected, by a wire under the cover, to the thick wire joining A and *a'*, previously mentioned. In order to provide the cell with a key, conveniently placed, one terminal of the cell is joined to the key on the right hand side of the box, the stud of which is joined permanently by thick wire to the point B The other battery terminal is joined to the binding-screw C.

In making a measurement, equal resistances are inserted in the ratio arms, and the resistance R is adjusted until a minimum deflection of the galvanometer is observed; this gives the resistance of X to the nearest whole number. The ratio P/Q is now made equal to 10, and the resistance of R again adjusted as nearly as possible; R is now approximately 10 times as great as X. Finally the ratio P/Q is increased to 100, and R again adjusted; X is then equal to R/100.

In Fig. 377, it will be seen that P = 10 ohms, Q = 10 ohms, and R = 1509 ohms; and if there is no deflection, X must be 1509 ohms. But, if P had been 100 ohms, and Q = 10 ohms, then, for no deflection, X = 150·9 ohms.

Resistance of metals.—In previous experiments (p. 426) with the pith-ball electroscope it was found that substances may be classified as good conductors, poor conductors and insulators. This property is expressed in the opposite sense by using the term **resistance**; thus, metals have *low resistance*, and silk has *high resistance* In those experiments, *all* metals, wood, water and the human body appeared to be equally good conductors. Yet, when handling voltaic cells and their circuits, no precautions have to be taken to avoid touching the cells and wires with the hands; and, except when using very sensitive apparatus, the wires may lie on the table without affecting the current traversing the circuit.

This apparent contradiction is due to the fact that in the case of the charged pith-balls, or the charged disc of an electrophorus, we are dealing with *potential-differences* which are extremely high, while the *quantity* of electricity is very small. and the P.D. is so great that the small 'quantity' involved leaks away very quickly even through a high resistance, such as the human body. In the case of a voltaic cell, or even a battery of accumulators, the opposite poles may be touched simultaneously by the two hands without any apparent consequence: there is a leakage through the body undoubtedly, but it is very slow, because the P.D. is comparatively so small, and the loss due to leakage is immediately balanced by the generation of an equal quantity by chemical action within the cell—the available 'quantity' is relatively enormous. But when *both* the P.D. and the available 'quantity' are

great, then there is danger in such an experiment: the discharge from a charged electrophorus disc does not hurt, but the discharge from a fully charged Leyden jar is distinctly painful—the P.D. in the two cases may be the same, but the 'quantity' is very different.

When the source of electricity provides a low potential-difference and an apparently unlimited supply of 'quantity' (*e.g.* an accumulator, or voltaic cell), it is found that metal wires differ very widely in the resistance which they exhibit. In all cases, the resistance depends

(i) *on the metal*: Silver has less resistance than copper, and copper less than iron;

(ii) *on the length*: Double the length, and the resistance is doubled;

(iii) *on the thickness*: Double the area of cross-section, and the resistance becomes one-half;

(iv) *on the temperature*: The hotter the wire the greater is its resistance. Thus, the resistance of a wire of pure iron increases by 60 per cent. when warmed from 0° to 100° C. The opposite effect is found in the case of carbon, *e.g.* the carbon filament of an incandescent lamp: its resistance is much greater when cold than when hot.

The relationship between length and resistance, and between cross-section and resistance, may be demonstrated by measuring the resistance (i) of different lengths of wire of the same metal and of the same cross-section, and (ii) of equal lengths of wires of different cross-sections, but of the same metal. Wire of the alloy *manganin* is suitable; or copper can be used —but as its resistance is relatively small, it is necessary to have longer lengths of wire and of small cross-section. The procedure is to cut a length of wire about 2 cm. longer than actually required, and to bend each end into the shape of the letter L; then measure accurately the length between the bends. The wire is fixed in the binding-screws of the apparatus so that it leaves the grip of the screws just at the bends. The cross-section of a wire is obtained by means of a micrometer screw-gauge.

Specific resistance.—The relationship between the resistance of a wire and the metal or alloy of which it is made is

expressed in its simplest form by a constant known as the *specific resistance* of the metal: this is defined as *the electrical resistance between the opposite faces of a one-centimetre cube of the metal.* Such a cube may be regarded as a wire 1 cm. long and 1 sq. cm. cross-section; if the dimensions of the wire are altered to l cm. in length and s sq. cm. in cross-section, then

$$\text{Resistance (R)} = \text{Specific resistance } (k) \times \frac{l}{s},$$

or

$$R = kl/\pi r^2.$$

The specific resistance of all the commoner metals and alloys can be obtained by reference to Physical Tables; and with this datum the resistance of a wire of given length and cross-section can be calculated. As the specific resistance is usually a very small fraction of one ohm, it is usually expressed in *microhms* (or millionths of one ohm).

EXAMPLE. The specific resistance of copper is $1{\cdot}77 \times 10^{-6}$ ohm. What is the resistance of 100 metres of copper wire (No. 24 S.W.G.; diameter=0·0559 cm.)?

Length $(l) = 10^4$ cm.
Cross-section $(s) = \pi \times (0{\cdot}02795)^2 = 0{\cdot}00245$ sq. cm.
$R = (1{\cdot}77 \times 10^{-6}) \times 10^4/0{\cdot}00245 = 7{\cdot}21$ ohms.

The temperature-coefficient of resistance.—The specific resistance of a metal always increases when the temperature of the metal is raised, and each metal or alloy has its own characteristic rate of increase. *The amount by which a 1-ohm resistance of the metal increases when warmed 1° C. is termed its temperature-coefficient* (α). Thus,

1-ohm, at 0° C., becomes $(1+\alpha)$ ohm at 1° C., and
,, ,, ,, $(1+\alpha t)$ ohm at t° C.

If the resistance at 0° C. is R_0 ohms, the resistance at t° C. is given by the equation

$$R_t = R_0\,(1+\alpha t).$$

(It will be noticed that this equation is identical in form with that which represents the change in *length* of a metal rod when its temperature is raised.)

The temperature-coefficients of all metals have been accurately determined; and it is evident, from the above equation,

that a measurement of the change in the resistance of a wire, of suitable metal, may be used as a means of calculating the change in the temperature of the wire. The device known as the *platinum resistance thermometer* consists of a coil of thin platinum wire wound on a mica frame, and enclosed in a long porcelain tube. The ends of the coil are joined by thick wires to an apparatus by which the resistance of the coil is measured: a Post-office box would be appropriate for the purpose. This type of thermometer is often used for determining the temperatures of furnaces and kilns.

Manganin, which is an alloy of copper, manganese and nickel, has an extremely small temperature-coefficient: for this reason, wire of this material is generally used for making resistance-coils.

EXAMPLE. (i) The specific resistance of platinum at 0° C. is 10·3 microhms. How long must a wire of No. 32 S.W.G. (diameter=0·0274 cm.) platinum be, to have a resistance of 4 ohms at 0° C.?

(ii) The temperature-coefficient of platinum is 0·0038. What will be the resistance of the above wire at 100° C.?

(i) Since $R_0 = k \times l/\pi r^2$,

$$l = R_0 \times \pi r^2/k.$$

Cross-section (πr^2) of wire $= \pi \times (0{\cdot}0137)^2$

$$= 0{\cdot}00059 \text{ sq. cm.}$$

Therefore $l = 4 \times 0{\cdot}00059/(10{\cdot}3 \times 10^{-6}) = \mathbf{229}$ cm.

(ii) $R_{100} = R_0(1 + 0{\cdot}0038 \times 100)$

$$= 4 \times 1{\cdot}38$$

$$= \mathbf{5{\cdot}52} \text{ ohms.}$$

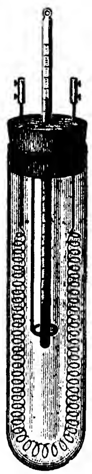


FIG. 379.—To illustrate Expt. 306.

EXPT. 306.—**The temperature-coefficient of iron.**—Fig. 379 represents a spiral of iron wire (No. 28), about 2 metres long, with its ends soldered to short pieces of thick copper wire which pass through a cork fitted in a glass boiling tube. The apparatus is fitted with a thermometer and stirrer, and the tube is nearly filled with paraffin oil.

Place a deep beaker full of water on a tripod, and fix the tube containing the wire spiral in the water. Connect up the ends of the spiral by means of thick copper wires to the binding-screws of a metre-bridge.

After the tube has been in the water for about five minutes, stir the paraffin oil, and note the temperature. Measure the resistance of the spiral. Slowly warm the water, and frequently stir the oil. When the temperature has risen about 10° C., remove the flame, stir the oil, and repeat the observations of temperature and resistance. Repeat these readings at higher temperatures.

Tabulate the observations; and calculate, from the first reading and each subsequent reading, separate values of the temperature-coefficient of iron.

Resistance of two or more conductors, joined in parallel.—If several conductors connect together the same points (Fig. 380), the total current I divides between the conductors, so that

$$I = i_1 + i_2 + \ldots .$$

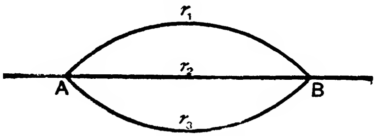


FIG. 380.

If E and R are the P.D. and the total resistance respectively between A and B, then $I = E/R$; also $i_1 = E/r_1$, $i_2 = E/r_2$, etc.

Hence $$\frac{E}{R} = \frac{E}{r_1} + \frac{E}{r_2} + \ldots,$$

and $$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots .$$

This last equation states that **the reciprocal of the total resistance is equal to the sum of the reciprocals of the resistances of the separate conductors.** When there are only *two* conductors joined in parallel, then

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$$

or $$R = r_1 r_2/(r_1 + r_2).$$

EXAMPLE.—The poles of an accumulator (E.M.F. = 2 volts) are connected by two wires in parallel, the resistances of which are 5 ohms and 6 ohms respectively. If the internal resistance of the accumulator is 0·1 ohm, find the total current traversing the circuit.

The external resistance $= \frac{r_1 r_2}{r_1 + r_2} = \frac{5 \times 6}{11} = \frac{30}{11} = 2 \cdot 73$ ohms approx.

The total resistance $= 2 \cdot 73 + 0 \cdot 1 = 2 \cdot 83$ ohms.

The total current $= \frac{E}{R} = \frac{2}{2 \cdot 83} = 0 \cdot 707$ ampere.

In circuits such as the above, it is sometimes useful to express the strength of current in each branch in terms of the total current I. Thus

$$i_1 = \mathrm{E}/r_1,$$

and
$$\mathrm{I} = \mathrm{E} \Big/ \frac{r_1 r_2}{r_1 + r_2}.$$

Hence,
$$\frac{i_1}{\mathrm{I}} = \frac{\mathrm{E}}{r_1} \times \frac{r_1 r_2}{\mathrm{E}(r_1 + r_2)} = \frac{r_2}{r_1 + r_2},$$

or
$$i_1 = \mathrm{I} \times \frac{r_2}{r_1 + r_2}.$$

Similarly,
$$i_2 = \mathrm{I} \times \frac{r_1}{r_1 + r_2}.$$

Shunts.—In measuring strong currents with a sensitive galvanometer, it is often necessary to connect a resistance (s, Fig. 381) in parallel with the coils of the galvanometer, so as to allow only a known fraction of the current to traverse the instrument. The resistance s is termed a **shunt** to the galvanometei. If s and g represent respectively the resistances of the shunt and galvanometer, and if i is the current traversing the latter, then

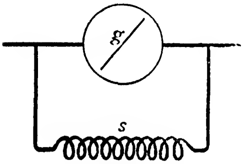


FIG. 381.

$$i = \mathrm{I} \times \frac{s}{g + s}.$$

In order that the ratio i/I may equal $1/n$, the values of s and g must be adjusted so that $s/(g+s) = 1/n$, or $ns = g + s$, or

$$s = \frac{1}{n - 1} \cdot g.$$

Hence, when
$$\frac{1}{n} = \frac{1}{10}, \quad s = \frac{1}{9} \times g,$$

,,
$$\frac{1}{n} = \frac{1}{100}, \quad s = \frac{1}{99} \times g, \text{ and so on.}$$

EXAMPLE.—A galvanometer of 100 ohms resistance requires a shunt in order to reduce the current traversing the coils to one-twentieth of the original amount. What must be the resistance of the shunt? Also, what additional external resistance must be added so that the total resistance remains unchanged?

$$s = \frac{1}{n - 1} \times g = \frac{100}{19} = 5{\cdot}263 \text{ ohms.}$$

The combined resistance of the galvanometer and shunt is

$$\frac{gs}{g+s}=\frac{526{\cdot}3}{105{\cdot}263}=5 \text{ ohms.}$$

Hence, additional resistance required is $(100-5)$ or 95 ohms.

Comparison of Electromotive Forces.

The potentiometer.—The simplest form of **potentiometer** consists of a long length of uniform thin wire mounted on a board, and with a graduated scale under the wire so as to measure *lengths* of it readily. For convenience, the wire is sometimes mounted in zig-zag fashion, as shown in Fig. 382.

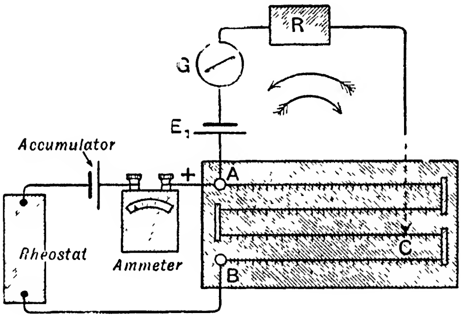


Fig 382 —The potentiometer.

Manganin wire is generally used, owing to its very low 'temperature-coefficient.'

A steady current, derived from an accumulator, passes from A to B, and through a rheostat. In order to ensure that the current strength remains constant during an experiment, it is well to include in the circuit a sensitive ammeter (or, failing this, a tangent-galvanometer of low resistance may be used).

Suppose that it is desired to compare the E.M.F.'s of two different cells, denoted by E_1 and E_2. The +ve pole of E_1 is connected to A; and its circuit includes a very sensitive galvanometer G and a high resistance R (*e.g.* 5000-10,000 ohms). The other end of its circuit terminates in a metal 'knife-edge'—preferably brass; steel is too hard, and copper

is too soft—which can be pressed gently on to any point of AB. Let this point of contact be at C.

The accumulator *tends* to send a current through the circuit of G in a clockwise direction, because A is at a higher potential than C; and the E.M.F. available is the P.D. between A and C. On the other hand, the cell E_1 *tends* to send a current through the same circuit in an anti-clockwise direction, and the E.M.F. available is that of the cell E_1. If the point of contact is adjusted until there is no deflection in the galvanometer G, the E.M.F. of the cell E_1 must be exactly balanced by the P.D. between A and C. Since the wire of the potentiometer is uniform, *the length of the wire between* A *and* C *is a measure of the* E.M.F. *of* E_1.

Then the cell E_2 is substituted for E_1, and the process of finding the point of contact when there is no deflection is repeated. If the two values of AC are denoted by l_1 and l_2, then

$$E_1/E_2 = l_1/l_2.$$

Standards of E.M.F.—In order to determine the actual E.M.F. of any voltaic cell, it is necessary to have some *standard* cell, the E.M.F. of which is known with accuracy, and then to compare the E.M.F. of the voltaic cell with that of the standard cell.

For ordinary purposes, the Daniell cell may be used as a standard: when the dilute sulphuric acid in this cell has a strength of about 1 part of acid to 12 parts of water the E.M.F. is 1·09 volt.

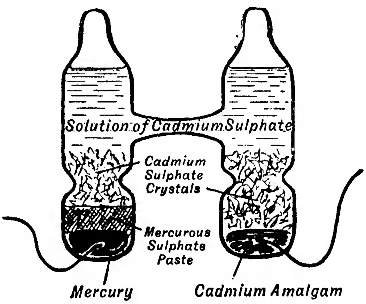


FIG. 383.—The Weston cadmium cell.

The **Weston cadmium cell** is a far more exact standard; and it is recognised as the international standard of E.M.F. A form of the cell is shown in Fig. 383. The glass container, which is permanently sealed, is shaped like the letter H. The positive element is some mercury, at the bottom of the left compartment; and the negative element is cadmium amalgam, at the bottom of the other compartment.

Chemically, cadmium is closely analogous to zinc, and fulfils the same part in a voltaic cell. The other components of the cell are a paste of mercurous sulphate and a saturated solution of cadmium sulphate. The E.M.F. of the cell is 1·0183 volt at 20° C.

As all *standard* cells are liable to polarisation if an appreciable current is allowed to pass through them, it is essential always to connect in series with them a high resistance of at least 10,000 ohms.

Direct determination of the E.M.F. of a standard cell.—The strength of a current traversing a coil, of known resistance, can be determined by means of a copper voltameter. Hence, since $E = I \times R$, the potential-difference between the ends of the coil can be calculated. As in the case of the potentiometer, this circuit can be adjusted so that the potential-difference between the ends of the coil is exactly equal to the E.M.F. of the cell.

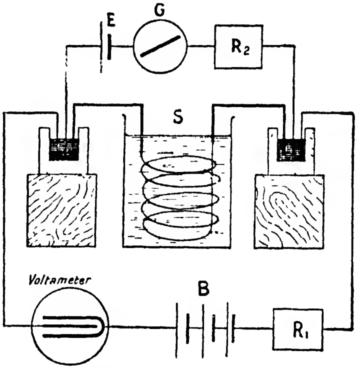


FIG. 384.—Method of determining the E.M.F. of a standard cell.

In Fig. 384, S is a coil of manganim wire, immersed in paraffin oil in a beaker; the terminals of thick copper wire dip into mercury. The resistance of the coil may conveniently be about 4 ohms, and must be measured as accurately as possible. The current is obtained from accumulators (B) connected in series with a rheostat (R_1). E is the standard cell, the E.M.F. of which is to be measured; G is a sensitive galvanometer, and R_2 is a high resistance of about 10,000 ohms.

When conducting the experiment, both circuits are completed, and R_1 is adjusted until there is no deflection in G. Both circuits are then broken, the cathode is removed, dried and weighed. Then it is replaced, the circuit is closed at a given instant, and the current allowed to continue for at least 30 minutes. During this period, R_1 is adjusted when necessary so as to maintain an absence of deflection in G. The subsequent handling of the voltameter is described in Expt. 299 (p. 505).

Comparison of the E.M.F. of two cells (method of 'sum and difference').—When two cells (E.M.F. denoted by E_1 and E_2) are connected in series with a tangent galvanometer, then $\tan\alpha$ is proportional to E_1+E_2. If E_2 is then reversed, $\tan\alpha$ will be less than before, and will be proportional to E_1-E_2 (assuming that E_1 is greater than E_2). Hence

$$\frac{\tan\alpha_1}{\tan\alpha_2}=\frac{E_1+E_2}{E_1-E_2}, \quad \text{or} \quad \frac{E_1}{E_2}=\frac{\tan\alpha_1+\tan\alpha_2}{\tan\alpha_1-\tan\alpha_2}.$$

EXPT. 307.—**Comparison of E.M.F.'s.**—This method can be used with a tangent galvanometer only when the cells differ considerably in their E.M.F. Connect the two cells *in series* with an adjustable resistance and a tangent galvanometer; use a commutator with the galvanometer. Adjust the resistance so that the deflection is about 60°. Note the deflection of both ends of the pointer, and also when the commutator is reversed. Reverse the cell having the lower E.M.F., and repeat the observations. Record the observations thus :

Cells	Deflections.		Mean deflection (α).	$\tan\alpha$
	East End.	West End.		
In conjunction (E_1+E_2)	(i)......	......	$\alpha_1=$......	$\tan\alpha_1=$...
	(ii)......	......		
In opposition (E_1-E_2)	(i)......	......	$\alpha_2=$......	$\tan\alpha_2=$...
	(ii)......	......		

Calculate the value of the ratio E_1/E_2.

GROUPING OF CELLS.

The various methods of grouping cells together so as to form a battery have been described already on p. 480.

Cells in series.—If n cells are connected together in series, and if E and r are the E.M.F. and the internal resistance of each cell, then

The total E.M.F. $=n$E.

,, internal resistance $=nr$.

Then, by Ohm's Law, $I=\dfrac{nE}{R+nr}$ ....(1)

Fig. 385 represents a battery of two cells in series. The continuous and the thick dotted lines are the potential

diagrams when the circuit is open and closed respectively. The lengths AB and BC represent the internal resistances of the cells, and CD represents the external resistance. AV (or CV′) is the total E.M.F. The current is represented by the ratio AV/AD (*i.e.* by $\tan\theta$). Before the circuit is closed the potential-difference between the terminals is CV′, but as soon as the circuit is closed the potential-difference at the terminals

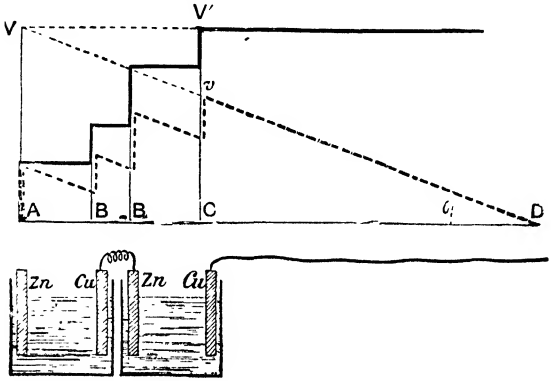


FIG. 385.—Potential diagram of a simple circuit containing two cells in series.

falls to Cv. The remainder of the total E.M.F. (viz. V′v) is used up in overcoming the internal resistance of the two cells and the resistance of the connecting wire BB. (The latter is usually very small, and may be disregarded.) In this case equation (1) becomes $I=\dfrac{2E}{R+2r}$.

Special Case.—Suppose that one of the n cells is accidentally reversed, so that it tends to send a current in the opposite direction. What will be the final result ? There are $n-1$ cells tending to send a current in one direction, and with an E.M.F.$=(n-1)E$, while there is one cell tending to reverse the current with an E.M.F.$=E$. The resultant E.M.F.$=(n-1)E-E=(n-2)E$.

Hence, by Ohm's Law, $I=\dfrac{(n-2)E}{R+nr}$.

Cells in parallel.—If m cells are connected together in parallel, the E.M.F. will be the same as that of one cell. The arrangement will be equivalent to one large cell, the plates of which are m times as large as those of a single cell, hence the

internal resistance will be $\frac{r}{m}$ (where r is the resistance of a single cell).

By Ohm's Law, $$I = \frac{E}{R + \frac{r}{m}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

If the cells are arranged in m rows, each row containing n cells in series, then the resistance of each row is nr. The effect of having m rows side by side will be equivalent to enlarging the plates of each cell m times, and the total internal resistance will be $\frac{nr}{m}$. The total E.M.F. will be nE (*i.e.* the same as n single cells in series).

By Ohm's Law, $$I = \frac{nE}{R + \frac{nr}{m}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Arrangement of cells for maximum current.—It is clear, from equation (1), when r is small compared with R, that the current obtained is approximately proportional to the number of cells used. But if R is small compared with r, then the current is scarcely increased by an increase in the number of cells, since the total resistance $(R + nr)$ will be increased almost in the same proportion as the E.M.F.; in this case it is advantageous to connect the cells in parallel so as to reduce the internal resistance. It can be proved mathematically that a maximum current is obtained when the cells are so arranged that the internal resistance is equal to the external resistance.

EXERCISES ON CHAPTER XL.

1. A cell having an E.M.F. of 2 volts and a resistance of 0·5 ohm is connected up with three lengths of wire having resistances of 1, 2, and 3 ohms respectively, the wires being in series. Find the difference in potential between the ends of the middle wire.

2. A Daniell cell is connected to a voltmeter, the reading being 1·1 volt. The terminals of the cell are now connected to a 2-ohm coil, and the voltmeter reading falls to 0·8 volt. Find the resistance of the cell. What would you expect the voltmeter to read if a 1-ohm coil were substituted for the 2-ohm coil?

(Lond. Matric.)

3. Upon what factor does the rate of production of heat in a wire carrying an electric current depend ?

A 4-volt battery of internal resistance 0·1 ohm sends a current of 0·5 ampere through a resistance of 3·8 ohms and a small electric lamp in series with it. Find the difference of the potential between the terminals of the lamp, and the amount of heat produced in it per second. (Joint Matric. Bd., S.C.)

4. The reduction factor of a tangent galvanometer is 0·24. It is connected in series with a battery, E.M.F. 7·2 volts, resistance 8·25 ohms, and a coil whose resistance is 20 ohms. The deflection is observed to be 45°. What is the resistance of the galvanometer and the connecting wires ? What additional resistance would be needed to reduce this deflection to 30° ? [Tan 30°=0·577.]
(Part of question) (Lond. Univ., G.S.)

5. The resistance of the telegraph wires joining two stations is 20 ohms, and the receiving instrument has a resistance of 80 ohms, and requires a current of $\frac{1}{20}$th of an ampere to work it. What is the smallest number of cells, each of E.M.F. 1·1 volts and resistance 2 ohms, which will suffice to send a signal from one station to the other ? (Oxf. and Camb., S.C.)

6. State Ohm's Law, and show how it leads to a definition of electrical resistance.

The E.M.F. of a cell is 2 volts, and when it is connected to a wire of 10 ohms resistance the potential-difference between the terminals is 1 volt. Calculate the internal resistance of the cell.
(Oxf. and Camb., S.C.)

7. The plates of a cell, the resistance of which is inappreciable, are connected by a platinum wire. How would the rate of development of heat in the wire, and the rate of consumption of zinc in the cell, be affected if the wire were drawn out uniformly to double its length ?

8. The resistance of 100 metres of copper wire (No. 24 S.W.G. ; diameter=0·0559 cm.) is 6·63 ohms at 0° C. What is the specific resistance of the copper ?

9. A column of mercury 106·3 cm. long and 1 sq. mm. cross-section has a resistance of 1 ohm at 0° C. What is its specific resistance ?

10. You are given a piece of German-silver wire (specific resistance $2{\cdot}08\times10^{-5}$ ohm per cm. cube) of diameter 0·5 mm. Calculate the length you will require to make a 5-ohm coil, and describe an experiment by which you can test the accuracy of your coil when you have made it. (Camb. S.C.)

11. The General Post Office employs bronze wire for the overhead lines of local telephone circuits. A sample of this wire has a diameter=1·27 mm. If the specific resistance of the alloy is 3·17 microhms, find the resistance of 1 mile of the wire. [1 mile=1·6093 km.]

12. Describe in detail the experiments you would make to determine the specific resistance of the material of a uniform wire.

What are the principal difficulties in the method described ? (Bristol, 1st S.C.)

13. The resistance of two wires in series is 15 ohms, and in parallel $3\frac{1}{3}$ ohms. What is the resistance of each wire ?

14. A wire has a resistance of 20·5 ohms. What must be the resistance of a wire joined in parallel with it so that the combined resistance is 20 ohms ?

15. A tangent galvanometer, resistance 6·5 ohms, is connected in parallel with a resistance of 3·25 ohms. The reduction factor of the galvanometer is 0·30. If the deflection is 30°, find the total current traversing the circuit.

16. How would you determine experimentally the relation between the electrical energy spent in a metallic conductor and the heat produced ?

Two coils, of resistance 1 and 10 ohms respectively, are connected in parallel across the terminals of a battery of negligible resistance. Compare the rates of production of heat in the two coils. (Bristol, 1st S.C.)

17. The terminals of a voltaic battery of resistance 1 ohm are connected by two wires *in parallel*, their resistances being 6 and 8 ohms respectively. The difference of potential between the terminals is 2 volts. Find the currents, and compare the rates at which energy is expended in the wires. Find also the electromotive force of the battery.

18. Describe some form of galvanometer which is useful for detecting very small electric currents.

A galvanometer of 30 ohms resistance is provided with a 3-ohm shunt. What effect has the use of this shunt on the current sensitiveness of the galvanometer ? (Lond. Matric.)

19. It is desired to use an ammeter, reading up to one ampere and having a resistance of 1·5 ohms, for measuring currents up to ten amperes. Explain carefully, giving all the calculations necessary, what would need be done to attain this end. (Lond. Matric.)

20. The poles of a battery consisting of three cells each of one ohm resistance and 1·6 volts E.M.F., joined in series, are connected by two wires in parallel. The wires have resistances of 5 ohms and 4 ohms respectively. Calculate the current flowing in each wire. How would the intensities of these currents be affected if the cells of the battery were joined in parallel instead of being in series ? (Joint Matric. Bd. S.C.)

21. Describe an experiment to show how the heat developed in a given wire varies with the current passing through it.

A cell which maintains a constant potential-difference of 2·1 volts between its terminals is used to send a current through two resistance coils, each of 2 ohms resistance. Compare the rates of production of heat by the current (*a*) when the coils are in series, and (*b*) when they are in parallel. (Oxf. and Camb., S.C.)

22. Two points A and B are maintained at a constant potential-difference of 110 volts. A third point C is connected to A by two resistances, of 100 and 200 ohms respectively, in parallel, and to B by a single resistance of 300 ohms. Find the current in each resistance, and the potential-differences between A and C, C and B. (Lond. Matric.)

23. A table galvanometer of 17 ohms resistance gives a deflection of one scale division for a current of 5×10^{-6} ampere. Explain how, by the use of appropriate resistances, the instrument can be converted (*a*) into a milliammeter, (*b*) into a millivoltmeter. Calculate in each case the value of the resistance required. (Oxf. and Camb., S.C.)

24. Deduce an expression for the combined resistance of two coils joined in parallel.

When a certain cell is connected to a coil of 10-ohms resistance, a current of 0·1 ampere flows through it. A 5-ohm coil is then connected in parallel with that of 10 ohms and the current through the cell becomes 0·18 ampere. Determine the resistance and E.M.F. of the cell. (Lond. Matric.)

25. What resistance must be placed in parallel with an eleven-ohm coil in order to reduce its resistance to ten ohms? How would you compare the resistance of this combination with that of a standard ten-ohm coil? (Lond. Univ., G.S.)

26. The two ends of a piece of wire of fairly high resistance are fastened together and points A, B, C, D, E are taken, dividing the ring so formed into five equal segments. A and C are connected with the poles of a battery by means of thick copper leads of negligible resistance. Compare the currents in the branches ABC and AEDC. Could you verify your answer without cutting the wire? (Joint Matric. Bd., Matric.)

27. How can the resistance of an electric lamp be measured? Draw a diagram indicating the apparatus that would be used and the manner in which it would be connected up.

(Lond. Univ., G.S.)

28. A carbon filament lamp and a metallic filament lamp are connected in series with an ammeter, a variable resistance and an accumulator battery. A voltmeter is used to determine the potential-difference between the terminals of each lamp. Draw a diagram of the connections. How would you proceed to find the resistances of the filaments as their brightness is altered? What results would you expect to find? Give reasons for your answer. (Durham S.C.)

29. Four cells have each an E.M.F. of 1·35 volts and a resistance of 5 ohms. Draw diagrams showing the various ways in which they might be grouped together, and find by actual calculation which arrangement would drive the largest current through a coil whose resistance is 4 ohms. (Lond. Matric.)

30. What do you understand by the electromotive force of a cell?

Explain the advantage gained by the use of a large cell instead of a small one of the same type.

Describe some practical method of showing, without using a voltmeter, that the E.M.F. of a large cell is the same as that of a small one of the same kind. (Durham S.C.)

31. What is Ohm's Law? Explain how you would join up three similar cells so that you might obtain the strongest possible current in a circuit whose external resistance is much less than the resistance of a cell. (Joint Matric. Bd., Matric.)

CHAPTER XLI.

ELECTRO-MAGNETIC INDUCTION.

THE DYNAMO, TRANSFORMER, INDUCTION COIL, AND TELEPHONE.

Faraday's experiments.—We have seen already that a magnetic field is set up in the space surrounding a wire conveying a current. If the current and magnetic field are connected indissolubly we might anticipate that the creation of a magnetic field round a closed circuit will cause a current to traverse the circuit. Faraday, in 1831, showed that whenever a circuit is moved in a magnetic field in such a manner as to alter the number of lines of force passing through the circuit, then an E.M.F. is set up, proportional in magnitude to the rate of change in the number of lines of force passing through it, and continuing so long as the change continues. This relationship between the rate of change in the number of lines of force and the magnitude of the E.M.F. set up, is known as **Faraday's Law.**

EXPT. 308.—**Induced currents, due to a moving magnet.**—Make a circular coil of many turns of thin cotton-covered copper wire, and tie the turns together with tape so as to make a compact bundle. Set up a sensitive galvanometer, and determine by means of a voltaic cell which terminal must be positive in order to give a deflection to the right. Connect the ends of the coil to the galvanometer, and place the coil on the bench *at a distance from it.*

Bring the N-seeking pole of a bar-magnet quickly towards the face of the coil and observe the deflection of the galvanometer needle (Fig. 386). Note whether it is to the right or to the left. Observe that the current ceases as soon as the bar-magnet is brought to rest. Now withdraw the magnet suddenly, and observe the deflection in the opposite direction.

By means of the direction of deflection of the galvanometer needle verify that (i) *when the N-seeking pole is approaching, the direction of the induced current is such that the near face of the coil has N-seeking polarity* (the direction of the current is anti-clockwise) and (ii), *when the N-seeking pole is receding, the direction of the current is clockwise and the polarity of the near face of the coil is S-seeking.* Using the S-seeking pole of the magnet, note that the direction of the induced currents is reversed. The same results are obtained when the magnet is fixed and the coil is moved so as to approach or recede from the magnet's pole.

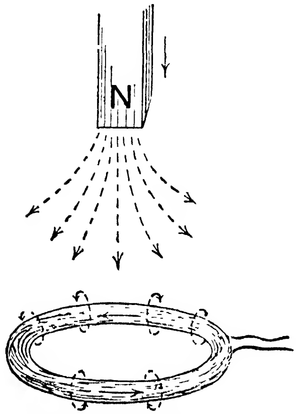


FIG. 386.—INDUCED CURRENTS. The approach of a N-seeking pole sets up in a coil of wire a momentary current, the direction of which is such that the near face of the coil has N-seeking polarity. When the magnet is suddenly withdrawn, the momentary induced current is reversed.

Make the relative movements more slowly, and notice that the induced currents are appreciably less than before.

Hold the coil vertically so that its edge is towards the magnet's pole. No induced current is obtained. In this position of the coil the approach of a magnet pole does not alter the number of lines of force passing through the *interior* of the coil. The experiments prove

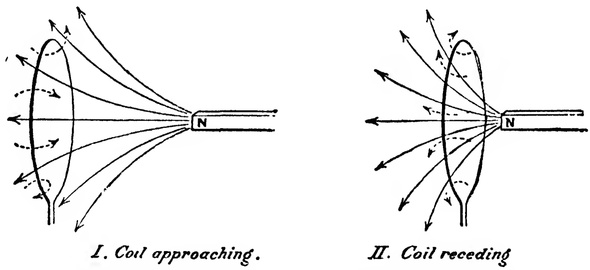


FIG. 387.—THE DIRECTION OF AN INDUCED CURRENT may also be deduced by the principle that when the number of magnetic lines passing through a coil alters, the direction of the current is such that it tends to keep the number of lines of force constant

that (i) **an induced current is obtained only when the number of lines of force passing through the coil is altered**; and (ii) **the induced current will be in that direction which tends to keep the number of lines of force**

constant. Thus, in Fig. 387, i, the coil is approaching a N-seeking pole, the number of lines of force passing through the coil being thereby increased; the induced current will generate lines of force in the opposite direction (shown by the dotted lines), thus tending to neutralise the increase due to the magnet, and creating N-seeking polarity at the near face of the coil. Fig. 387, ii, represents the coil receding from the magnet.

Induction by a moving current.—As explained previously (p. 484), a coil of wire conveying a current exhibits magnetic polarity at its opposite faces, and all the phenomena of Expt. 308 ought, therefore, to be obtained when such a coil is used instead of a permanent magnet.

EXPT. 309.—**Induced currents, due to another current.** (i) Connect one coil of wire A (Fig. 388) with a small battery, and the other coil B with a sensitive galvanometer. Coil A is usually called the **primary circuit**, and B the **secondary circuit.** With the current traversing the primary in the direction shown, hold it above B and bring it rapidly towards B. Notice the momentary current in B, and verify that its direction is as shown in the diagram and in the *opposite* direction to that in A. Quickly withdraw coil A, and notice that the momentary current in B is now in the *same* direction as that in A.

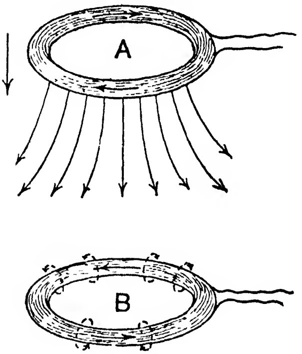


FIG. 388.—INDUCED CURRENTS. The bar-magnet of Fig. 386 may be replaced by a coil of wire A traversed by a steady current. The effect on coil B is the same.

(ii) Break the circuit of coil A, and lay it on the top of coil B. Close the circuit of A and note the momentary *reversed* current in B. Break the circuit A, and observe that the momentary current in B is in the *same* direction as that in A. The effects are the same as when the primary coil approached or receded.

If the coils of wire were unwrapped so as to form two long straight wires near together, the effects obtained would be similar; but the experiment would be difficult to perform because the effects would be much smaller.

(iii) Hold a short bar of soft iron within the coil A, and notice that the induced currents in B are stronger than before. This

result might be anticipated, because the lines of magnetic force through the secondary coil include those due to the magnetised iron in addition to those due to the current in the coil A.

Remember that the effect produced in the secondary circuit is really an electromotive force, and this happens whether the circuit is closed or open. When the circuit is closed the E.M.F. causes a current to flow, but the magnitude of this depends both upon the induced E.M.F. *and upon the resistance* of the circuit. The induced E.M.F. would be set up even if the secondary circuit consisted of one single turn of wire, but probably it would be too small for the current to be detected. If the circuit consisted of two turns of wire, the same E.M.F. is induced in *each* turn, and the total E.M.F. between the extreme ends of the coil would be twice as great as that generated in a single turn; with 50 turns of wire in the coil the induced E.M.F. would be 50 times as great. Similarly, 50 small voltaic cells joined in series gives an E.M.F. 50 times as great as that obtained with one cell.

It can be proved that **the induced E.M.F., expressed in absolute units, is equal to the rate of change in the number of lines of force enclosed within the circuit.** Thus, if the number of lines of force through the circuit changes from N_1 to N_2 in an interval of time t, then

$$E = (N_1 - N_2)/t\,; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and, if the circuit be closed, since $I = E/R$,

$$I = (N_1 - N_2)/Rt. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Equation (1) may be used as the basis of a definition of the absolute unit of E.M.F., from which is derived the practical unit—the volt (p. 513). The absolute unit of E.M.F. is then defined as that which is set up in a single circuit when the number of lines of force through the circuit changes by unity per second.

Lenz's Law.—Energy is required in order to generate the induced currents observed in the preceding experiments. Since there is no source of energy in the secondary coil, the energy represented by the induced currents must be due to some external agency; in fact, in Expts. 308 and 309, it

originates from the **mechanical work** done in overcoming the mutual forces of attraction or repulsion which exist between the currents in the two circuits. Thus, when the primary circuit (or a magnet pole) is approaching, the direction of the induced current is such as to exert a repulsion ; also, when receding, the induced current will exert an attraction. Lenz's law may be stated thus: **The induced current is in such a direction that its reaction tends to stop the motion or change to which the induced current is due.**

It has been explained (p. 511) that the energy of a current traversing any simple circuit is converted into its equivalent quantity of heat energy. The same applies to *induced currents ;* and the *mechanical work* required to create an induced current re-appears as heat in an equivalent quantity. Reference has been made previously (p. 229) to the application of this principle in a method of determining the **mechanical equivalent of heat.**

Generation of electric current.—A slight modification of the previous experiments will explain the principle of the

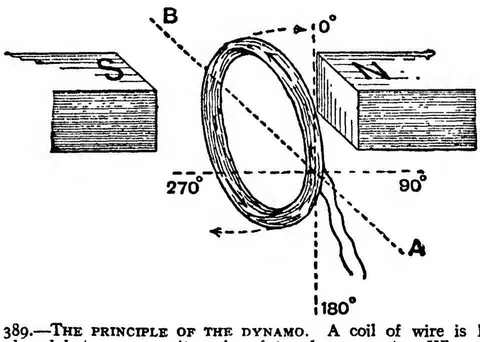


FIG. 389.—THE PRINCIPLE OF THE DYNAMO. A coil of wire is held in the hand and between opposite poles of two bar-magnets. When rapidly rotated round an axis AB into a horizontal position, an induced current is generated.

dynamo. In Fig. 389 two bar-magnets are supported horizontally with opposite poles pointing inwards, and a coil of wire, with its ends connected to a galvanometer, is held in a vertical position midway between the poles, so that it can be rotated round the horizontal axis AB. Consider the rotation from 0° to 90° : the number of lines of force passing through

the coil is reduced from a maximum to zero, and an induced E.M.F. is set up in the direction shown. The rate at which lines of force are cut by the coil at first is but slight, but the rate rapidly increases as the coil approaches the horizontal position. When the rotation is continued from 90° to 180°, the induced E.M.F. is in the same direction as before, and rapidly diminishes from a maximum to zero. When rotated in

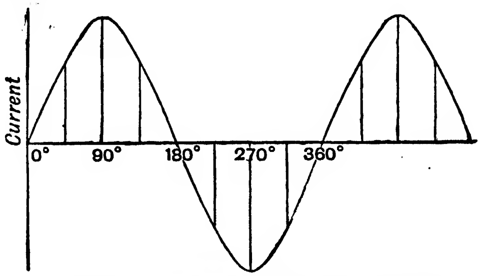


FIG. 390.—VARIATION OF CURRENT OBTAINED BY ROTATING A SINGLE COIL IN A MAGNETIC FIELD. The curve shows the growth, decay and reversal of the current induced in a rotating coil (Fig. 389). It is characteristic of an *alternating current.*

the same direction from 180° through 270° to its original position the induced E.M.F. is in the opposite direction; it gradually increases to a maximum at 270°, and becomes zero when the coil has again become vertical.

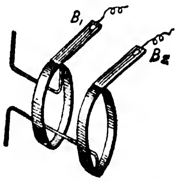


FIG. 391.—The method of 'collecting' the current generated in the armature of an alternating-current dynamo.

Thus, at every half-revolution the current is reversed: it is called an **alternating current.** Fig. 390 represents how the current set up in the circuit changes during one complete revolution of the coil; and the number of times this cycle of change is repeated in each second is called the **frequency** of the alternating current.

The apparatus is a model of the kind of dynamo called an **alternator,** and the coil of wire represents the **armature.** The rotation of the coil could not be continued indefinitely, because the wires leading from it would be twisted and broken. So, in practice, the ends of the coil are joined to metal rings (Fig. 391), which are firmly attached to, but insulated from, the armature; and two brushes (B_1 and B_2), of flexible metal

or of hard carbon, rub on these rings and convey the current to the external circuit.

Fleming's **Right-hand Rule** (Fig. 392) is a convenient method of deducing the direction of the induced current in a conductor

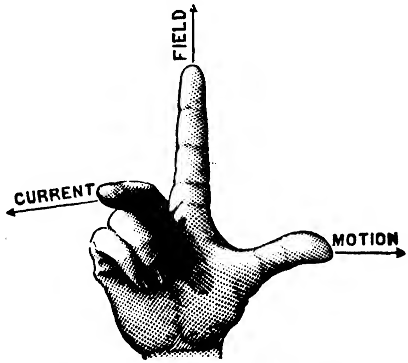


FIG. 392.—Fleming's Right-hand Rule.

which is moving across the lines of force of a magnetic field: **Hold the thumb and first finger of the right hand as fully extended as possible and bend the second finger at right angles to the palm. If the first finger represents the direction of the lines of force, and the thumb that of the motion, then the second finger will point in the direction of the induced current.**

(It will be remembered that Fleming's **Left-hand Rule** applies to the principle of the electric *motor* (p. 492), in which a conductor *conveying a steady current* experiences a mechanical force when placed in a magnetic field.)

Principle of the dynamo.—In the simplest type of dynamo the wire of the armature is wound on a soft-iron core (Fig. 393), which in shape resembles a weaving-shuttle, and its cross-section is somewhat like a dumb-bell. The iron core serves (i) as a rigid support for the coil of wire, and (ii) it causes a great increase in the number of magnetic lines of force which are cut by the coil of wire when it rotates. This type of armature is now used only on very small machines; larger dynamos have armatures of much more elaborate construction. In Fig. 393, i, a 'shuttle-wound' armature is rotating between the soft iron pole-pieces of a large horse-shoe

magnet. Remembering (i) that soft iron, when placed in a magnetic field, becomes strongly magnetised, and (ii) that the lines of force appear to prefer a path through soft iron, although this may not be the shortest, we readily understand why the magnetic lines from N are bent upwards, in their effort to enter the soft iron and to avoid the air ; they emerge through the lower end of the core, and proceed through S into the permanent magnet. A moment later the armature will have rotated into the position shown in Fig. 393, ii. The iron core is again traversed by magnetic lines, *but their direction is reversed.* Hence, in the very brief interval between the two

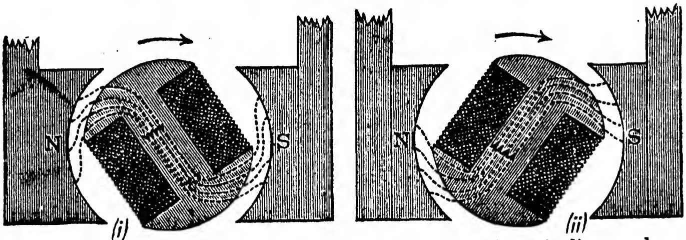


FIG. 393.—A SIMPLE FORM OF DYNAMO-ARMATURE. The coil of wire is wound on a core of soft iron. In position (i) the magnetic lines pass *downwards* through the core; and when the armature has rotated into position (ii) the direction of the lines is reversed. This sudden reversal of the magnetic field generates in the coil a considerable electro-motive force.

positions shown in the diagram, the magnetic lines have not only been removed, but re-established in the reverse direction ; and this explains the fact that during this interval the maximum E.M.F. is generated in the coil. There is, of course, another pulsation of maximum E.M.F. when the armature has rotated through one-half of a revolution.

Direct-current dynamo.—By a slight modification of the armature the alternating current may be changed into a *direct* current (*i.e.* a current always passing in one direction). Instead of the two rings of Fig. 391, the ends of the coil are joined to the two halves of a split tube (Fig. 394). With the magnet poles arranged as shown, an E.M.F. is induced in the arm AB of the coil and in the direction from A to B, when the coil is rotated in the direction shown. Similarly, an E.M.F. is induced in the lower arm CD and in the direction from C to D. Hence, when the two carbon brushes B + and B −

are joined to any conducting circuit, a current will leave the armature through B +, and return through B −. At any instant when the coil is rotating B + will always be in contact

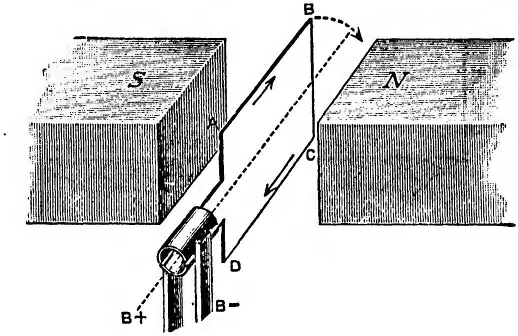


FIG. 394.—THE PRINCIPLE OF A DIRECT CURRENT DYNAMO. The ends of the rotating coil are joined to the two halves of a split tube which rotates with the coil. Thus the induced current always leaves through the same brush (B+). The split tube is called a ' commutator.'

with that half of the split ring through which the current is coming from the armature. The current thus obtained can be represented graphically as in Fig. 395. The alternating current is thus *commuted* (or changed) into a direct current, and the split ring is called a **commutator.** By having a number of coils distributed uniformly round the armature each coil gives a current-curve like Fig. 395, but separated from the

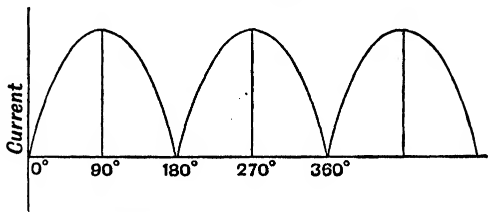


FIG. 395.—VARIATION OF CURRENT OBTAINED BY ROTATING A SINGLE COIL FITTED WITH A COMMUTATOR. This is characteristic of a *direct current.*

others by a short interval of time, and the resultant of all this is a current of practically uniform strength. The slightly waving line in Fig. 396 indicates the result which might be obtained with five separate coils; with a still greater

number of coils the current would approach more nearly to a straight line.

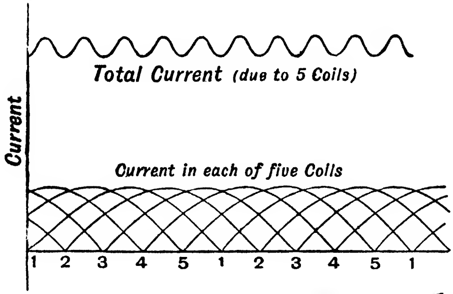


FIG. 396.—THE ADVANTAGE OF USING AN ARMATURE OF SEVERAL COILS EQUALLY SPACED ROUND THE CORE. Each coil generates a current resembling Fig. 395, and the *total* current is much more uniform.

The essential parts of a *dynamo*, for generating direct-current, are shown in Fig. 397. The armature-coils are wound

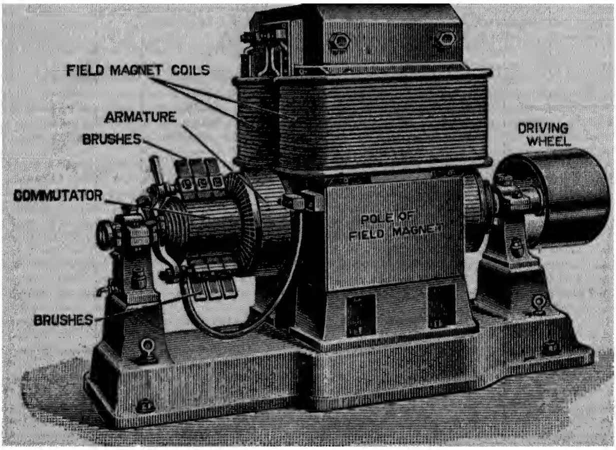


FIG. 397.—A DYNAMO FOR GENERATING DIRECT-CURRENT. In large dynamos, an electromagnet is used instead of permanent magnets, for setting-up the magnetic field in which the armature rotates. The current for exciting the electromagnet is obtained from the dynamo itself.

on a cylindrical *drum* of thin sheets of soft iron fixed together : this drum serves both to support the coils and to increase the magnetic field by substituting for air a continuous iron path—

except for the narrow air-gaps required for free rotation. Just as the number of coils is increased, so must the number of parts into which the commutator ring is split increase; when there are 16 coils, the commutator is divided into 16 separate parts. The commutator consists of copper bars separated from each other by thin layers of insulating material.

The magnetic field in which the armature rotates is obtained by means of an electromagnet, of horse-shoe shape, excited by a current traversing the *field-magnet coils*. This current is generated by the dynamo itself: in some types of machine, the whole of the current passes through the coils—the dynamo is then said to be 'series-wound.' A method which is more frequently adopted is to have alternative paths for the current, just after it leaves the brushes—one path is through the coils, and the other path is through the 'external circuit' (*i.e.* the cable, the lamps, motors, etc.); thus the field-magnet coils act as a 'shunt' to the external circuit, and the dynamo is then said to be 'shunt-wound.'

Transformers.—It will be remembered that electrical power is expressed by the product *amperes* × *volts*, and this product gives the power in terms of the *watt*. Thus, if it is desired to transmit 10,000 watts (that is 10 kilowatts or 13½ horse-power), it may be done by sending a current of 100 amperes at a pressure of 100 volts, or a current of 10 amperes at a pressure of 1000 volts, and so on. So large a current as 100 amperes would require for its transmission a *thick* copper wire; and if the distance were considerable the cost of the wire would be prohibitive. But high voltage does not require a thick wire. Hence, to reduce the cost of equipment it is desirable to use as high a voltage as possible and a correspondingly small current.

A voltage exceeding 500-600 volts is impossible if a direct-current dynamo is used, owing to the difficulty of ensuring the insulation of the sections of the commutator; for this reason alternating currents are generally used for transmitting electric power to a distance. There is the additional advantage that, as the current is continually varying, a device called a **transformer** may be used by which the voltage is increased

enormously (and the current reduced correspondingly); this small current at high voltage is then sent along *thin* overhead wires to the distant station, where it is passed through another transformer, so as to reduce the voltage and increase the current correspondingly, to a degree which is appropriate for lighting lamps or driving motors in buildings. The transformer is based upon an experiment first carried out by Michael Faraday, which is easy to repeat.

EXPT. 310.—**The transformer.**—In Fig. 398, C is a circular ring of soft iron about $\frac{3}{4}$ inch in thickness; it may be made of a coil of thin soft-iron wire, such as is used by florists. P is a coil

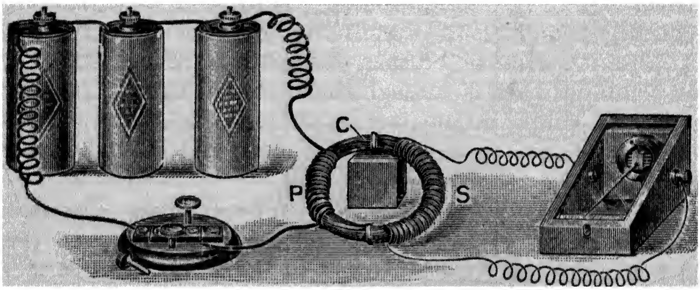


FIG. 398.—THE PRINCIPLE OF A TRANSFORMER. Two coils of cotton-covered wire, P and S, are wound round portions of a soft-iron ring C. The coil P has a few turns of thick wire, and S has many turns of thin wire. When a current through P is started, stopped or reversed an induced current is generated in S.

of fairly thick copper wire (cotton-covered) consisting of comparatively *few* turns wound on the soft-iron core; it is called the **primary** coil. S is a similar coil, but of *many* turns of thin copper wire; it is called the **secondary** coil. Connect the ends of the coil S to a galvanometer, and the ends of coil P to the terminals of a voltaic cell. When the primary circuit is closed or completed so that a current can pass along it, a momentary current is set up in the secondary circuit; and when the former is broken an induced current in the opposite direction is obtained in the latter. When the primary current begins or ceases, increases or diminishes, induced currents are set up in the secondary.

The result is due to the fact that the current in the primary coil magnetises the soft-iron ring, setting up circular magnetic lines of force *within the iron.* All of these lines of force pass through each turn of the secondary coil, thus creating a

momentary E.M.F. in each turn. There are many turns of wire in the secondary coil, and the total E.M.F. between its extreme ends will be equal to the sum of the electromotive forces set up in each turn: it may be compared to the total E.M.F. obtained when a number of voltaic cells are joined together *in series.* When S contains 10 times as many turns as P, the E.M.F. at the ends of S will be 10 times as great as that between the ends of P. But the power obtained from S cannot be greater than that which is put into P; in practice, it is slightly less. Hence, if the voltage is increased 10 times, then the current is reduced to slightly less than one-tenth.

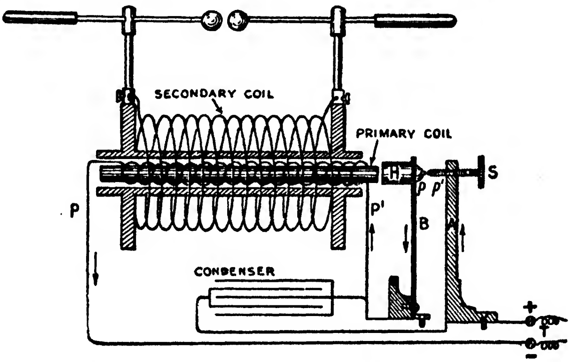


FIG. 399.—AN INDUCTION COIL CLOSELY RESEMBLES A TRANSFORMER. The soft-iron core becomes magnetised when a current traverses the primary coil PP'; and this coil is surrounded by the *secondary coil,* which consists of many turns of thin wire. The primary circuit is rapidly opened and closed at the platinum contacts *pp'* by the movement of the soft-iron hammer H; and each of these operations generates a high E.M.F. in the secondary coil.

The induction coil.—The induction coil differs from the transformer (Fig. 398) only in the fact that the soft-iron core is straight instead of circular. Fig. 399 represents the essential parts of the instrument. The primary coil consists of two or three layers of thick copper wire wound round a core of soft-iron wire; the secondary coil, of many turns of thin wire, is wound on an ebonite tube, and its ends terminate in adjustable brass rods and knobs. The poles of the battery are joined to the terminals T, and the current passes up the metal pillar A, across the platinum contacts *p'p*, down the brass spring B, through the primary coil, and back to the battery. The screw S is adjusted until the contacts just touch. The current

magnetises the iron core, and this attracts the soft-iron hammer H fixed to the top of the spring, and thereby the circuit is broken at $p'p$. The core ceases to be a magnet, and the sudden withdrawal of the magnetic lines of force generates in the secondary coil a high E.M.F., sufficient to cause a spark to pass between the knobs. The hammer returns to its initial position, the current again passes through the primary coil, and the sudden magnetisation of the core again sets up an E.M.F. in the secondary coil. This process is continued automatically by the hammer. With large coils it is possible to obtain sparks 10-15 inches long.

There is always more or less sparking at the contacts p' and p when the coil is working, and it may be sufficient to destroy the contacts unless precautions are taken, when the coil is made, to add a device for reducing the sparking to a minimum. Even in a simple electrical circuit, worked by a small battery, a small spark may be observed at the moment when the circuit is broken. The reason for this is that, owing to its *inertia* effect, it takes an appreciable time to set up a current; and, when once set up, it tends to continue. So, when a circuit is suddenly broken, the current tends to leap across the small air-gap formed at the point where the break occurs, thus delaying the cessation of the current. The sparking is diminished by connecting the uprights A and B to a condenser, which is usually placed inside the wooden case under the coil. When the contacts p and p' separate, the current *surges* into the condenser instead of leaping across the air-gap; the next instant the charge surges back through the battery and through the primary coil from P to P′, thus helping to demagnetise the soft-iron core. All this happens in the short time before the hammer returns to its original position.

The earliest induction coil was made, in 1851, by H. D. Ruhmkorff (1803-1877), a native of Hanover: for this reason, the apparatus is sometimes termed the Ruhmkorff coil.

The telephone.—In the telephone, the air-waves set up by speech are converted into feeble fluctuations of an electric current which is transmitted along a wire to a distant apparatus. There the fluctuating current is re-converted into

audible sound-waves which are identical with those generated at the other end of the wire.

The main principles of a telephone are shown in Fig. 400. At the base of the mouthpiece of the **transmitter** is a very thin disc of flexible carbon, supported round its edge. Behind this is a shallow circular cavity loosely packed with granulated carbon ; it is like a pill-box, with a thin carbon lid, a hard carbon bottom and cotton-wool sides. The lid and the bottom are connected by wires to a small battery of dry cells and to the **receiver** at a distant station. The receiver consists of a cylindrical bar-magnet with a short extension of soft iron

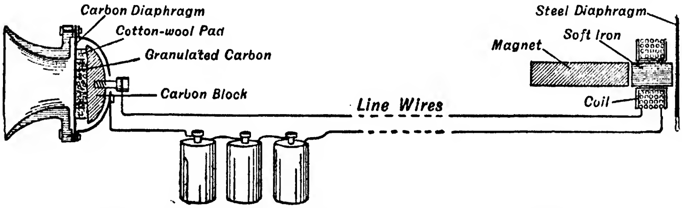


FIG. 400.—THE TELEPHONE. The sound-waves fall on the carbon diaphragm of the *transmitter*, shown on the left. The vibrations of the diaphragm vary the pressure on, and therefore the resistance of, the loosely packed carbon granules behind it. This varies the current traversing the line-wires to the *receiver*. Here the current passes through a coil wrapped round a soft-iron core already magnetised by a permanent magnet, and the varying pole-strength of the iron sets up vibrations in the steel diaphragm.

round which is wrapped a coil of thin silk-covered wire, with its ends joined to the *line-wires*. Just in front of the soft iron is a disc of flexible steel : a special alloy of steel, known as 'Stalloy,' is now generally used.

Action of a telephone.—When arranged as shown, a small steady current is passing through the whole apparatus. If sound-waves fall on the diaphragm of the transmitter it vibrates to-and-fro ; when it moves inwards, the carbon granules are pressed together more tightly, the resistance which they offer to the current is diminished, and a pulse of *stronger* current flows along the line-wires ; when the diaphragm moves outwards, the pressure on the granules is diminished, their resistance increases, and a pulse of diminished current is set up in the line-wires. These variations of current, passing through the coil of wire in the receiver, cause

corresponding variations in the strength of the polarity of the soft iron, and the attraction which it exerts on the diaphragm fluctuates in the same way. Hence, vibrations are set up in the diaphragm, and these cause audible air-waves, which resemble in every particular the air-waves which affected the diaphragm of the transmitter. Thus, if middle C of the musical scale is sung into the transmitter, its diaphragm will vibrate to-and-fro 256 times per second. The current flowing along the line-wires will fluctuate with the same frequency, the diaphragm of the receiver will vibrate at the same rate, and the audible note heard will coincide in pitch with that which was sung into the transmitter.

In telegraphy, only one wire is used, and the current returns to the sending station through the earth. In telephones, the earth cannot be used in this way, because the fluctuations of

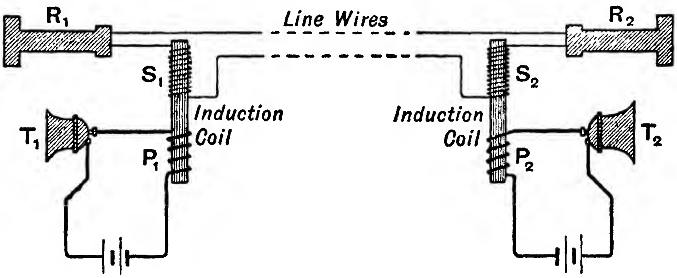


FIG. 401.—THE TELEPHONE. The apparatus of Fig. 400 is not efficient when the line-wires are long, because the changes in resistance of the carbon granules are then small compared with the total resistance of the circuit. So, an induction-coil (or telephone-*transformer*) is inserted at each end, and the transmitter-current passes through only the primary coil P of the transformer. Magnified fluctuations of E.M.F. are set up in the secondary coil S of the transformer, and these determine the current in the line-wire.

current which operate the receivers are extremely small, and they would be completely disturbed by the stray currents which are often passing through the earth, especially in districts where electric tram-lines or electric-power cables are laid. Hence, there are always *two* line-wires to each telephone equipment.

The variations in the resistance of the carbon granules are very slight, and if these are to produce sufficiently great

variations in the current, it is essential that the *total* resistance of the circuit be small. This is impossible if the line-wires are very long. The difficulty is overcome by inserting in the circuit near to the transmitter a small **induction coil** (P_1S_1, Fig. 401), and sending the varying current from the transmitter only through the primary circuit P_1 of the coil. Thus, the resistance of the transmitter-circuit can be made very small, and the fluctuations of current therefore are considerable. The fluctuating current in P_1 sets up much greater fluctuations of E.M.F. in the secondary coil S_1 (because there are far more turns of wire in S_1 than there are in P_1), which cause sufficient variations of current along the line-wires to give rise to audible sounds in the distant receiver R_2. It is evident that the person speaking into his transmitter T_1, with R_1 applied to his ear, will hear his own conversation ; but this is always the case, even in ordinary conversation, and it causes no confusion.

EXERCISES ON CHAPTER XLI.

1. Give an account of the laws which govern the production of induced currents.

Describe a simple arrangement of apparatus by means of which these currents can be exhibited. (Bristol 1st S.C.)

2. Describe three simple experiments to illustrate three different ways of obtaining an induced current in a coil connected to a galvanometer. Show clearly how you would arrange the experiments so that the deflection of the galvanometer would be in the same direction in each experiment. (Lond. G.S.)

3. A loop of wire whose ends are attached to a delicate galvanometer is carried slowly up to one end of a stationary coil of insulated wire, slipped over it without stopping and taken some distance beyond ; it is then brought back over the same path to the starting point. How will the galvanometer behave supposing a steady current has been flowing in the stationary coil all the time ? How do you account for this behaviour ?

(Joint Matric. Bd., S.C.)

4. Describe experiments illustrating the induction of currents, and explain how arrangements can be made to obtain by such means a current through a wire always in one direction. (If the candidate wishes he can answer the second part by describing some simple form of dynamo.) (Joint Matric. Bd., S.C.)

5. Supposing that a railway line is laid in England on insulating material, and that the two rails are connected at a certain

station by a cross-wire, show that a current will flow in the cross-wire and the rails, when a train is moving on the line, and draw a figure showing its direction when the train is moving from the station. Give a reason for the direction you assign.

6. A length of insulated wire is coiled up to form a hoop of about 15 inches diameter, leaving long free ends which are attached to a delicate galvanometer. The hoop is held horizontally at first and then turned over on an axis which lies on a line approximately East and West. Explain the behaviour of the galvanometer :

(*a*) As the hoop passes from the horizontal to a vertical position.

(*b*) As it passes over to the horizontal.

(*c*) As, still going on, it approaches to the vertical again.

(*d*) As it approaches its original position, having made a complete turn.

Each movement is made rapidly and is followed by a pause.
(Joint Matric. Bd., Matric.)

7. Explain how the secondary current is produced in a Ruhmkorff coil. In what respects do the secondary and primary currents differ ? (Joint Matric. Bd., S.C.)

8. Explain the working of an induction coil, pointing out the ways in which the different parts of it contribute to increase the length of the spark. (Lond. Matric.)

9. Two circular coils are placed near to and facing one another. Through one of them is passed a continuous current in a right-handed sense as viewed by a particular observer. What happens in the other coil, when (*a*) the first coil is moved nearer to it ; (*b*) the first coil is turned right round ; (*c*) the current in the first coil is increased ; (*d*) the current in the first coil is rapidly made and broken ? (Joint Matric. Bd., S.C.)

10. Describe experiments to illustrate the laws of electromagnetic induction of current.

A copper ring placed on one pole of an electro-magnet is thrown off when the magnet is excited. Explain this observation.
(Bristol 1st S.C.)

CHAPTER XLII.

RADIO COMMUNICATION. X-RAYS AND RADIOACTIVITY.

Electric oscillations.—The fundamental principle of the generation and propagation of electric waves through space is based upon the fact, discovered many years ago, that under special conditions the spark-discharge of a Leyden jar (or any other form of condenser) is **oscillatory.** The discharge does not consist of a momentary transference of electrons in one direction, and in just sufficient quantity to discharge the jar: a quantity of electrons greater than this are transferred, and the charges on the plates of the condenser are momentarily reversed; another discharge takes place, and the charges are again reversed. This process is repeated several times before the discharge finally ceases; but at each discharge the quantity of electrons transferred is much less than in the preceding discharge—in other words, the electric oscillation is rapidly 'damped down.' The current passing between the terminals during the process may be represented by Fig. 402. The surging to and fro of the electrons is much like an *alternating current* which is rapidly decreasing; but instead of each oscillation taking 1/50 sec. or 1/100 sec., the time is as short as 1/20,000 sec., or even one-millionth of a second, the time occupied depending upon the capacity of the condenser and upon the dimensions and arrangement of the circuit.

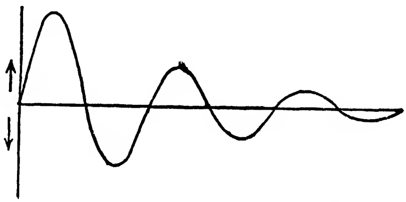


FIG. 402.—A damped electric oscillation.

The oscillatory character of the discharge has been compared to the vibrations of a flexible steel strip clamped at one end. After the free end has been deflected and then released, it returns to its position of rest; but having acquired speed, its *inertia* causes it to 'overshoot the mark'; thus it continues to vibrate to and fro several times, the amplitude (or extent of swing) gradually becoming less. The steel strip would not vibrate thus if immersed in treacle or thick oil, because the 'resistance' of the medium would be too great; a *low* resistance is necessary. Nor can the experiment be carried out with a strip of lead, because lead has not the 'elasticity' of steel. It seems then that there are three essentials if vibrations are to be obtained, viz. (i) low resistance, (ii) elasticity, and (iii) inertia.

In the case of a spark-discharge low resistance is obtained by having the spark-gap as short as possible; and the 'capacity' of the condenser imparts what is equivalent to 'elasticity.' The quality of 'inertia' in ordinary matter has its equivalent in the **inductance** of a circuit in which a varying current is passing. The simplest circuits have a small amount of inductance; but it can be greatly increased by inserting in the circuit a wire coil of few turns. Inductance is so important that a fuller explanation is required.

Inductance.—Imagine a coil of wire (Fig. 403), the ends of which are connected to the terminals of a voltaic cell. The current does not acquire instantaneously its maximum strength for this reason: the lines of force due to the current pass through the coil from left to right, and their sudden increase in number is equivalent in its effect to the rapid approach of the N-seeking pole of a magnet towards the left-hand end of the coil. By Lenz's Law (p. 566), this approach must *tend* to set up N-seeking polarity in that end of the coil, and the direction of this induced current is 'anti-clock-wise,' or *opposite* to that shown by arrow-heads in the diagram. Hence the actual current is weaker than it would be, and an appreciable fraction of a second is required for the current to acquire its final strength.

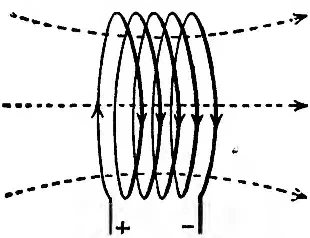


FIG. 403.—Inductance delays the growth and decay of a current.

Similarly, when the circuit is broken, the number of lines of force through the coil diminishes : this is equivalent to removing the N-seeking pole away from the coil ; and, by Lenz's Law, this tends to induce a current such that the near end of the coil has S-seeking polarity, *i.e.* to induce a current in the *same* direction as that of the current already traversing the coil. In other words, the decay of the current is delayed.

Electro-magnetic waves.—The electric condition of the medium near to a spark-gap, across which an oscillatory discharge is about to take place, is indicated by Fig. 404 (i). Before the electrostatic lines of force can shrink back into the circuit, the charge has 'overshot the mark' (Fig. 404 (ii)): a closed loop of electrostatic force has been established and travels outwards into space. Meanwhile, a second loop is generated—this is shown in process of formation in Fig. 404 (iii). Notice that, in this loop, the direction of the electrostatic force is reversed. The discharge, of course, has the character of a rapidly oscillating current, and has associated with it groups of circular lines of *magnetic* force. Hence each loop of electrostatic force is accompanied by a group of these lines of magnetic force (Fig. 405), the directions of the force through two consecutive loops being *opposite* to each other.

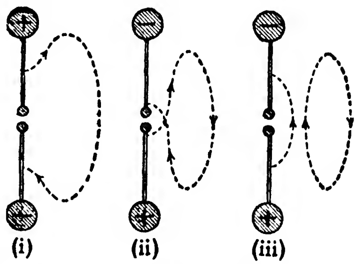


FIG. 404.—Generation of loops of electric strain.

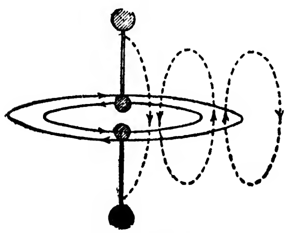


FIG. 405.—A group of electric and magnetic waves travelling outwards.

So early as 1867, James Clark Maxwell (1831-1879) proved theoretically, by means of a remarkable mathematical investigation, that a rapid oscillatory discharge must give rise to a system of electro-magnetic waves, and that these must travel through space with a velocity equal to that of light (186,300 miles per second, or 3×10^8 metres per second) ; further, he

deduced that these waves are identical in their nature with those of light, and differ from them only in wave-length.

It was not until 1888 that electromagnetic waves were generated and detected at a distance from their source, and their wave-lengths measured. This was done by Heinrich Hertz (1857-1894).

Frequency and wave-length.—An important point is the relation between the 'frequency' (or number of oscillations per sec.) and the 'wave-length' (or distance between the centres of alternate 'loops' of electric force). Suppose that the electric oscillations in a discharge take place at the rate of n complete oscillations per second, and that the discharge lasts for at least one second: n complete waves will be generated in that time. At the end of one second the first wave will have travelled to a distance of 3×10^8 metres, and the last wave will be quite near to the circuit. Over this distance there will be n waves equally distributed, and the wave-length will be equal to $3 \times 10^8/n$ metres. Thus, when the number of waves generated per second is 822,000, the wave-length is $3 \times 10^8/822{,}000 = 365$ metres (or 400 yards approximately). This is the wave-length used by the London station of the British Broadcasting Company. If the waves were 45,000 billion times shorter the human eye would be sensitive to them, and would detect a red light coming from the aerial over the station!

Principles of 'spark' transmission.—Signor Marconi found that the electric waves are transmitted to a much greater distance when one side of the spark-gap is connected to the earth, and the other to a long vertical wire (now called the **aerial**). This is shown in Fig. 406, which represents the fundamental principles of what is termed 'spark transmission.' A sequence of sparks can be generated across the spark-gap by means of an induction-coil, and an oscillatory character is given to the discharge by connecting the spark-gap to an 'oscillating circuit.' Notice that this circuit contains the essentials for oscillation, viz. a condenser, a short coil, and a short spark-gap. An oscillatory current is thus set up in the aerial, from which electromagnetic waves are generated at a

considerable height above the earth's surface. As one of the sparking terminals is connected to earth, the waves emitted

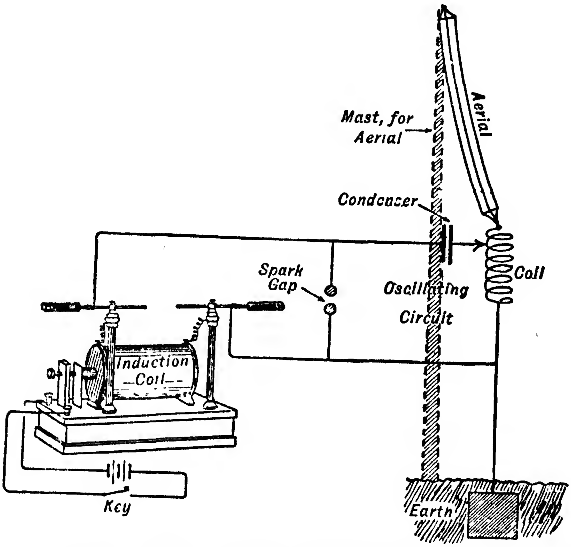


FIG. 406.—A SIMPLE FORM OF WIRELESS TRANSMITTER. Each discharge across the spark-gap is accompanied by electric oscillations in the *oscillating circuit*, and these set up oscillations in the aerial.

by the aerial consist of only *half-loops* of electrostatic strain and circular magnetic lines of force, as shown in Fig. 407.

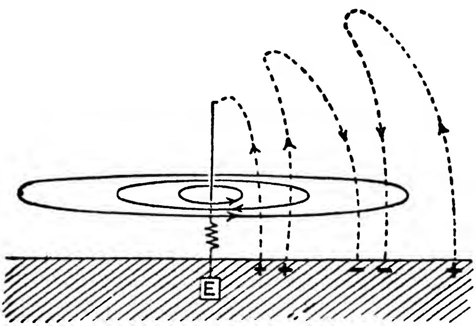


FIG 407.—Radiation from an aerial.

The free ends of the loops terminate on the earth's surface,

and they travel forwards at the same speed as the loops themselves. The frequency of the waves, and therefore the wave-length, can be varied by altering the capacity of the condenser and the number of turns of wire in the coil included in the oscillating circuit. By means of the key, long or short sequences of sparks can be generated, and by this means signals can be transmitted in accordance with the Morse Code of ordinary telegraphy (p. 488). The apparatus shown in Fig. 406 does not represent the method of generating the powerful waves required for transmitting to distant receiving-stations and in 'broadcasting': much more elaborate appliances are required for this purpose.

The detection of electromagnetic waves.—It is well known that when the dampers are lifted away from the wires of a piano, by depressing the loud pedal, and a brief single note is sung loudly near to the instrument, the *same note* of the piano will respond in sympathy. This phenomenon of resonance is due to the fact that when the sound-waves impinge upon the wires, that wire which when struck or plucked gives out waves of the same frequency will be made to vibrate by the sequence of waves striking its surface; but none of the other wires are affected.

The same phenomenon is found when a sequence of electromagnetic waves travels past the aerial of a receiving station. It is easy to realise how the moving magnetic lines of force, with their directions alternating as each wave passes by, will tend to set up a feeble oscillating current in the aerial. But the result will be satisfactory only when the aerial, and the circuit to which it is connected, is adjusted so that its natural 'frequency' of electric oscillation is the same as the 'frequency' of the arriving waves. The aerial, and its attachments, have to be *tuned*, so as to respond to the wave-length which it is required to detect.

The crystal receiving-set.—A simple receiving-set, in which a crystal-valve is used, is shown in Fig. 408. The waves arriving from the transmitting-station set up oscillations in the aerial, which is roughly adjusted so as to be 'in tune' with the transmitter, and these *induce* oscillations in the coil

of the neighbouring 'oscillating circuit,' which must be adjusted so as to be 'in tune' with the oscillating circuit at the sending-station. As these oscillating currents are so rapid, it serves no purpose to send them through a telephone, because the diaphragm would not respond to such rapid alternations of current. But when the telephone-circuit includes a contrivance which allows the current to pass *in one direction only*, a telephone may be used. Such a contrivance is called a valve; and an effective one is obtained by means of a copper, silver, or brass wire with its point touching a crystal of galena or of grey carborundum. By this means one-half of each oscillation is obliterated, and a single spark at the sending-station sends through the telephone a series of half-waves (Fig. 409) which merge together into a single slight pulse of current, as represented by the dotted line in the diagram.

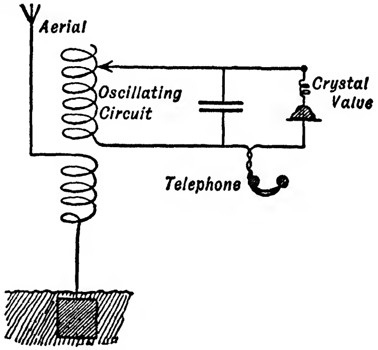


FIG. 408.—A SIMPLE WIRELESS RECEIVING-SET, in which the high frequency oscillations are *rectified* by means of a crystal-valve.

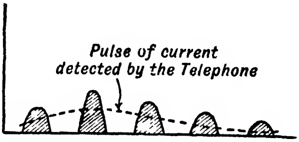


FIG. 409.—The oscillations due to a single spark, after being rectified.

The process of obliterating one-half of each oscillation is termed *rectification*. This single pulse of current makes the diaphragm of the telephone move. If, at the transmitting-station, there are 200 sparks per second, then the diaphragm of the receiving telephone will vibrate 200 times per second, and an audible note of this pitch will be heard.

Thermionic valves.—The **Fleming valve**, introduced by Dr. J. A. Fleming in 1904, is more sensitive and effective than the crystal-valve in 'rectifying' the oscillations set up in the receiving-circuit. This valve consists simply of a small electric glow-lamp with its tungsten filament surrounded by

a cylinder of copper or nickel supported from a platinum wire sealed through the glass. For simplicity in diagrams, this metal cylinder is usually represented as a small flat *plate* supported above the filament (Fig. 410). The size of the filament is such that it can be made incandescent by two or three small accumulators.

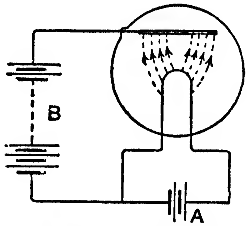


FIG. 410.—THE PRINCIPLE OF THE FLEMING VALVE. The dotted lines are the paths of electrons escaping from a glowing filament towards a +ly charged plate.

In order to understand the action of the Fleming valve it is only necessary to remember that the current traversing the filament consists of millions of electrons travelling at great speed: the speed is so great that some of them escape from the surface of the filament—just as steam would escape through small holes in a canvas pipe. Thus the space round the filament becomes more or less occupied by these negatively charged electrons. Suppose, then, that a +ve charge be given to the *plate*; some, or all, of the electrons which escape from the filament will be attracted upwards to the plate, and the movement will continue until the +ve charge on the plate is neutralised by the electrons which reach its surface. This flow of electrons *upwards* is equivalent to an ordinary electric current flowing downwards. If the plate should be charged *negatively*, it will repel the electrons, and no current will pass.

Fig. 410 shows how the +ve charge on the plate may be maintained by connecting it to the positive terminal of a battery B of several cells, the negative terminal being joined to the negative terminal of the battery A which is heating the filament. In this way a steady stream of electrons will move upwards to the plate, down through the battery, into the filament, and again up to the plate. This current is easily detected by inserting a galvanometer in the circuit. Suppose, now, that the battery B is reversed, so that its negative terminal is joined to the plate; the electrons are repelled by the plate's charge, and no current passes. Thus the lamp allows current to pass *in one direction only*, and it is capable therefore of fulfilling the same purpose as a crystal-valve.

In Fig. 411 a receiving-circuit is shown, with a Fleming valve included for *rectifying* the electric oscillations. The

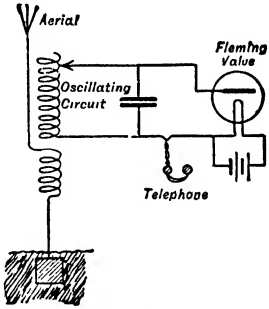


FIG. 411.—A wireless receiving-set, in which the oscillations are rectified by a Fleming valve.

oscillations charge the plate alternately +ve and −ve; when it is +ve a current passes through the telephone, but no current passes when the plate is −ve.

The triode valve.—In 1907, Dr. Lee de Forest modified the Fleming valve by introducing inside the lamp a third electrode, which is placed between the filament and the plate

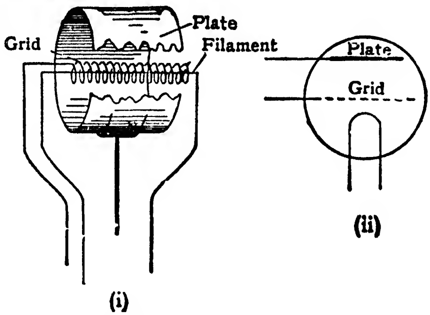


FIG. 412.—A THREE-ELECTRODE (OR TRIODE) VALVE. (i) In some patterns the filament, grid, and plate are arranged as shown. The grid consists of an open spiral of wire surrounding the filament; and the plate is a cylinder of thin metal. (ii) One of the conventional methods of representing the valve in diagrams.

(Fig. 412). This additional electrode is made of fine copper gauze or a spiral of thin wire; and it is called the **grid**. The device is known as a **three-electrode,** or **triode, valve.** When a slight +ve charge is given to the grid it encourages the

upward flow of electrons from the filament, and the majority of these escape through the grid, proceeding onwards to the plate; thus the 'plate-current' is increased. When the grid has a slight −ve charge the movement upwards of electrons is discouraged, and the plate-current is diminished. It was found that the smallest variation of charge on the grid produced great variation in the plate-current. Thus the addition of the grid enables the valve to be used for magnifying or *amplifying*, small oscillations.

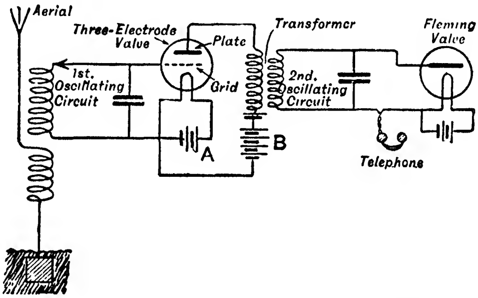


FIG. 413.—A WIRELESS RECEIVING-SET, in which the high-frequency oscillations are *amplified* by a 'triode valve,' then transmitted through a transformer to a second oscillating circuit, and rectified by a Fleming valve.

The diagram (Fig. 413) suggests how very feeble oscillations in the receiving-aerial can be made audible in the telephone. The grid of the three-electrode valve is joined to one end of the first oscillating-circuit, and sets up big oscillations in the plate-current. This current passes through the primary coil of a transformer, the secondary coil of which is part of a second oscillating-circuit; and these oscillations may be *rectified* by means of a Fleming valve. In actual practice the two oscillating-circuits are 'tuned,' by altering the capacities of the condensers, so that the frequency of oscillations in the two circuits is the same: oscillations in one of them will then readily set up oscillations in the other, like the vibrations of a tuning-fork setting up vibrations in another fork, when the rate of vibration of the two forks is the same.

Continuous wireless waves.—The wireless transmission of speech, or of music, would be impossible if the electric spark discharge were the only method available for the generation of wireless waves. The sparks themselves, as previously

described, cause in the receiving telephone an audible note, the pitch of which is determined by the frequency of the sparks. A note becomes inaudible to the human ear only when its frequency exceeds 16,000, and it is impossible to generate sparks at a frequency approaching this number. Hence the note due to the sparks themselves would always be audible. Also, the fact that the sparks are intermittent would prevent their use in transmitting the continuous waves which are characteristic of sound.

Wireless telephony has become possible only by the discovery of methods for generating electric waves which are perfectly uniform in amplitude and not broken into isolated groups, as when the waves are generated by sparks. Such waves are termed 'continuous.' The difference between the two types of wave may be compared to the difference between a note sounded on a piano and the same note sounded on an organ. When a key of a piano is depressed and held down, the intensity of the sound rapidly diminishes and finally

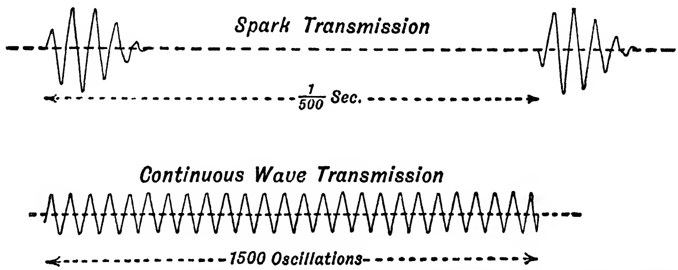


FIG. 414.—THE DIFFERENCE BETWEEN SPARK-TRANSMISSION AND CONTINUOUS WAVE TRANSMISSION. The diagram indicates the difference between waves generated by sparks of frequency 500 per sec., and continuous waves of wave-length 400 metres.

becomes silent : the sound becomes audible again only when the key is again depressed. In the organ, a key depressed causes a sound which is uniform in loudness and continues so long as the key is depressed (or so long as there is any compressed air remaining in the air-chamber).

Fig. 414 will help to explain the difference more clearly. Suppose that the spark-frequency is 500, then two consecutive sparks will be separated by a time-interval of 1/500 sec. If

continuous waves, of wave-length 400 metres, are being generated, then, since the velocity is 3×10^8 metres per second, the frequency of the waves will be $(3 \times 10^8)/400 = 750{,}000$; and the number of waves generated in 1/500 sec. will be $750{,}000/500 = 1500$.

Triode valves as generators of continuous waves.—The generation of continuous waves by means of triode valves is the method now generally adopted in broadcasting stations, and it is gradually being extended to all transmitting-stations. In practice the equipment is far more

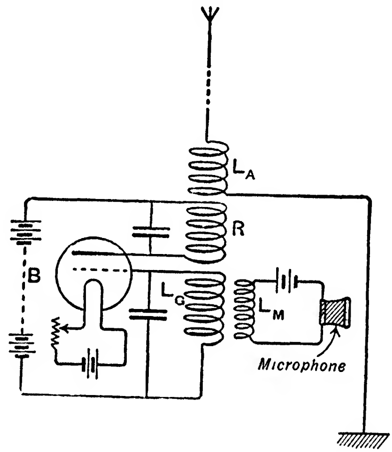


FIG. 415.—A triode valve arranged as a generator of continuous waves.

complicated than is shown in Fig. 415, but the diagram may suffice to explain the principle. The 'plate circuit' includes the high-tension battery B (or a dynamo) and a coil R, which is in close proximity to a coil L_G included in the 'grid circuit.' Suppose that the current in R increases slightly, and that the windings of R and L_G are so related to each other that this increasing current, reacting upon L_G, reduces the potential of the grid, and thus reduces the current in R. The diminishing current in R then reacts upon L_G, so that the potential of the grid rises; this will cause the current in R to increase again. When once started, the system continues to oscillate in this

manner, and the rate of oscillation is regulated by the condensers in parallel with the coils.

The oscillations in R induce corresponding oscillations in the coil L_A, which is connected directly to the aerial, from which 'continuous' waves are transmitted in all directions. If the wave-form characteristic of any given note or sound can be superimposed upon these continuous waves, then it becomes possible to transmit sounds and speech by means of wireless. In the diagram a *microphone* (or telephone transmitter) is connected in series with the coil L_M, which is closely associated with the coil in the grid-circuit of the valve. Any fluctuations of current in L_M will cause corresponding fluctuations in the grid-circuit and, therefore, in the amplitude of the continuous waves sent out from the aerial. These fluctuations are faithfully reproduced in the telephone of any receiving-set tuned for the same wave-length.

Electric discharge through gases.—The insulating power of air and other gases, which is so evident when these are

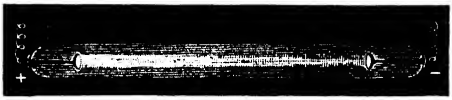


FIG. 416.—Electric discharges in rarefied gases.

under normal conditions, ceases to hold good when they are rarefied by means of an air pump. Thus, when the gas is enclosed in a glass tube (Fig. 416), fitted with an exit tube which can be attached to an air-pump, and the pressure is reduced to about 1/200 of 'one atmosphere,' it is possible to set up an electric discharge between two metal electrodes, at opposite ends of a long tube, which are supported on wires fused through the walls of the tube. The colour of the discharge depends upon the gas used: thus air, hydrogen, nitrogen and other gases, each exhibit a characteristic colour when traversed by the discharge; and when this is examined by a spectroscope, the spectrum consists, in each case, of coloured lines which afford an extremely delicate method of identifying the gas, or mixture of gases contained within the tube.

Cathode rays.—When the pressure is reduced still further the gas ceases to be luminous, and the discharge consists of an invisible stream of particles projected from the cathode. These particles travel with great speed (about 18,000 miles per second—or, about one-tenth the speed of light), and in straight lines at right angles to the cathode's surface. Where they strike the inner surface of the glass tube, they cause a bright fluorescence of the glass, the colour of which depends upon the composition of the glass. These rays are known as **cathode rays**; and it has been proved that they consist simply of *electrons* torn from the atoms of gas remaining within the tube. *The effect is the same whatever gas is contained within the tube*: indeed, this would be expected in view of the fact that the electrons contained in the atoms of all forms of matter are identically the same.

FIG. 417.—An X-ray photograph of the hand, showing a needle embedded in the flesh near the wrist-bones.

X-rays.—Although the mass of each electron is only 1/1845 of that of the hydrogen atom, yet its speed is so great that it

possesses considerable energy ; and the stream is like that of a torrent of rifle-bullets striking a target. When the obstacle in the path of the electrons consists of a very hard metal, such as tungsten, the sudden impact of each electron sets up a single electromagnetic wave, or pulse, resembling a single wave of light, but of so short a wave-length as to be invisible. The difference between such pulses and waves of light may be compared to the difference between solitary whip-cracks and the deepest notes of an organ.

These electromagnetic pulses, known as X-rays, were observed by Prof. W. C. Röntgen, in 1895, to affect a photographic plate in the same manner as light, and to pass through many solid substances with comparatively little absorption. Metals, and compounds of heavy metals (*e.g.* lead glass) are opaque to the rays ; but non-metallic substances are comparatively transparent. Prof. Röntgen observed also that flesh is more transparent than bones ; consequently, if the hand be laid on a photographic plate (protected, if necessary, from ordinary light by being enclosed in an opaque envelope) and held in the path of the rays, a ' negative ' or ' radiograph ' is obtained which shows clearly the details of the bones (Fig. 417).

Fig. 418 represents a type of tube now used frequently for producing X-rays. The cathode is of aluminium and is made concave so as to bring the cathode rays to a focus at a point

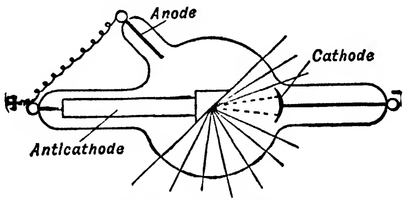


Fig. 418.—An X-ray tube.

within the tube, the path of a ray being at right angles to the surface from which it is generated. A plate of hard infusible metal, such as tungsten, is supported at the point where the rays meet, and it serves as a ' target ' for the rays. The X-rays are generated where the bombardment occurs.

Since much heat is generated by the phenomenon, the target is usually mounted on a solid support of copper, which assists in absorbing the heat. The target is called the *anticathode* ; and it is joined externally to another electrode, called the *anode*, the function of which is not clearly understood, but it appears to make the discharge more steady.

Ionisation of gases.—In addition to their effect on a photographic plate, X-rays exhibit to a remarkable degree the property of *ionising the air*, or other gas, through which they pass. The energy of an electromagnetic pulse, of which X-rays consist, when it impinges on a gas molecule, appears to be partly used up in expelling an electron from the molecule, thus converting it into a *+ly charged ion*. The electron, thus liberated, sooner or later attaches itself to a normal molecule, and converts it into a *−ly charged ion*. Thus the air or gas becomes a 'conductor'; and a +ly charged insulated conductor in the neighbourhood will attract −ve ions and thus gradually lose its charge. Similarly, a −ly charged insulated conductor will be discharged by +ve ions.

The effect can be shown readily by placing a charged gold-leaf electroscope near to an X-ray tube which is in operation ; and, as the degree to which ionisation is established in a gas depends entirely upon the intensity of the beam of X-rays passing through it, the *rate* at which an electroscope is discharged serves as a means of comparing the intensities of X-ray beams obtained either from different sources or under different conditions.

Chemical action, and especially **combustion**, is a frequent cause of ionisation: thus, the gases within and near to the flame of a Bunsen burner or of a taper are extensively ionised. A charged gold-leaf electroscope is rapidly discharged by holding a flame near to it ; but the effect ceases almost completely when an earth-connected metal plate is supported between the electroscope and the flame.

Another method of ionising a gas will be mentioned in a later paragraph.

Radioactivity.—Uranium is a metal, the compounds of which have long been used for giving the characteristic colours

to canary-glass and to pottery glazes: its chief source is the mineral pitchblende, which consists chiefly of an oxide of uranium. In 1896, M. Henri Becquerel observed that when a thin layer of *any* compound of uranium was spread over a black-paper envelope containing a photographic plate, and left in the dark for a short time, the plate when developed showed the same darkening as though it had been exposed to light. Evidently, the uranium gave off some kind of radiation which was capable of penetrating the opaque paper; and, in this sense, the radiation resembled X-rays.

At a later date, Prof. and Madame Curie observed that a certain specimen of pitchblende exhibited the property to a peculiarly marked degree; and, by subjecting the original specimen to chemical separation, they obtained a very minute residue of extremely great radioactive power, which proved to be a chloride closely resembling the chloride of barium. In fact, it was a chloride of a new element, to which the name **radium** was given.

In more recent times it has been proved that a number of elements exhibit radioactive properties: and *all of these have high atomic weights.* Of all known elements, uranium has the heaviest atom—its Atomic Weight is 238, and its Atomic Number is 92. Hence its nucleus must have 238 *protons*, while there are only 92 free electrons rotating round the nucleus. Hence, if the atom is to be electrically neutral, its nucleus must also contain $(238-92)=146$ electrons. It would seem, therefore, that the nucleus of the uranium atom is a very complicated structure, containing 238 *protons* and 146 *electrons.* As in many other things, complication is accompanied by instability; and it is instability which gives rise to the phenomenon of radioactivity.

Atomic disintegration.—We have to imagine that, through long periods of time, the nucleus of each atom of a radioactive element is subject to a regular process of disintegration. In the case of the uranium atom, at one moment there is thrown off from its nucleus a portion which has been identified as the *nucleus* of the helium atom, consisting of two protons and two electrons (Fig. 295 ii); at a later moment, individual

electrons are ejected, followed by other helium nuclei. By such a process the uranium atom becomes changed to the radium atom (A.W. = 226); and, during the passage of a further lapse of time, other portions of the nucleus are ejected, and the weight of the atom is reduced to 208, which is approximately the Atomic Weight of *lead*. The nucleus of the lead atom does not appear to be subject to any disintegration.

Alpha-, beta-, and gamma-rays.—The radiations which have been observed are of three kinds:

(i) Positively-charged particles—termed **alpha-rays** (or α-rays)—which are identical with the nuclei of helium atoms.

(ii) Negatively-charged particles—termed **beta-rays** (or β-rays)—which are separate electrons.

(iii) **Gamma-rays** (or γ-rays)—which are identical with X-rays of very short wave-length.

It has been proved that the α-particle is thrown off from a radium nucleus with a velocity of 10,000 miles per second, which is 20,000 times greater than the swiftest bullet. On leaving the nucleus, it comes into collision with atoms of gases in the surrounding air; and, as it is positively-charged, it soon appropriates two electrons from these atoms and becomes an ordinary atom of helium gas. In its violent motion through the air it may disturb the equilibrium of many atoms in its path, knocking out electrons from some, leaving these with an excess of +ve-charge, while the expelled electrons, sooner or later, attach themselves temporarily to other atoms which have not been bombarded and giving to them an excess of −ve charge.

The air near to a radioactive substance, therefore, becomes *ionised*; and this effect is utilised in measuring the degree of radioactivity of a substance. For this reason the gold-leaf electroscope is an important instrument in researches on radioactivity. Fig. 419 represents a modern form of electroscope, designed by Sir Ernest Rutherford, for such experiments. The upper metal plate, supported by an insulating plug of sulphur, is connected to a narrow metal strip, from the upper end of which is suspended a strip of gold-leaf. The

radioactive substance is spread evenly over the lower plate. The leaf and plate are charged by removing the cap, and the gradual diminution of divergence is observed through a window by means of a low-power microscope.

Each of the three types of rays have this power of ionising a gas; but they can be distinguished, and separately observed, by the fact that they have very different power of penetrating obstacles. Thus, when a thin sheet of tin-foil is laid over the radioactive substance, all the α-rays are stopped, and the effect on the electroscope is due to β-rays and γ-rays only. When a fairly thick sheet of lead is used, only the γ-rays penetrate through to the other side, and the rate of discharge is a measure of the intensity of the γ-rays.

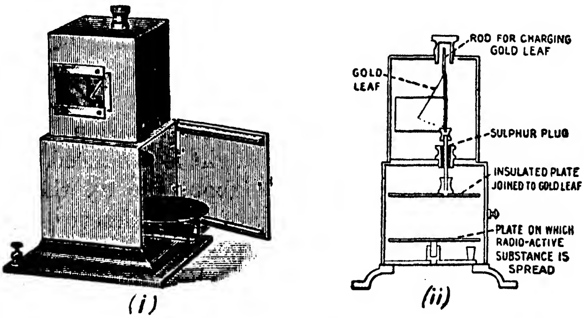


FIG. 419.—A gold-leaf electroscope, specially designed for investigating radioactive phenomena, and made by Messrs. J. J. Griffin and Sons, Ltd.

The gamut of ether waves.—The phenomena of wireless waves, radiant heat, visible light, ultra-violet rays, X-rays and γ-rays are transmitted through space, in each case, by means of transverse vibrations in the ether, which differ only in wave-length. It must be remembered, however, that X-rays and γ-rays are not continuous waves, but are more correctly described as *pulses* in the ether. The former are emitted when fast-moving electrons are stopped, and γ-rays are the result of the disintegration of the nuclei of atoms.

The complete gamut consists of about sixty octaves, beginning with the shortest wave-length, namely, γ-rays, of which the properties are known, and proceeding to waves

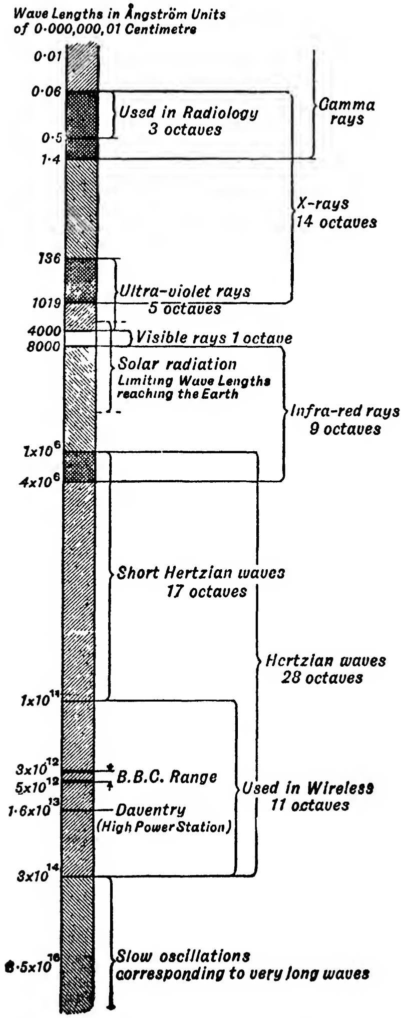


FIG. 420.—THE 'GAMUT' OF ETHER WAVES, from the shortest X-ray to the longest waves, used in wireless telegraphy. Adapted from a Chart prepared for the Royal Society exhibit at Wembley, 1925, and published in "Phases of Modern Science."

longer than those used in radio-communication by steps in which the wave-lengths are doubled, like going down the key-board of a piano from the highest to the lowest octave (Fig. 420). There are 14 octaves of **X-rays,** the shortest of which are γ-rays; these are followed by 5 octaves of **ultra-violet rays,** which produce photographic, chemical and similar action, but the wave-lengths of which are so short that they do not affect human vision. The sensation of violet light is produced by waves about the 25 thousandth of a centimetre (16 millionths of an inch) in length; another slight increase in wave-length gives *blue* then *yellow* and finally *red,* but all the waves giving **visible light** are included between 0·00004 and 0·00008 cm. (16 and 30 millionths of an inch).

Next on the scale are 9 octaves of **infra-red** rays, or heat radiations from hot bodies. These wave-lengths overlap short **Hertzian** waves, of which 17 octaves are known, and then come 11 octaves of

other Hertzian waves used in radio-communication. All the Hertzian waves are generated and detected by electrical methods. Signor Marconi has recently devised a method of long-distance transmission by means of Hertzian waves having a wave-length of only about 30 metres.

The only difference, therefore, between the waves from one end of the gamut to the other is that of wave-length, though the effects on matter vary so greatly. Visible light is thus due to electromagnetic waves detected by the human eye, while various artificial means are used to detect similar waves above or below them in the scale. We have seen that matter consists of positive particles called protons around which negative particles called electrons are in rapid revolution. Changes in the number or position of these electrons give rise to disturbances of the medium called ether, and are detected as radiations of one kind or another. The whole science of Physics is concerned with the interrelations between the electrically-charged atomic particles and the ether, and the knowledge which has been gained in recent years as to the connection between these constituents of the physical universe has led to such remarkable results that we may confidently look forward to even greater achievements in the future.

EXERCISES ON CHAPTER XLII.

1. Explain the process of conduction in an ionised gas. Describe carefully any experiment which illustrates this mode of conduction. (Bristol 2nd S.C.)

2. What is meant by the statement that a certain sample of air is ionised? Describe two ways of causing air to become ionised. How may its ionised state be detected? (Bristol 1st S.C.)

3. Describe the process of conduction in an ionised gas. An insulated metal rod AB is mounted on the axis of a vertical metal tube CD. AB is charged and connected to a gold-leaf electroscope, while CD is connected to earth. Describe and explain what happens (*a*) when the apparatus is exposed to ordinary pure air, and (*b*) when gases from a Bunsen flame are allowed to pass up through CD. (Bristol 2nd S.C-)

4. Give an account of the methods of producing (*i*) cathode rays, (*ii*) X-rays, and a general comparison of their properties. (Joint Matric. Bd., Higher S.C.)

5. It is desired to receive wireless telephony from transmitting stations (*a*) within 10 miles, (*b*) distant more than 100 miles. Describe our receiving set in each case suitable for the purpose and of the simplest kind compatible with efficiency. Explain as fully as you can the action of the sets you describe.
(Durham H.C.)

6. Describe a simple arrangement for the reception of wireless signals. (Lond. G.S.)

PHYSICAL TABLES.*

(1) Equivalents of Metric Weights and Measures in terms of Imperial Units.

METRIC TO IMPERIAL.

Linear Measure:

1 millimetre (mm.) ($\frac{1}{1000}$ m.) - - -	= 0·03937 inch.
1 centimetre ($\frac{1}{100}$ m.) - - - -	= 0·3937 "
1 decimetre ($\frac{1}{10}$ m.) - - - -	= 3·937 inches.
1 metre (m.) - - - - - -	= **39·37 inches.** **3·28 feet.** **1·09 yards.**
1 dekametre (10 m.) - - - -	= 10·936 yards.
1 hectometre (100 m.) - - - -	= 109·36 "
1 kilometre (1000 m.) - - - -	= 0·62 mile.

Square Measure:

1 square centimetre - - - - -	= 0·155 square inch.
1 square decimetre (100 square centimetres) - - - - - -	= 15·50 square inches.
1 square metre (100 square decimetres) -	= 10·76 square feet. 1·19 square yards.

Cubic Measure:

1 cubic centimetre - - - - - -	= 0·06 cubic inch.
1 cubic decimetre (c.d.) (1000 cubic centimetres) - - - - - -	= 61·02 cubic inches.
1 cubic metre (1000 cubic decimetres) -	= 35·31 cubic feet. 1·31 cubic yards.

Measures of Capacity:

1 centilitre ($\frac{1}{100}$ litre) - - - -	= 0·070 gill.
1 decilitre ($\frac{1}{10}$ litre) - - - -	= 0·176 pint.
1 litre - - - - - - -	= **1·76 pints.**

* Chiefly compiled from "Physical Tables," published by the Smithsonian Institution, Washington, U.S.A.

Mass:

		Avoirdupois.
1 milligram ($\frac{1}{1000}$ gm.) - - - -	=	0·015 grain.
1 centigram ($\frac{1}{100}$ gm.) - - - -	=	0·154 „
1 decigram ($\frac{1}{10}$ gm.) - - - -	=	1·543 grains.
1 gram (1 gm.) - - - - -	=	15·432 „
1 dekagram (10 gm.) - - - -	=	5·644 drams.
1 hectogram (100 gm.) - - - -	=	3·527 oz.
1 kilogram (1000 gms.) - - - -	=	**2·20 lb. or 15432·356 grains.**

(2) Equivalents of Imperial Weights and Measures in terms of Metric Units.

IMPERIAL TO METRIC

Linear Measure:

1 inch - - - - - -	=	25·4 millimetres.
1 foot (12 inches) - - - -	=	0·30 metre.
1 yard (3 feet) - - - - -	=	**0·914 metre.**

Square Measure:

1 square inch - - - - -	=	6·45 sq. centimetres.
1 square foot (144 square inches) -	=	9·29 sq. decimetres.
1 square yard (9 square feet) - -	=	0·836 square metre.

Cubic Measure:

1 cubic inch - - - - -	=	16·387 cub. centimetres.
1 cubic foot (1728 cubic inches) - -	=	0·028 cub. metre.
1 cubic yard (27 cubic feet) - -	=	0·764 cub. metre.

Measures of Capacity:

1 gill - - - - - - -	=	1·42 decilitres.
1 pint (4 gills) - - - - -	=	0·568 litre.
1 quart (2 pints) - - - -	=	1·136 litres.
1 gallon (4 quarts) - - - -	=	**4·546 litres.**

Apothecaries Measure:

1 minim - - - - - -	=	0·059 millilitre.
1 fluid drachm (60 minims) - -	=	3·552 millilitres.
1 fluid ounce (8 drachms) - - -	=	2·841 centilitres.
1 pint (20 fluid ounces) - - -	=	0·568 litre.
1 gallon (8 pints or 160 fluid ounces) -	=	**4·546 litres.**

Avoirdupois Weight:

1 grain - - - - - -	=	0·065 gram.
1 dram - - - - - -	=	1·77 grams.
1 ounce (16 drams or 437·5 grains) -	=	28·35 „
1 pound (16 ounces or 7000 grains) -	=	**0·4536 kilogram.**

(3) Mensuration.

$\pi = 3.14159.$	$\frac{1}{\pi} = 0.3183.$
$2\pi = 6.28318.$	
$\pi^2 = 9.8696.$	$\sqrt{\pi} = 1.7724.$

Lengths.

Circumference of circle of radius r - - - - -	$=2\pi r.$
" " diameter d - - -	$=\pi d.$
" ellipse with semi-axes a and b -	$=2\pi\sqrt{\frac{a^2+b^2}{2}}.$

Areas.

Triangle, base b, perpendicular h - - - - -	$=\frac{bh}{2}.$
Rectangle, sides b, h - - - - - - - -	$=bh.$
Parallelogram, base b, perpendicular h - - -	$=bh.$
Circle, radius r - - - - - - - - -	$=\pi r^2.$
Circle, diameter d - - - - - - - -	$=\frac{\pi d^2}{4}.$
Ellipse, semi-axes a, b - - - - - - -	$=\pi ab.$
Surface of cube, edge a - - - - - - -	$=6a^2.$
Surface of sphere, radius r - - - - - -	$=4\pi r^2.$
Curved surface of right cylinder, radius r, height h -	$=2\pi rh.$
Total surface of right cylinder, " " -	$=2\pi r(r+h).$
Curved surface of right cone, radius r, altitude h, slant height s	$=\pi rs.$ $=\pi r\sqrt{r^2+h^2}.$
Total surface of right cone - - - - - -	$=\pi r(s+r).$

Volumes.

Cube, edge a - - - - - - - - - -	$=a^3.$
Rectangular parallelopiped, edges a, b, c - - - -	$=abc.$
Pyramid, area of base a, altitude h - - - - -	$=\frac{ah}{3}.$
Cone, with circular base, radius r, altitude h - - -	$=\frac{\pi r^3 h}{3}.$
Cylinder or prism, area of base a, altitude h - -	$=ah.$
Sphere, radius r - - - - - - - - -	$=\frac{4}{3}\pi r^3.$

(4) Density, or Mass per Unit Volume.

Common Solids.

Substance.	Mass of 1 Cubic Centimetre in Grams.	Mass of 1 Cubic Foot in Lbs.
Anthracite	1·4–1·8	87–112
Beeswax	0·96-0·97	60–61
Brick	1·4–2·2	87–137
Butter	0·86–0·87	53–54
Caoutchouc	0·92–0·99	57–62
Chalk	1·9–2·8	118–175
Ebonite	1·15	72
Felspar	2·55–2·75	159–172
Flint	2·63	164
Glass (Common)	2·4–2·8	150–175
„ (Flint)	2·9–5·9	180–370
Granite	2·64–2·76	165–172
Graphite	2·30–2·72	144–170
Ice	0·917	57·2
Limestone	2·68–2·76	167–171
Magnetite	4·9–5·2	306–324
Marble	2·6–2·84	160–177
Paper	0·7–1·15	44–72
Paraffin	0·87–0·91	54–57
Pitch	1·07	67
Porcelain	2·3–2·5	143–156
Pumice Stone	0·37–0·9	23–56
Quartz	2·65	165
Rock Salt	2·18	136
Sand (Silver)	2·63	164
Sandstone	2·14–2·36	134–147
Slate	2·6–3·3	162–205
Starch	1·53	95
Sugar	1·61	100
Tallow	0·91–0·97	57–60

Metals and Alloys.

Substance.	Mass of 1 Cubic Centimetre in Grams.	Mass of 1 Cubic Foot in Lbs.
Metals: Aluminium	2·56–2·80	160–175
Copper (Cast)	8·30–8·95	517–558
,, (Drawn)	8·93–8·95	556–558
Gold	19·3	1205
Iron (Cast)	7·03–7·73	438–482
,, (Wrought)	7·80–7·90	486–493
,, (Steel)	7·60–7·80	474–486
Lead	11·36	709
Magnesium	1·74	108·5
Mercury (at 0° C.)	13·596	848
Nickel	8·6–8·90	536–555
Platinum	21·37	1331
Silver	10·40–10·57	649–659
Tin	7·30	455
Zinc	7·04–7·19	439–449
Alloys: Brass	8·44–8·70	526–542
Bronze	8·74–8·89	545–555
Coins (British): Gold	17 72	1106
Silver	10·31	643
Bronze	8·96	559
German Silver	8·30–8 77	518–547
Gunmetal	8·0–8·4	499–524

Liquids.

Substance.	Mass of 1 Cubic Centimetre in Grams.	Mass of 1 Cubic Foot in Lbs.
Alcohol (Ethyl)	0·807	50·4
,, (Methylated Spirit)	0·823 (approx.)	51·2 (approx.)
,, (Proof Spirit)	0·916	57·2
Benzene	0·899	56·1
Carbon Bisulphide	1·293	80·6
Ether	0·736	45·9
Glycerine	1·260	78·6
Oil, Castor	0·969	60·5
,, Linseed	0·942	58·8
,, Mineral	0·900–0·925	56·2–57·7
,, Olive	0·918	57·3
,, Turpentine	0·873	54·2
Petrol	0·68–0·72	42·4–44·9
Petroleum	0·878	54·8
Sea Water (Mean)	1·0275	64·14
Water (at 4° C.)	1·000	62·43
,, (at 15° C.)	0·9991	62·37

Aqueous Solutions.

[The strength of the solutions is expressed as the weight of the dissolved substance in 100 parts by weight of the solution.]

Substance.	Weight, in gms. per c.c.	Weight, in lbs. per c. ft.
Sodium Chloride (5 %) -	1·035 (at 15° C.)	64·62
,, ,, (10 %) -	1·072 ,,	66·92
,, ,, (15 %) -	1·110 ,,	69·29
,, ,, (20 %) -	1·150 ,,	71·80
,, ,, (25 %) -	1·191 ,,	74·35
Sulphuric Acid (5 %) -	1·032 (at 20° C.)	64·43
,, ,, (15 %) -	1·102 ,,	68·80
,, ,, (25 %) -	1·178 ,,	73·55
,, ,, (35 %) -	1·260 ,,	78·66
,, ,, (45 %) -	1·348 ,,	84·16
,, ,, (55 %) -	1·445 ,,	90·22
,, ,, (98 %) -	1·836 ,,	114·7

Gases.

Substance.	Specific Gravity.	Mass of 1 Litre in Grams.	Mass of 1 Cubic Foot in Lbs.
Air - - -	1·000	1·293	0·0807
Carbon Dioxide	1·530	1·977	0·1234
Hydrogen -	0·0695	0·0899	0·00561
Nitrogen - -	0·967	1·251	0·0781
Oxygen - -	1·105	1·429	0·0892
Steam at 100°C.	0·463	0·598	0·0373

Woods

Substance.	Mass of 1 Cubic Centimetre in Grams.	Mass of 1 Cubic Foot in Lbs.
Ash - - -	0·65–0·85	40–53
Beech - - -	0·70–0·90	43–56
Birch - - -	0·51–0·77	32–48
Box - - -	0·95–1·16	59–72
Cedar - - -	0·49–0·57	30–35
Cork - - -	0·22–0·26	14–16
Ebony - - -	1·11–1·33	69–83
Elm - - -	0·54–0·60	34–37
Lignum Vitae - -	1·17–1·33	73–83
Lime - - -	0·32–0·59	20–37
Mahogany (Spanish)	0·85	53
Maple - - -	0·62–0·75	39–47
Oak - - -	0·60–0·90	37–56
Sycamore - -	0·40–0·60	24–37
Teak - - -	0·66–0·98	41–61
Walnut - - -	0·64–0·70	40–43

(5) Acceleration due to gravity.

Locality.	Latitude.	Increase of Velocity per Second, due to the Earth's attraction.	
Equator - - -	0° 0′	978·04 cm.	32·09 ft.
	45° 0′	980·61 cm.	32·17 ft.
Greenwich - -	51° 29′	981·18 cm.	32·19 ft.
Edinburgh - -	55° 55′	981·54 cm.	32·20 ft.
North Pole - -	90° 0′	983·21 cm.	32·26 ft.

(6) Length of Seconds Pendulum.

Locality.	Latitude.	Length (in cms.)	Length (in ins.)
Equator - - -	0° 0′	99·09	39·01
	45° 0′	99·36	39·12
Greenwich - -	51° 29′	99·41	39·14
Edinburgh - -	55° 55′	99·45	39·15
North Pole - -	90° 0′	99·62	39·22

(7) Pressure.

A standard **atmosphere** is the pressure of a vertical column of pure mercury, having a height of 760 mm. and temperature 0° C. under standard gravity at latitude 45° and at sea level.

1 standard atmosphere = 1033 grams per sq. cm.
$= (1033 \times 980{\cdot}61) = 1{\cdot}013 \times 10^6$ dynes per sq. cm.
$= 14{\cdot}7$ lb. per sq. in.
$= 2116$ lbs. per sq. ft.

The **Bar** = 10^6 dynes per sq. cm.
= 1000 **millibars**
= 75·01 cm. (or 29·53 in.) of mercury
= 0·987 'atmosphere.'

A column of water 2·3 ft. high corresponds to the pressure of 1 lb. per sq. in.

(8) Imperial Standard Wire Gauge.

Descriptive Number.	Diameter.		Area of Cross Section.	
	Inches.	Centimetres.	Square Inches.	Square Centimetres.
14	0·080	0·203	0·00503	0·0324
16	0·064	0·163	0·00322	0·0207
18	0·048	0·122	0·00181	0·0117
20	0·036	0·0914	0·00102	0·00657
22	0·028	0·0711	0·000616	0·00397
24	0·022	0·0559	0·000380	0·00245
25	0·020	0·0508	0·000314	0·00203
26	0·018	0·0457	0·000254	0·00164
27	0·0164	0·0417	0·000211	0·00136
28	0·0148	0·0376	0·000173	0·00111
29	0·0136	0·0345	0·000145	0·00094
30	0·0124	0·0315	0·000121	0·00078
32	0·0108	0·0274	0·000092	0·000591
34	0·0092	0·0234	0·000066	0·000429
36	0·0076	0·0193	0·000045	0·000293

(9) Melting Points and Latent Heats of Fusion.

	Melting Point.	Latent Heat.
Alloys :		
Soft Solder (77·8 lead + 22·2 tin)	176°·5 C.	9·54
Wood's Alloy (25·8 lead + 52·4 bismuth + 14·7 tin + 7 cadmium) - - - -	75·5	8·40
Beeswax - - - - -	61·8	42·3
Butter - - - - -	28–33	—
Ice - - - - - -	0	79·6
Metals :		
Copper - - - - -	1083	42
Iron - - - - -	1530	—
Lead - - - - -	327	5·36
Mercury - - - -	−38·9	2·8
Platinum - - - -	1755	—
Tungsten - - - -	3400	—
Naphthalene - - - -	79·8	35·6
Paraffin Wax (ordinary white solid) - - - - -	50–55	35·1
Sulphur - - - - -	115	—

(10) Boiling Points and Latent Heats of Vaporization.

	Boiling Point.	Latent Heat.
Alcohol (Ethyl) - - -	78° C.	205
Ammonia - - - -	−33·5	—
Benzene (or Benzol) - - -	80–83	92·9
Carbon Bisulphide - - -	46·1	84
Ether - - - - -	35	90·4
Mercury - - - - -	357	—
Steam - - - - -	100	538·7
Sulphur - - - - -	444·7	—
Sulphuric Acid - - -	338	—
Turpentine - - - -	159	74

(11) Coefficients of Linear Expansion of Solids.

	Expansion per Degree C.
Aluminium	0·0000222
Brass	0·0000187
Copper	0·0000168
Glass (Tube)	0·0000083
Glass (Crown)	0·0000090
Ice	0·0000510
Iron (Wrought)	0·0000114
,, (Annealed Steel)	0·0000109
Lead	0·0000271
Platinum	0·0000090
Silver	0·0000192
Zinc	0·0000292

(12) Coefficients of Cubical Expansion of Liquids.

Alcohol (Ethyl), (0° to 80°)	0·00104
Benzene	0·00118
Carbon Bisulphide	0·00114
Ether (−15° to +38°)	0·00151
Glycerin	0·000485
Mercury	0·000182
Olive Oil	0·000682
Petroleum	0·00090
Sulphuric Acid	0·00058
Turpentine	0·00090
Water (10°–100° C.)	0·00047
,, (10°–30°)	0·000203
,, (30°–50°)	0·000385

(13) Volume and Density of Water at different Temperatures.

(Metric Units.)

t°.	Volume of Unit Mass.	Density.	t°.	Volume of Unit Mass.	Density.
0°	1·0001	0·9999	55°	1·0145	0·9857
4°	1·0000	1·0000	60°	1·0170	0·9832
10°	1·0003	0·9997	65°	1·0198	0·9806
15°	1·0009	0·9991	70°	1·0227	0·9778
20°	1·0018	0·9982	75°	1·0258	0·9749
25°	1·0029	0·9971	80°	1·0290	0·9718
30°	1·0043	0·9957	85°	1·0324	0·9686
35°	1·0060	0·9941	90°	1·0359	0·9653
40°	1·0078	0·9922	95°	1·0396	0·9619
45°	1·0098	0·9902	100°	1·0434	0·9584
50°	1·0121	0·9881			

(14) Coefficients of Expansion of Gases.

	Increase of Pressure at Constant Volume.	Increase of Volume at Constant Pressure.
Hydrogen	0·003669	0·003660
Air	0·003665	0·003671
Carbon Dioxide	0·003686	0·003710

(15) Specific Heats.

Solids.

Aluminium	0·209	Lead	0·0315
Beeswax	0·64	Magnesium	0·245
Brass	0·0900	Marble	0·21
Gas Coal	0·3145	Nickel	0·1092
Graphite	0·1604	Paraffin	0·694
Copper	0·094	Platinum	0·0275
Glass, Crown	0·161	Silver	0·0559
,, Flint	0·117	Steel	0·118
Ice	0·463	Sulphur	0·184
Indiarubber	0·481	Vulcanite	0·3312
Iron	0·1150	Zinc	0·0931

Liquids.

Alcohol (Ethyl) (at 0° C.)	0·547	Glycerine	0·576
,, ,, (at 40° C.)	0·648	Mercury	0·033
Benzene (at 10° C.)	0·340	Olive Oil	0·471
,, (at 40° C.)	0·423	Petroleum	0·712
Ether	0·530	Turpentine	0·411

(16) Maximum Pressure of Aqueous Vapour.

(The pressure p is given in mm. of mercury, at 0° C.)

Temp. °C.	p.	Temp. °C.	p.	Temp. °C.	p.
0	4·58	21	18·66	96	657·8
1	4·92	22	19·84	97	682·2
2	5·29	23	21·09	98	707·4
3	5·68	24	22·40	98·2	712·3
4	6·10	25	23·78	98·4	717·4
5	6·54	26	25·24	98·6	722·6
6	7·01	27	26·77	98·8	727·9
7	7·51	28	28·38	99	733·3
8	8·04	29	30·08	99·2	738·5
9	8·61	30	31·86	99·4	743·8
10	9·21	40	55·40	99·6	749·2
11	9·85	50	92·60	99·8	754·5
12	10·52	60	149·6	100	760·0
13	11·24	70	233·9	100·2	765·2
14	11·99	80	355·4	100·4	771·0
15	12·79	90	526·0	100·6	776·5
16	13·64	91	546·3	100·8	782·1
17	14·54	92	567·2	101	787·5
18	15·49	93	588·8	102	815·9
19	16·49	94	611·1	103	845·0
20	17·55	95	634·1	104	875·1

(17) Mass of the Aqueous Vapour contained in a Cubic Metre of Saturated Air.

Temp. ° C.	Mass in Grams.	Temp. ° C.	Mass in Grams.	Temp. ° C.	Mass in Grams.	Temp. ° C.	Mass in Grams.
0	4·85	11	10·01	21	18·34	31	32·05
1	5·19	12	10·66	22	19·43	32	33·81
2	5·56	13	11·35	23	20·58	33	35·66
3	5·95	14	12·07	24	21·78	34	37·58
4	6·36	15	12·83	25	23·05	35	39·60
5	6·80	16	13·63	26	24·38	36	41·71
6	7·26	17	14·48	27	25·77	37	43·91
7	7·75	18	15·37	28	27·23	38	46·21
8	8·27	19	16·31	29	28·76	39	48·61
9	8·82	20	17·30	30	30·37	40	51·10
10	9·40						

(18) Boiling Point of Water at different Pressures.

Press. (mm.).	Boiling Point.	Press. (mm.).	Boiling Point.	Press. (mm.).	Boiling Point.	Press. (mm.).	Boiling Point.
740	99°·256	750	99°·630	760	100°·000	770	100°·366
1	·293	1	·667	1	·037	1	·402
2	·331	2	·704	2	·074	2	·439
3	·368	3	·741	3	·110	3	·475
4	·406	4	·778	4	·147	4	·511
5	·443	5	·815	5	·184	5	·548
6	·481	6	·852	6	·220	6	·584
7	·518	7	·889	7	·257	7	·620
8	·555	8	·926	8	·293	8	·656
9	·593	9	·963	9	·330	9	·692
750	99°·630	760	100°·000	770	100°·366	780	100°·728

(19) Boiling Point of a Solution of Common Salt in Water.

(Pressure = 76 cm.)

Number of Grams of Salt dissolved in 100 Grams of Water.	Boiling Point.	Number of Grams of Salt dissolved in 100 Grams of Water.	Boiling Point.
6·6	101° C.	25·5	105° C.
12·4	102° C.	33·5	107° C.
17·2	103° C.	40·7	108°·8 C.
21·5	104° C.		

(20) Coefficients of Thermal Conductivities.

Substance.	Coefficient (k).	Substance.	Coefficient (k).
Air - - -	0·000057	Marble - - -	0·0071
Aluminium 18°	0·480	Leather (Cow-hide)	0·00042
Aluminium 100°	0·492	Linen - - -	0·00021
Asbestos paper -	0·00017	Mercury - 0°	0·0148
Beeswax - -	0·00009	Mercury - 50°	0·0189
Brass - 0°	0·204	Olive Oil - -	0·000395
Brass - 100°	0·254	Paper - - -	0·0003
Carbon Dioxide -	0·000307	Rubber (Vulcanised)	0·00034–0·00054
Concrete - -	0·0022		
Copper 18°	0·918	Sawdust - -	0·00012
Copper 100°	0·908	Silk - - -	0·000095
Firebrick - -	0·00028	Silver (at 18° C.) -	1·006
Flannel - -	0·00023	Slate - - -	0·0034
Glass (Window) -	0·0025	Snow (Fresh) - -	0·00026
Granite - -	0·0053	Soil - - -	0·0037
Ice - - -	0·0050	Vulcanite - -	0·00087
Iron (Wrought) 18°	0·144	Water - - 0°	0·0015
Iron (Wrought) 100°	0·142	Water - - 25°	0·00136
Lead - - 18°	0·083	Zinc - - -	0·265
Lead - - 100°	0·081		

(21) Indices of Refraction, Relative to Air.

Solids.			
Crown Glass - -	1·52	Fluor Spar - -	1·43
Diamond - - -	2·42	Ice - - - -	1·31
Emerald - - -	1·58	Rock Salt - -	1·54
Flint Glass -	1·58–1·66	Ruby - - -	1·77
Liquids.			
Alcohol - - -	1·36	Olive Oil - - -	1·47
Benzene - - -	1·50	Sulphuric Acid - -	1·42
Carbon Bisulphide -	1·63	Turpentine - -	1·46
Glycerin - - -	1·47	Water - - -	1·33

(22) Velocity of Sound.

Substance.	Metres per Second.	Feet per Second.
Aluminium	5104	16740
Brass	3500	11480
Copper (at 20° C.)	3560	11670
Iron ,,	5130	16820
Silver ,,	2610	8553
Marble	3810	12500
Slate	4510	14800
Glass	5000–5300	16410–17380
Ash, along the fibre	4760	15310
,, across the rings	1390	4570
,, along the rings	1260	4140
Oak	3850	12620
Pine	3320	10900
Alcohol (95%) at 12°·5 C.	1241	4072
Turpentine	1326	4350
Water (Fresh) at 13° C.	1441	4728
,, (Seawater)	1520	4990
Temp. 0° C. Air	331·7	1088
Temp. 0° C. Carbon Dioxide	258	846
Temp. 0° C. Hydrogen	1286	4221
Temp. 0° C. Illuminating Gas	490	1609
Temp. 0° C. Oxygen	317	1041

(23) Electro-Chemical Equivalents.

Element.	Atomic Weight (O = 16).	Chemical Equivalent.	Electro-Chemical Equivalent (in Grams per Coulomb).
Aluminium	27·1	8·96	0·0000936
Copper	63·6	31·54	0·0003294
Gold	197·2	65·21	0·0006812
Hydrogen	1·008	(1)	0·00001046
Oxygen	16·00	7·935	0·0000829
Nickel	58·7	29·12	0·0003041
Silver	107·93	107·73	0·0011180
Zinc	65·4	32·45	0·0003387

(24) Mean Values for the Year 1920, of the Magnetic Elements at various Observatories.

Place.	Latitude.	Longitude.	Declination.	Inclination.	Force in C.G.S. Units. Horizontal.	Force in C.G.S. Units. Vertical.
	° ′	° ′	° ′	° ′	γ*	γ
N. Magnetic Pole	70 5 N	96 45 W	—	90·0N	—	—
Sitka	57 3 N	135 20 W	30 28·2 E	74 22·1N	15574	55662
Rude Skov	55 51 N	12 27 E	7 57·2 W	68 59·6N	17124	44596
Eskdalemuir	55 19 N	3 12 W	16 49·7 W	69 39·5 N	16706	45084
Stonyhurst	53 51 N	2 28 W	15 52·9 W	68 43·5 N	17300	44433
Potsdam	52 23 N	13 · 4 E	7 29·4 W	66 33·5 N	18606	42912
Seddin	52 17 N	13 1 E	7 31·2 W	66 30·6 N	18645	42899
De Bilt (Utrecht)	52 5 N	5 11 E	11 24·2 W	66 51·8 N	18397	43056
Valencia (Ireland)	51 56 N	10 15 W	19 17·9 W	68 5·3 N	17840	44353
Kew (Richmond)	51 28 N	0 19 W	14 31·0 W	66 57·9 N	18410	43297
Greenwich	51 28 N	0 0	14 8·6 W	66 53·6 N	18454	43249
Val Joyeux (near Paris)	48 49 N	2 1 E	12 53·0 W	64 41·6 N	19666	41591
Munich	48 9 N	11 37 E	8 3·8 W	—	—	—
Agincourt (Toronto)	43 47 N	79 16 W	6 45·4 W	74 44·6 N	15865	58166
Tortosa	40 49 N	0 30 E	11 59·3 W	57 39·4 N	23291	36781
Coimbra	40 12 N	8 25 W	15 21·5 W	58 22·8 N	23087	37496
Cheltenham (Maryland)	38 44 N	76 50 W	6 18·5 W	70 55·4 N	19118	55285
Tsingtau	36 4 N	120 19 E	4 12·9 W	52 7·0 N	30817	39610
Tucson (Arizona)	32 15 N	110 50 W	13 48·0 E	59 27·6 N	26910	45610
Lu-ka-pang	31 19 N	121 2 E	3 21·4 W	45 30·7 N	33175	33773
Dehra Dun	30 19 N	78 3 E	1 52·0 E	44 59·9 N	32951	32949
Hongkong	22 18 N	114 10 E	0 20·8 W	30 46·4 N	37174	22137
Honolulu (Hawaii)	21 19 N	158 4 W	9 53·2 E	39 25·1 N	28847	23711
Toungoo	18 56 N	96 27 E	0 23·7 W	23 7·7 N	39114	16707
Alibag (Bombay)	18 39 N	72 52 E	0 20·3 E	24 54·7 N	36922	17147
Vieques (Porto-Rico)	18 9 N	65 26 W	3 46·1 W	51 22·7 N	27827	34832
Antipolo	14 36 N	121 10 E	0 35·9 E	16 11·7 N	38100	11065
Kodai-Kanal	10 14 N	77 28 E	1 49·9 W	4 36·1 N	37787	03042
Mauritius	20 6 S	57 33 E	10 20·3 W	52 40·1 S	23093	30278
Christchurch (N.Z.)	43 32 S	172 37 E	17 1·7 E	68 9·2 S	22261	55525
S. Magnetic Pole	72 25 S	154 0 E	—	90·0S	—	—

* 1γ corresponds to 1×10^{-5} C.G.S. unit.

(25) Electromotive Force of Voltaic Cells.

Name of Cell.	Solution for Negative Pole.	Solution for Positive Pole.	E.M.F. in Volts.
Lead Accumulator	H_2SO_4 Solution of density 1·1	—	2·2
Bichromate -	12 parts $K_2Cr_2O_7$ to 25 parts H_2SO_4 and 100 parts H_2O	1 part H_2SO_4 to 12 parts H_2O	2·00
Clark Standard	Saturated $ZnSO_4$	Paste of Hg_2SO_4 and $ZnSO_4$	1·434 at 15° C.
Daniell (i) -	1 part H_2SO_4 to 4 parts H_2O	Saturated solution of $CuSO_4\ 5H_2O$	1·06
,, (ii) -	1 part H_2SO_4 to 12 parts H_2O	,,	1·09
Leclanché -	Sal-ammoniac	—	1·46
Weston Normal	Saturated $CdSO_4$	Paste of Hg_2SO_4 and $CdSO_4$	1·0183 at 20° C.

(26) Specific Resistance of Metals and Alloys.

Substance.	Specific Resistance (in Microhms or Millionths of an Ohm).	Temperature Coefficient.
Elements—		
Silver (hard drawn) (at 18° C.) - - -	1·629	0·0038
Copper (hard drawn) (at 20° C.) - - -	1·77	0·0038
Tungsten (at 20° C.) -	5·51	0·0045
Iron (soft wire) (at 20° C.)	10·0	0·0050
Platinum (at 20° C.) -	10·0	0·0030
Mercury (at 0° C.) -	94·07	0·00089
Alloys—		
German Silver (various)	20–33	0·00044
Phosphor-bronze - -	5–10	0·0007
Platinoid - - -	43·60	0·00025
Manganin {Cu 84%, Mn 12, Ni 4}	43·5	{at 12° C., $+(6·0 \times 10^{-6})$; at 25° C., nil}
Nichrome - - -	100·0	0·0004
Constantan (or 'Eureka')	49·0	{at 12° C., $+(8·0 \times 10^{-6})$; at 25° C., $+(2·0 \times 10^{-6})$}
Non-metal—		
Carbon (lamp filament) -	$(0·004 \times 10^6)$	−0·0003

(27) Radiation: Wave-length Limits.

Electromagnetic waves (as used in wireless telegraphy, etc.)	longest -	1 000 000·0 cm.
	shortest -	0·2 cm.*
'Infra-red' - - - -	longest -	0·04 cm.
	shortest -	0·000 08 cm.
Visible Spectrum - - -	longest -	0·000 08 cm.
	shortest -	0·000 04 cm.
'Ultra-violet' - - -	shortest -	0·000 001 4 cm.
X-rays - - - -	longest -	0·000 010 cm.
	shortest -	0·000 000 000 6 cm.
Gamma-rays - - - -	longest -	0·000 000 013 cm.
	shortest -	0·000 000 000 7 cm.

(28) Dimensions of Atoms, and of the Electron.

Mass of a hydrogen atom - - - - - -	$1{\cdot}662 \times 10^{-24}$ gm.
Number of molecules in 2 gm. of hydrogen - -	$6{\cdot}062 \times 10^{23}$
Number of molecules in 1 c.c. of hydrogen (at 0° C. and 76 cm. pressure) - - - - -	$2{\cdot}705 \times 10^{19}$
Radius of hydrogen molecule - - - -	10^{-8} cm. approx.
Radii of atoms: Hydrogen - - - - -	$0{\cdot}52 \times 10^{-8}$ cm.
Oxygen - - - - -	$0{\cdot}65 \times 10^{-8}$ cm.
Copper - - - - -	$1{\cdot}37 \times 10^{-8}$ cm.
Lead - - - - - -	$1{\cdot}90 \times 10^{-8}$ cm.

Negative Electron: Mass of electron - - - $9{\cdot}01 \times 10^{-28}$ gm.

$$= \frac{\text{mass of hydrogen atom}}{1845}.$$

Radius of electron - - - 2×10^{-13} cm.

$$= \frac{\text{atomic radius}}{26000}.$$

$$\left[\text{Compare } \frac{\text{Radius of earth}}{\text{Radius of orbit of Mars}} = \frac{1}{35700}.\right]$$

The Proton (or **Positive electron**):

$$\text{Radius} = \frac{\text{radius of an electron}}{2000}.$$

* E. F. Nichols and J. D. Tear have generated and measured electromagnetic waves of length varying from 0·7 cm. to 0·022 cm. (*Nature*, August 11, 1923, p. 228). A. Glagolewa-Arkadiewa describes in *Nature* of May 3, 1924, her measurements of electromagnetic waves so short as 82 microns (*i.e.* 0·0082 cm.). It would appear therefore that electromagnetic waves have been generated which have wave-lengths overlapping those of 'infra-red' radiation.

(29) Trigonometrical Ratios.

Angle: Deg.	Sine.	Tangent.	Cotangent.	Cosine.	
0°	0	0	∞	1	90°
1	0·0175	0·0175	57·2900	0·9998	89
2	0·0349	0·0349	28·6363	0·9994	88
3	0·0523	0·0524	19·0811	0·9986	87
4	0·0698	0·0699	14·3006	0·9976	86
5	0·0872	0·0875	11·4301	0·9962	85
6	0·1045	0·1051	9·5144	0·9945	84
7	0·1219	0·1228	8·1443	0·9925	83
8	0·1392	0·1405	7·1154	0·9903	82
9	0·1564	0·1584	6·3138	0·9877	81
10	0·1736	0·1763	5·6713	0·9848	80
11	0·1908	0·1944	5·1446	0·9816	79
12	0·2079	0·2126	4·7046	0·9781	78
13	0·2250	0·2309	4·3315	0·9744	77
14	0·2419	0·2493	4·0108	0·9703	76
15	0·2588	0·2679	3·7321	0·9659	75
16	0·2756	0·2867	3·4874	0·9613	74
17	0·2924	0·3057	3·2709	0·9563	73
18	0·3090	0·3249	3·0777	0·9511	72
19	0·3256	0·3443	2·9042	0·9455	71
20	0·3420	0·3640	2·7475	0·9397	70
21	0·3584	0·3839	2·6051	0·9336	69
22	0·3746	0·4040	2·4751	0·9272	68
23	0·3907	0·4245	2·3559	0·9205	67
24	0·4067	0·4452	2·2460	0·9135	66
25	0·4226	0·4663	2·1445	0·9063	65
26	0·4384	0·4877	2·0503	0·8988	64
27	0·4540	0·5095	1·9626	0·8910	63
28	0·4695	0·5317	1·8807	0·8829	62
29	0·4848	0·5543	1·8040	0·8746	61
30	0·5000	0·5774	1·7321	0·8660	60
31	0·5150	0·6009	1·6643	0·8572	59
32	0·5299	0·6249	1·6003	0·8480	58
33	0·5446	0·6494	1·5399	0·8387	57
34	0·5592	0·6745	1·4826	0·8290	56
35	0·5736	0·7002	1·4281	0·8192	55
36	0·5878	0·7265	1·3764	0·8090	54
37	0·6018	0·7536	1·3270	0·7986	53
38	0·6157	0·7813	1·2799	0·7880	52
39	0·6293	0·8098	1·2349	0·7771	51
40	0·6428	0·8391	1·1918	0·7660	50
41	0·6561	0·8693	1·1504	0·7547	49
42	0·6691	0·9004	1·1106	0·7431	48
43	0·6820	0·9325	1·0724	0·7314	47
44	0·6947	0·9657	1·0355	0·7193	46
45	0·7071	1·0000	1·0000	0·7071	45
	Cosine.	Cotangent.	Tangent.	Sine.	Angle: Deg.

TYPICAL EXAMINATION PAPERS.

EXAMINATION QUESTIONS FROM SCHOOL LEAVING CERTIFICATE PAPERS.

I. Mechanics and Hydrostatics.

1. (*a*) What is a pendulum? Explain the terms—length, period of vibration, and amplitude, of a pendulum. State the relation between the first two when the third is small, and describe the experiment you would perform to prove the relation.

(*b*) If the length of the seconds pendulum is 100 cm., find the length of the pendulum which makes 48 vibrations in a minute. (Madras.)

2. How would you verify experimentally the relation between the length and the period of a simple pendulum?

A simple pendulum 85 cm. in length makes 50 complete oscillations in 92·5 seconds. Find (i) the value of *g*, (ii) the length of the seconds pendulum. (Mysore.)

3. (*a*) Make a sketch of a burette, and describe briefly the uses to which this apparatus can be put.

(*b*) Mention how you would proceed to ascertain whether a burette has been accurately graduated. (United Provinces.)

4. Explain how you could find the internal volume of a small bottle by means of the physical balance. How could you use the bottle to find the density of oil? If the volume of the bottle is 45 c.c., calculate how many grams of oil (of specific gravity 0·8) it will hold. (United Provinces.)

5. Distinguish between mass and weight.

Will a given body weigh more at the poles or at the equator? Give reasons.

A force acting on a gram of matter produces an acceleration of 2 cm. per sec. per sec. Find the magnitude of the force. (Punjab.)

6. Enunciate the principle of Archimedes.

A man carries a 60 lb. bucket of water in his right hand and a 4 lb. fish in his left hand. The specific gravity of the fish is 1. He puts the fish into the bucket of water. How much does his right hand carry? (Punjab.)

7. A sphere of wood of density 0·5 and radius 3 cm. is gently placed in a beaker full of water, so as to float in it. How much water will run over ? ($\pi=3{\cdot}14$.) (United Provinces.)

8. (*a*) State the principle of Archimedes, and explain how you would apply it to find the specific gravity of turpentine.

(*b*) A piece of lead weighs 33 gm. in air. A piece of wood, which weighs 120 gm. in air, is fastened to the lead, and the two together weigh 20 gm. in water. Find the specific gravity of the wood, given that the specific gravity of lead is 11. (Madras.)

9. 1 c.c. of lead (specific gravity=11·4) and 21 c.c. of wood (specific gravity=0·5) are fixed together. Show whether they will float or sink in water. (United Provinces.)

10. (*a*) A cylindrical disc of wood 2 ft. 6 in. in diameter and 8 inches in height floats half submerged in a tank of water ; calculate in c.cm. the volume of water displaced.

(*b*) If the wooden disc is loaded uniformly with pieces of lead until the top surface of the disc corresponds with the surface of the water, what weight of lead, expressed in kilograms, must be added ? (United Provinces.)

11. State Boyle's Law, and describe how you would proceed to verify it, showing clearly the method of entering the results of your observations in your record of the experiment.

In a single-piston air pump, the barrel has a capacity equal to one-third of that of the bell-jar. The bell-jar is initially full of air at atmospheric pressure. Calculate the pressure in the bell-jar at the end of three full strokes. (Mysore.)

12. State Boyle's Law.

A uniform glass tube closed at one end and bent into a U-form is set up vertically as a closed air manometer. When the open end is exposed to the atmosphere the mercury levels in the two limbs are the same, and the enclosed air column is 20 cm. long. On connecting the open end to the water supply, the air column is found to occupy a length of the tube equal to 8 cm. Find the pressure of the water supply in centimetres of mercury, it being given that the barometric height is 68 cm. of mercury. (Mysore.)

13. Explain how you would measure the pressure of the gas contained in a gasholder by using an open-air manometer.

A uniform narrow tube with one end closed has a thread of mercury 30 cm. long enclosing a column of air at the closed end. When the tube is held vertically (i) with the open end downwards, the length of the air column is found to be 35 cm., (ii) with the closed end downwards it is 15 cm. Calculate the atmospheric pressure. (Mysore.)

14. (*a*) Describe an ordinary cistern barometer. In what important respect is Fortin's an improvement on it ?

(*b*) When the barometer stands at 75 cm. a quantity of air, 10 c.c. in volume at the same pressure, is introduced into the

vacuum of the barometer. The mercury immediately falls, and is now found to stand at 25 cm. What volume does the air occupy inside the barometer tube ? (Madras.)

15. State the law of the parallelogram of forces. Show how you would apply the law to find graphically the resultant of a number of forces acting at a point by the polygon method.

The following forces act at a point :—20 lbs. weight due East, 16 lbs. weight 60° North of East, 25 lbs. weight North-West, 40 lbs. weight 75° South of West. Find graphically the resultant force at the point. (Mysore.)

16. (*a*) Find the magnitude and point of application of the resultant of two given like parallel forces.

(*b*) A steel-yard is constructed of a uniform bar AB one metre long, weighing 150 gm., with the fulcrum at C, the distance AC being 30 cm. At A is hung a pan weighing 120 gm.; and a sliding weight P moves over the arm CB. P is so adjusted that for every gram added to the pan at A, P has to be moved on through 1 cm. to keep the bar horizontal. Find P, and its position when no weights are placed in the pan. (Mysore.)

17. A kilogram weight is suspended from a point A by a fine string AB. To a point C in AB another string CD is attached. The string CD is kept always stretched in a horizontal position. Find the tensions of AC and CD when the angle ACD is 120°. (The kilogram weight is attached to the end B of the string.) (Mysore.)

18. (*a*) Define the terms 'resultant' and 'equilibrant' of forces. Explain by means of an example.

(*b*) State the law of the triangle of forces, and describe an experiment to verify it.

(*c*) Three forces of 4, 5 and 6 gm. weight respectively act at a point and are in equilibrium. What are the angles between their lines of action ? (Madras.)

19. Define *power*, and state the C.G.S. and the F.P.S. practical units of power.

Water is pumped up from a well through a height of 30 feet by means of a 5 horse-power motor. If the efficiency of the pump is 85%, find in gallons the quantity of water pumped up per minute. (1 gallon of water weighs 10 lb.) (Mysore.)

II. Heat.

1. How could you show that brass expands more than iron when rods of these two metals are heated through the same temperature ?

When hot water is thrown on the bulb of a thermometer, the mercury column first falls and then rises. Why is this ? (United Provinces.)

2. (*a*) What do you understand by the expression 'coefficient of expansion' of a liquid? Explain clearly the various steps in your determination.

(*b*) Describe an experiment you have performed to find the 'apparent coefficient of expansion' of a liquid. Explain clearly the various steps in your determination.

(*c*) A small flask weighs 12·2 gm. when empty, and 159·6 gm. when filled with mercury at 10° C. At a temperature of 95° C., 2·02 gm. of mercury are expelled. Find the apparent coefficient of expansion of mercury. (Madras.)

3. It is required to find the manner in which the pressure of a given mass of gas alters when only its temperature is varied. Describe with a sketch the apparatus you would employ, and the readings you would take for the purpose. (Mysore.)

4. (*a*) State precisely how the melting-point of a substance like naphthalene can be determined. Sketch the apparatus required.

(*b*) What precautions are necessary in the foregoing experiment? (United Provinces.)

5. Define unit of heat, capacity for heat, and specific heat.

A piece of iron weighing 100 grams is warmed 10° C. How many grams of water could be warmed 1° C. by the same amount of heat? The specific heat of iron is 0·10. (Punjab.)

6. (*a*) Explain how you have determined the specific heat of a liquid like alcohol.

(*b*) A mass of 147·5 gm. of copper was heated to 100° C. and placed in 117·2 gm. of water contained in a copper calorimeter of mass 54·2 gm. and at a temperature of 18° C. The temperature of the mixture was 26°·3 C. Find the specific heat of copper. (Madras.)

7. Why is mercury used as the liquid in an ordinary thermometer? What reading on the Centigrade thermometer will be the same as 22° F.? (United Provinces.)

8. (*a*) There are two thermometers, of which one has the larger bulb, and the other the finer bore. Explain the advantages and disadvantages in each case.

(*b*) Convert −32° Fahrenheit into degrees Centigrade and Réaumur. (United Provinces.)

9. Define specific heat and latent heat.

How much ice at 0° C. will be melted by 500 gm. of hot oil at a temperature of 88° C. (Latent heat of ice=80, specific heat of oil=0·6.) (United Provinces.)

10. Describe fully all that happens when ice, say at −8° Fahrenheit, is slowly heated until a temperature of 212° F. is reached. (United Provinces.)

11. One hundred c.c. of water are heated in a beaker from 27° C. to 72° C. What is the effect on (i) its density, (ii) its quantity of heat, (iii) its latent heat, (iv) its volume? In case (ii), calculate the amount of the change. (United Provinces.)

12. (*a*) What is meant by the 'saturated vapour of a liquid'?

(*b*) How would you measure the saturation pressure of aqueous vapour at ordinary temperatures?

(*c*) A little water lies on the top of the mercury column in a barometer. What error would be made if this barometer were used to measure the pressure of the atmosphere? Would the error be the same on a cold day as on a warm day?

13. Describe experiments to show that (1) the vapour pressure of ether is greater than that of water, (2) that the vapour pressure of water at boiling point is equal to atmospheric pressure. (Madras.)

14. (*a*) Explain how you would determine the boiling-point of a liquid, of which a small quantity only is available.

(*b*) A calorimeter contains 100 gm. of water at 30° C., and on adding 8 gm. of ice at 0° C. and stirring the contents, the temperature falls to 23° C. Given that the latent heat of water is 79, find the water-equivalent of the calorimeter and stirrer. (Mysore.)

15. Describe and distinguish between the three modes of propagation of heat. (Punjab.)

16. (*a*) Can you show from the cooling curve of a liquid that it cools more rapidly at higher temperatures than at lower temperatures?

(*b*) In a cup P we put 100 c.c. of tea at 80° C.; it is allowed to cool for 10 minutes; we then add 20 c.c. of milk at 20° C. But in cup Q we put 100 c.c. of tea at 80° C. as before and add at once 20 c.c. milk at 20° C., and then allow it to cool for 10 minutes. Will the result be the same in each case? (United Provinces.)

III. Light.

1. Show by a diagram (carefully drawn as large as your paper will allow) how many reflections of a pin can you obtain in two plane mirrors placed at an angle of sixty degrees.
(United Provinces.)

2. A glass cube is placed over a pencil mark on a sheet of paper. The mark is viewed through the cube. Explain, by means of a diagram, the apparent position of the pencil mark.
(United Provinces.)

3. (*a*) State the laws of refraction of light.

(*b*) What do you understand by 'total internal reflexion'? Why is the reflexion so called? What are the conditions for such a reflexion?

(*c*) How would you use a glass block to find the index of refraction by an experiment in which the light is totally reflected from one face?

(*d*) Illustrate by a diagram how objects above the surface are seen by an observer submerged in the water. (Madras.)

4. (*a*) With reference to a concave mirror, define the following terms:—Centre of curvature, focal length and conjugate foci.

(*b*) Describe how you would find the focal length of a concave mirror.

(*c*) Determine graphically the position of the image formed of an object placed 60 cm. from a concave mirror, of which the focal length is 40 cm. (Madras.)

5. The focal length of a concave mirror is 15 cm. If the object is placed at a distance of 20 cm. from the mirror, where will the image be formed? Draw a diagram to show the position of the object and its image. (Punjab.)

6. What is meant by the radius of curvature of a spherical mirror? Explain, with a sketch, how you would find by experiment the radius of curvature of a concave mirror. (United Provinces.)

7. Show by means of neat diagrams how (i) a real and enlarged image, (ii) a real and diminished image, can be produced by a convex lens. (United Provinces.)

8. How would you find the focal length of a convex lens? An object is placed 20 cm. from a convex lens, and an inverted image is formed 4 times as large as the object. Find the focal length of the lens. (United Provinces.)

9. Describe in detail the kind of lens you would use in a bicycle lamp, and explain the purpose of placing a reflector behind the flame. (United Provinces.)

10. What is the difference between a real and a virtual image? A luminous object 10 mm. high is placed at a distance of (*a*) 20 mm., (*b*) 10 mm., from a convex lens whose focal length is 15 mm. Show by neat diagrams the nature, size and position of the image in each case. (United Provinces.)

11. (*a*) Define the terms *principal focus* and *conjugate foci*.

(*b*) Describe with a diagram the arrangement of the lenses in a magic lantern, and explain how a bright enlarged image is thrown on the screen. (Madras.)

12. Describe the principle of the astronomical telescope *or* the photographic camera, giving diagram. (Punjab.)

13. (*a*) What is meant by dispersion and spectrum? Describe a method of obtaining a pure solar spectrum.

(*b*) In a dark room are placed three bodies painted white, blue and green respectively, and a beam of blue light is allowed to fall on these bodies. What effect will the bodies produce on the eye of a person inside the room? (Madras.)

14. (*a*) Explain the terms 'deviation' and 'dispersion' as applied to the path of light through a prism.

(*b*) Draw the section of a prism. Draw also the section of a beam of sunlight passing through the prism, and show by your sketch how this light is acted on by the prism. Show also by your sketch the effect of introducing a red glass in the path of the beam emerging from the prism. (Madras.)

IV. Magnetism and Electricity.

1. We take a number of rods of different metals and a magnet. How can we find out which of the rods can be magnetized ? If some have become magnetized, how can we find out which of these is steel and which is soft iron ? (United Provinces.)

2. Two bar-magnets are arranged on a table parallel to each other and about one inch apart, (i) with the north-poles pointing in the same direction, (ii) with the north-poles pointing in opposite directions. Draw two sketches, and show by dotted lines how iron filings will arrange themselves about the magnets. (United Provinces.)

3. Define unit magnetic pole.
Two bar magnets each 5 cm. long and of pole strength 50 units are placed with their axes in the same straight line, the positive pole of one being nearest to the negative pole of the other and at a distance of 5 cm. from it. Find the force between the magnets. (Mysore.)

4. Define ' pole-strength ' and ' strength of magnetic field.'
A magnet, the distance between whose poles is 30 cm., is placed with its axis in the magnetic meridian, and it is found that the horizontal component of the resultant magnetic field strength is zero at a point whose distance measured from either pole is the same and equal to 20 cm. If the horizontal component of the earth's magnetic field is 0·38 C.G.S. unit, calculate the pole-strength of the magnet. (Mysore.)

5. What do you understand by the *magnetic moment* of a bar-magnet ?
How would you compare experimentally the magnetic moments of two short bar-magnets ? (Mysore.)

6. Describe the construction and explain the action of a gold-leaf electroscope. How can you ascertain with the help of this instrument whether a body is charged or uncharged ; and, in the former case, how would you proceed to determine the nature of the charge ? (United Provinces.)

7. How would you prove that positive and negative electricities are produced in equal quantities (i) by friction, and (ii) by induction ? (United Provinces.)

8. State Ohm's Law.
You are given an ammeter, a voltmeter, a battery, a rheostat, and a piece of thin platinum wire. How would you proceed to show that when the wire is heated by passing a current through it the resistance of the wire increases as the current through it increases ? Draw a diagram of the arrangement of the apparatus, and show clearly how you would record the results of your observations. (Mysore.)

9. (*a*) Describe an experiment you would perform to determine the direction of the force on a conductor carrying a current and placed in a magnetic field.

(*b*) Describe a moving-coil galvanometer, stating clearly the function of each part. (Mysore.)

10. Define the *reduction-factor* of a tangent galvanometer, and find the relation between it and the dimensions of the coil.

Describe how you would find the reduction-factor of a tangent galvanometer experimentally. (Mysore.)

11. Explain the action of the 'tangent galvanometer.'

Two cells, A and B, are connected in series with a tangent galvanometer, and the deflection is found to be $47°{\cdot}5$. The cell B, of smaller E.M.F., is reversed, and the deflection is now found to be $27°{\cdot}5$. If the E.M.F. of A is 2 volts, calculate the E.M.F. of B. (Mysore.)

12. (*a*) Describe the procedure you would employ to compare the E.M.F.'s of two cells, using a potentiometer.

(*b*) A potentiometer wire has a resistance of 8 ohms and is connected to the terminals of a battery of 3·2 volts, having an internal resistance of 12 ohms. Calculate the potential-difference between the ends of the potentiometer wire. (Mysore.)

13. Give a neat diagram of the arrangement of apparatus when a potentiometer is used for the comparison of the E.M.F.'s of two cells, and state briefly the manner of carrying out the experiment.

A potentiometer wire is one metre long and has a resistance of one ohm. It is connected up in series with a resistance of 99 ohms and a cell of negligible resistance and of E.M.F. 2 volts. The connecting wires are also of negligible resistance. The apparatus is now used in determining the E.M.F. of a cell (of low E.M.F.), and the balance point is found to be 45 cm. from one end of the wire. Calculate the E.M.F. of the cell. (Mysore.)

14. Define 'the Joule' and 'the Watt.'

A current of two amperes flows for 1·5 minutes through a coil of german-silver wire immersed in a copper calorimeter containing 75 gm. of oil of specific heat 0·5. The difference of potential between the ends of the wire is found to be 5 volts, and the water-equivalent of the calorimeter is 5. Calculate the rise in temperature.

Draw a neat diagram of the electrical connections in the above experiment, given that an ammeter and voltameter are used for measuring the current strength and the difference of potential respectively. (Mysore.)

EXAMINATION QUESTIONS FROM INTERMEDIATE PAPERS.

I. Mechanics and Hydrostatics.

1. Explain how you would deduce the value of g with a simple pendulum.

A clock which keeps correct time when its pendulum beats seconds, was found to be losing 4 minutes a day. On altering the length of the pendulum it gained $2\frac{1}{2}$ minutes a day. By how much was the length altered, if the length of the seconds pendulum is 99·177 cm.? (Madras.)

2. Distinguish between absolute and relative density.

A ring is made of 900 parts by weight of gold and 100 of copper, and is beaten hard so as to contract 5%. Find the density of the alloy. Density of gold = 19·32 gm. per c.c.; density of copper = 8·93 gm. per c.c. (Madras.)

3. When a horizontal surface is immersed in a liquid, the resultant thrust on the surface is proportional to the depth below the free surface of the liquid and to the density of the liquid. Describe experiments to verify the truth of the above law.

A layer of petroleum 25 cm. in depth, and of specific gravity 0·875, floats on a quantity of water which by itself forms a layer 30 cm. in depth. Find the difference between the pressures on the top surface of the oil and on the bottom surface of the water. (Madras.)

4. How would you demonstrate Archimedes' Principle by experiment?

A submarine boat, of weight 224 tons, lies damaged at the bottom of the sea. With the exception of an air chamber, which remains uninjured, it is full of water. Chains are employed to raise the boat to the surface. If the air chamber has a capacity of 20 c.ft., the specific gravity of the material of the boat is 7·8, and that of the water is 1·025, calculate the total tension on the chains. (Madras.)

5. Give practical instructions for the use of Nicholson's hydrometer in measuring the specific gravity of (1) a small piece of rock, (2) a piece of wood, (3) kerosene oil. (Madras.)

6. How do you find the specific gravity of a solid lighter than water?

A piece of cork whose weight is 19 grammes is attached to a bar of silver weighing 63 grammes and the two together just float in water. The specific gravity of silver is 10·5. Find the specific gravity of cork. (Calcutta.)

7. Show how to prove that the resultant vertical thrust on any body immersed in a fluid at rest is equal to the weight of the fluid displaced by the body.

A block of ice of specific gravity 0·915 floats in sea-water of density 1·025. What fraction of the volume of the ice will be exposed by the water? (Madras.)

8. How would you find the specific gravity of a solid less dense than water, such as a piece of wood, in the laboratory?

The following results of certain weighings are given:

Weight of a piece of brass in air,	22·68 gm.
,, ,, wood ,,	1·595 ,,
,, the brass in water,	20·02 ,,
,, the brass and wood together in water,	14·275 ,,

Find the specific gravity of the brass and of the wood. (Madras.)

9. Describe a barometer, and explain what happens when

(*a*) the liquid is changed;

(*b*) the tube is inclined;

(*c*) a drop of alcohol is introduced in the tube;

(*d*) a hole is bored at the top. (United Provinces.)

10. Obtain a formula which will enable you to calculate the distance a body has travelled in a given time, when the initial velocity and the acceleration are given.

A stone is thrown vertically upwards with a velocity of 160 ft. per second from the top of a cliff 120 feet high. How high will the stone rise above the cliff, and after how long will it fall to the foot of the cliff? What will be the velocity of the stone when it is 80 ft. above the point of projection? (Madras.)

11. Distinguish between pressure and force.

A horizontal bar of square cross section, 4·08 metres long, and 8 cm. wide, rests on two uprights of the same size, the whole forming three sides of a square. The density of the material is 2·5 gm. per c.c. A weight of 20 kilogrammes is placed at a distance of one meter from the middle point of the horizontal bar. Find the force and pressure on each support, assuming that the bar does not bend. (Punjab.)

12. Define Young's modulus, and describe how it can be experimentally determined in the case of a wire. (United Provinces.)

13. Define the terms *work*, *power*, *horse-power*; and explain how their units are related.

An engine is employed to pump 6000 gallons of water per minute from a well, through an average height of 21 feet. Find the horse-power of the engine, if 45% of the power is wasted. (Madras.)

14. What are kinetic energy, potential energy, and work?

Find the energy stored in a train weighing 250 tons and travelling at 60 miles per hour. How much energy must be added to the train to increase its speed to 65 miles per hour? (Calcutta.)

15. Explain what is meant by the moment of a force about a point.

A rod, the weight of which may be neglected, is suspended horizontally from two spring balances 12 inches apart. Two weights of 3 lb. and 4 lb. respectively are hung from the rod 20 inches from each other, neither weight being between the balances. Find the readings on the two balances (*a*) when the 3 lb. weight is 5 inches from one of them, (*b*) when the weights are interchanged in this position. (Madras.)

II. Heat.

1. The loss of weight of a weighted bulb when immersed in liquid at 0° C. is w_0; show that the loss w at t° C. is given by

$$w=w_0\{1+(\alpha-\beta)t\},$$

where α and β are the coefficients of expansion of the bulb and of the liquid respectively.

Sketch an apparatus to determine the apparent coefficient of expansion of a liquid by the above formula.

(United Provinces.)

2. Enumerate the principal errors of mercury-in-glass thermometers.

Find the mass of mercury expelled from a glass bulb containing 950 grams of mercury at 0° C., when heated to 100° C. (Coefficient of expansion of mercury$=0·18\times10^{-3}$; coefficient of linear expansion of glass$=0·083\times10^{-4}$.) (United Provinces.)

3. Describe how to measure the absolute expansion of a liquid with the weight thermometer. A weight thermometer contains 43·218 grams of liquid at 15° C., but only 42·922 grams at 40° C. The coefficient of linear expansion of the glass is 0·000009. Find the absolute coefficient of expansion of the liquid. (Punjab.)

4. Define specific heat, and describe a method of determining the specific heat of a liquid.

200 grammes of water at 98° C. are mixed with 200 cubic centimetres of milk of density 1·03 at 30° C. contained in a brass vessel of thermal capacity equal to that of 8 grammes of water, and the temperature of the mixture is 64° C. Assuming there is no loss of heat due to radiation, find the specific heat of milk.

(Calcutta.)

5. The diameter of the capillary tube of a Bunsen ice calorimeter is 1·4 mm. On dropping into the instrument a piece of metal whose temperature is 100° C. and mass 11·088 grams, the mercury thread is observed to move through 10 cms. Calculate the specific heat of the metal.

(Given latent heat and density of ice to be 80 and 0·9 respectively, and $\pi=\frac{22}{7}$.) (United Provinces.)

6. Distinguish between boiling and evaporation. What condition determines whether a liquid will boil or evaporate ?

A glass bottle and a jug of porous earthenware are both filled with water and exposed to air, side by side. What difference do we notice between the temperatures of the water in the two vessels after a few hours ? Explain why this happens. If there is very little or no differemce of temperature, what conclusion may we draw as to the state of the atmosphere, and why ?

(Calcutta.)

7. What is meant by *latent heat* and *specific heat* ?

A copper vessel weighing 300 grams contains 50 grams of ice mixed with 50 grams of water. A ball of iron weighing 500 grams heated to 300° C. is dropped in. What happens ?

[Specific heat of iron	0·112.
,, ,, copper	0·094.
Latent heat of water	80.
,, ,, steam	540.]

(Madras.)

8. Define latent heat of ebullition.

A building is heated by radiators heated by the condensation of steam. If the radiators radiate heat to the room at the rate of 297 million calories per hour, find the weight of the steam to be supplied per minute. The latent heat of steam plus the loss of heat per gram of water in falling to the temperature of radiation may be taken as 550. (Punjab.)

9. (*a*) Define thermal conductivity.

(*b*) Find how much steam per minute is generated in a boiler made of boiler-plate 0·5 cm. thick, if the area of the walls of the fire-chamber is 2 sq. metres, the mean temperature of the plate-faces 200° C. and 120° C. respectively, the latent heat of steam 522, and the conductivity of the steel plate 0·164. (Punjab.)

10. How would you experimentally determine the thermal conductivity of a substance ? The earth is found to be about 1° C. hotter for every 30 metres of vertical descent. If the coefficient K for rock is, on the average, 0·0045, what is the approximate loss of heat from the surface of the earth, in calories per sq. metre per annum ? (United Provinces.)

11. State the laws which govern the transmission of heat by radiation.

You are given two ordinary clear glass thermometers, and the bulb of one of them is coated with lamp-black. Compare their readings when exposed (1) on a damp cloudy night, (2) on a clear dry night in the cold weather, (3) in the sun. (United Provinces.)

12. What is meant by the statement that heat is a form of energy ?

An iron ball, having fallen from rest through 25 metres, contains kinetic energy sufficient to raise its temperature through 0°·6 C. Calculate the value of the mechanical equivalent of heat. (Specific heat of iron=0·1.) (United Provinces.)

13. What is meant by 'the mechanical equivalent of heat'? How much work is done in supplying the heat necessary to convert 10 grams of ice at $-5°$ C. into steam at $100°$ C.?

[Specific heat of ice is 0·5.
Latent heat of fusion of ice is 80.
Latent heat of steam is 537.
Mechanical equivalent of heat is $4{\cdot}2 \times 10^7$ ergs.]

(United Provinces.)

14. Describe briefly a method that has been used to determine the 'mechanical equivalent of heat.'

A bullet of lead strikes a target with a velocity of 200 metres per second. Find the rise of temperature produced in the bullet by the impact, supposing that the bullet absorbed only half the total heat generated by the impact. The specific heat of lead is 0·032, and the value of J is $4{\cdot}2 \times 10^7$ ergs per calorie. (Madras.)

III. Light.

1. Define umbra and penumbra. The diameter of the sun being taken as 900,000 miles, and its distance from the earth 90,000,000 miles, and the diameter of the moon 2100 miles, find the distance of the earth from the moon at the time of a solar eclipse when the eclipse is total only at a single point on the earth. Also find the diameter of the area on the earth within which the eclipse is total when the distance of the moon from the earth is 209,000 miles. The earth is assumed flat for this purpose. (Punjab.)

2. State the laws of reflection of light. How could you verify them?

Show by means of a diagram that a man can see the whole of his person in a plane mirror, the length of which is half his own height. (Calcutta.)

3. A cubical block of glass, each of whose edges is 10 cm., is placed over a dot on a piece of cardboard placed on a horizontal table, and an observer views the dot, placing his eye vertically above it. If the refractive index of the glass is 1·52, calculate the apparent position of the dot. Trace the course of a pencil of rays from the dot to the eye of the observer. Show how this apparent shift in the position of an object may be experimentally determined and then used in determining the refractive index of a liquid. (Madras.)

4. You are given a microscope and a thick block of glass. State how you would proceed to find the refractive index of glass.

A mark is made on the bottom of a beaker, and a vertical miscroscope is focussed on it. The microscope is then raised through a distance of 1·5 cm. What height of water must be poured into the beaker in order to bring the mark again into focus? (μ for water$=\frac{4}{3}$.) (United Provinces.)

5. State the laws of refraction of light. Deduce the relation between the distances of a point and its image formed by direct refraction through a plane surface.

A body is viewed through a glass plate 4 inches thick, the body being 1 inch behind the plate. Where will the body appear to be? (Madras.)

6. A thick glass plate rests below the surface of water at an angle of 15° to the horizontal. A ray of light falls on the surface of the water at 60° to the normal, and is refracted so as to pass through the glass plate. Find graphically the path of the ray in the water and in the glass, taking the refractive index from air to water as 1·33, and that from air to glass as 1·55. (Madras.)

7. Describe how you would determine the angle of a prism with a spectrometer.

In an experiment done with a prism, the angle of the prism is 59° 20′, the direct reading of the slit through the telescope is 168° 18′, and the reading of the telescope in the minimum deviation position is 216° 36′. Find the refractive index of the material of the prism. (Madras.)

8. An object 3 cm. high is placed at a distance of 120 cm. from a convex spherical mirror of 30 cm. focus. Find the size of the image. (Punjab.)

9. What is the difference between a real and a virtual image of an object formed by a spherical mirror? Illustrate by diagrams.

A pin 3 cm. long is placed with its middle point at a distance of 1·5 metres from a concave spherical mirror whose radius of curvature is 50 cm.; find the position and the size of the image formed. (Calcutta.)

10. Give two methods for determining the focal length of a concave lens. (United Provinces.)

11. A prism is held in the hand with its refracting edge horizontal and uppermost. Will the top of an object seen through it be red or blue? Give reasons for your answer. (United Provinces.)

12. Define the principal focus and the optical centre of a lens. Draw diagrams to show the positions of the images formed by a convex lens, when an object is placed (i) within and (ii) beyond its focal length. (Madras.)

13. What is meant by each of the following terms in relation to a thin lens: *focal length, focal power, centre, conjugate focus, principle focus*?

Give a geometrical construction for finding the positions of conjugate foci in the case of a thin lens.

A thin converging lens of focal length 20 cm. produced an image which is three times as long as the object. What will be the distance between the object and the image; and which will be the nearer to the lens? (Madras.)

14. A luminous source and screen are placed at a fixed distance apart along a scale, and a convex lens can be moved between them. Explain how you will utilise this arrangement to determine the focal length of the lens. If A and B be the sizes of the image for the two positions of the lens, show that the size of the object is equal to $\sqrt{ab}$. (United Provinces.)

15. Describe the chief refractive errors of the eye, and their correction, illustrating your answer by suitable diagrams. (Punjab.)

16. Explain by means of diagrams the following instruments: (1) magic lantern, (2) microscope, (3) astronomical telescope. (Punjab.)

17. Describe a spectroscope.

What will be observed in the spectroscope when the light passing through the slit comes from (*a*) a spirit lamp with a salted wick; (*b*) an electric light; (*c*) an electric light in a red glass globe? (United Provinces.)

IV. Sound.

1. Explain the formation of echoes. A man stationed between parallel cliffs fires a gun. He hears the first echo after two seconds and the next after five seconds. What is his position between the cliffs, and when will he hear the third echo? (United Provinces.)

2. Explain how two bridges should be placed in order to divide a stretched string 100 cm. long into three segments whose fundamental frequencies are in the ratio of 1 : 2 : 3 (United Provinces.)

3. How does the frequency of a vibrating string or wire depend on its linear density? Describe an experiment which may be performed to verify this law.

A stretched wire under a tension of 1 kilogram weight is in unison with a tuning-fork of frequency 320. What alteration in the tension would make the wire vibrate in unison with a fork of frequency 256? (Madras.)

4. How does the frequency of the fundamental vibration of a stretched string depend upon its length, its tension and its mass per unit length?

A wire 50 cm. long and of mass 6·5 gm. is stretched so that it makes eighty vibrations per second. Find the stretching force in grams-weight.

How would you double the frequency, (1) by changing the length of the wire; (2) by changing the tension in the above case? (United Provinces.)

5. Upon what does the vibration frequency of a stretched string depend?

A steel wire 60 cm. long and 5 mm. diameter gives a note of 240 vibration frequency when stretched with a certain weight.

A second steel wire bears the same weight, but is 40 cm. long and 6 mm. diameter. Find the periodic time of its fundamental note. (United Provinces.)

6. State the laws of vibration of strings, and describe experiments to verify them. A wire of length 140 cm. and mass 52 grams is stretched by means of a load of 16 kilograms. Calculate the frequency of the fundamental vibration. (United Provinces.)

7. Upon what does the frequency of the note sounded by a string vibrating transversely depend ?

A copper wire (density 8·8 gm. per c.c.), one metre long and 1·8 mm. in diameter, is stretched by a weight of 20 kilograms. Calculate the frequency of the fundamental note. (United Provinces.)

8. Explain the production of beats when two forks of nearly the same pitch are sounded together. How would you determine which fork has the greater vibration number ? (United Provinces.)

9. What is a tuning-fork ? A vibrating tuning-fork is held at the mouth of an open jar, and water is poured into the jar gradually. Explain what will happen. How could you determine the velocity of sound in air by an experiment of this kind ? (Calcutta.)

10. What is an 'end correction' ? A certain tuning-fork first produced resonance in a glass tube with an air column of 33 cm., and it could again produce resonance with a column of 100·5 cm. in the same tube. Calculate the 'end correction.' (United Provinces.)

11. In building an organ for use in a warm climate it is necessary, in order to produce notes of a given pitch, to make the pipes longer than if they were to be used in England. Explain why this is so. (United Provinces.)

V. Magnetism and Electricity.

1. What is magnetic induction ? A magnet is placed horizontally in the magnetic meridian due south of a compass-needle. How will its action on the latter be affected if the needle is surrounded by a spherical shell of soft iron ? (United Provinces.)

2. Obtain a formula for the magnetic force at a point situated on the axis produced of a small bar magnet. (Punjab.)

3. Define the terms unit magnetic pole and strength of a magnetic field at a given point. Calculate the field due to a bar magnet 10 cm. long, and having a pole strength of 100 units, at a point 20 cm. from each pole. (United Provinces.)

4. (*a*) Distinguish between the terms magnetic potential and magnetic force in the neighbourhood of a magnet.

(*b*) A magnet of moment M lies with its axis making an angle θ with the direction of the surrounding uniform field of strength H. Find the amount of the force acting on the magnet about its centre. (Punjab.)

5. Describe a method of comparing the magnetic moments of two magnets.

A magnet whose pole strength is 25 and magnetic moment 250 is placed on a horizontal table with its axis in the magnetic meridian and its north pole pointing magnetic north. It is found that a small compass needle placed on the table at a point whose distance from each of the poles is 10 cm. shows no tendency to rest in any particular direction. Calculate the horizontal component of the earth's magnetic field. (Madras.)

6. What is meant by the magnetic moment of a magnet? A bar magnet whose poles are 10 cm. apart is placed in the meridian with its north-seeking-pole to the north. A neutral point is found 10 cm. due east of the centre of the magnet. Calculate the moment of the magnet, given that H is equal to 0·35 C.G.S. unit. (United Provinces.)

7. Explain the method of comparing the horizontal magnetic force at two different places by means of the vibration magnetometer. Show how the total intensity of the earth's magnetic field can be calculated if the intensity of the horizontal field and angle of dip are known. (United Provinces.)

8. Two steel bars in all other respects alike are magnetised to different degrees. How would you discover which is the stronger magnet without the aid of any third magnet but the earth? (United Provinces.)

9. A thin bar magnet is provided with a horizontal axis through its centre of gravity perpendicularly to its length, about which it can rotate freely. Describe and explain the positions taken by the magnet, (*a*) when the axis of rotation is in the magnetic meridian, (*b*) when it is at right angles to the meridian. If the magnet is made to oscillate, in which position will the rate of oscillation be the greater? (Punjab.)

10. Explain what is meant by an electric line of force.

Two small insulated metallic spheres A and B, of the same size, are given charges of $+50$ and -50 units respectively and placed at a distance of 20 cm. from each other. Indicate by means of a diagram the distribution of the electric lines of force in the field of the two charges. Calculate the electric force on a small body carrying three units of + charge placed at a distance of 10 cm. from B on AB produced. (Madras.)

11. Explain why a conductor, which is required to retain an electric charge for a long time, should be rounded and without sharp points. Describe experiments illustrating the action of sharp points on a conductor. (Calcutta.)

12. Describe a condenser, and demonstrate experimentally how the capacity and potential can be altered. Explain the terms 'capacity' and 'potential' fully. (United Provinces.)

13. Define dielectric constant.

The inner coating of a Leyden jar is connected to a gold-leaf electroscope. If the jar rests on a piece of ebonite one charge from an electrophorus produces a large divergence of the leaves of the electroscope. If the ebonite be removed and the jar is held in the hand, several charges of the electrophorus are needed to produce the same divergence. Explain this. (Punjab.)

14. Describe the construction of a simple voltaic cell, and give an account of the chemical changes which take place in the cell when in action. What are the chief defects of such a cell, and how can they be remedied ?

Or,

Discuss, with full experimental details, what you know about the following :—(*a*) Magnetic induction ; (*b*) the law of magnetic attraction and repulsion ; (*c*) the method of determining the poles of a magnet ; and (*d*) magnetic lines of force. (Punjab.)

15. State and prove the formula for the relationship between the current and the deflection produced by it in a tangent galvanometer.

Describe what improvements can be made in such a galvanometer to make it more sensitive, and show how these can be inferred from the formula. (Punjab.)

16. Obtain a formula which will enable you to calculate the resistance of a number of wires in parallel.

Three wires of resistance 2, 6 and 12 ohms respectively are connected in parallel and are inserted in a circuit with a cell and tangent galvanometer. The deflection is 60° ; the 2 ohms wire is removed, and the deflection becomes 45°. Calculate the resistance of the galvanometer. (Neglect the resistance of the cell.) (United Provinces.)

17. State Ohm's law. The same current passes through a metre of copper wire 1 mm. diameter and two metres of a thinner copper wire. The difference of potential between the ends of the first wire is 1 volt and that between the ends of the second wire 20 volts. Find the diameter of the thinner wire. (United Provinces.)

18. State Ohm's law, and describe experiments to illustrate it.

Determine the number of cells required to send a current of half an ampere through a body whose resistance is 30 ohms, if each cell has an E.M.F. of 1·25 volts and a resistance of 2 ohms. (Calcutta.)

19. State Ohm's law.

How would you arrange 30 cells, in each of which the resistance is 5 ohms, so as to send the most powerful current through an external circuit of 6 ohms resistance ? (United Provinces.)

20. Enunciate Ohm's law.

A battery is connected to a tangent galvanometer of resistance 9 ohms, and produces a deflection of 60°. An extra resistance of 7 ohms is then placed in series in the circuit, and the deflection falls to 45°. Calculate the resistance of the battery.
(United Provinces.)

21. A group of fifty 100-volt lamps, joined in parallel and taking 0·6 ampere each, are connected to a battery of 54 accumulators, 2 volts each, and internal resistance 0·005 ohm per cell. Find if the lamps will be correctly, over, or under lighted.
(Punjab.)

22. Distinguish between P.D. and E.M.F. A circuit is formed of six similar cells in series and a wire of 10 ohms resistance. E.M.F. of each cell is one volt, and its internal resistance 5 ohms. Determine the P.D. between the +ve and −ve poles of any one of the cells. (United Provinces.)

23. State the laws of electrolysis. A plate of copper and a plate of platinum are dipped into a solution of copper sulphate; describe the effect of passing a current from the copper to the platinum plate. What happens when the current is afterwards reversed? (United Provinces.)

24. Describe carefully the experiments you would make to ascertain whether an ammeter (graduated up to 2 amperes) gives correct indications. (United Provinces.)

25. State Joule's law.

A current of 5 amperes is passed for 10 minutes through a coil of 10 ohms resistance immersed in water. Calculate the energy spent in the coil, stating your unit. How would you find J based on this experiment? (United Provinces.)

26. State Joule's law relating to the production of heat in a wire carrying a current, and describe briefly how the law may be verified experimentally.

Calculate the amount of heat produced in 5 minutes in a wire of resistance 5 ohms, if a steady difference of potential of 1 volt is maintained between its ends. (Madras.)

27. Describe an electrical method of determining the mechanical equivalent of heat. (United Provinces.)

28. Describe the construction of a simple telephone, and explain its action. (United Provinces.)

29. Explain the working of carbon microphone transmitters in telephones; and show why an induction coil is generally connected with the transmitter. (United Provinces.)

TYPICAL INTERMEDIATE EXAMINATION PAPERS.

CALCUTTA UNIVERSITY.

First Paper.

Only Six *questions are to be attempted.*

The questions are of equal value.

1. State the laws of the pendulum. Will the period of vibration of a pendulum be affected if it be taken to the top of a hill? Give reasons for your answer.

2. How would you show experimentally that the resultant vertical thrust on a body immersed in a heavy liquid is equal to the weight of the liquid displaced?

The apparent weight of a piece of platinum in water is 60 grammes, and the absolute weight of another piece of platinum twice as big as the former is 126 grammes. Determine the specific gravity of platinum.

3. State Boyle's law. How may it be experimentally verified for pressures greater than the atmospheric pressure?

An accurate barometer reads 30 in. when one containing air above the mercury reads 24 in. If the tube of the latter be raised 3 in. the reading becomes 25 in. Find what length of the tube the air would occupy if brought to atmospheric pressure.

4. Describe in detail, with a diagram, the common pump and its mode of action. Is there any limit to the depth from which it can raise water? State reasons.

5. Explain how the height of a mountain can be determined experimentally by finding the boiling-point of water at its top and bottom.

6. Distinguish between temperature and quantity of heat.

Explain what is meant by the statement 'that the latent heat of fusion of ice is 80.'

Dry ice at 0° C. is dropped into a copper can at 100° C., the weight of the can being 60 grammes and the specific heat of copper 0·1. How much ice would reduce the temperature of the can to 40° C.?

7. Distinguish between saturated and unsaturated vapours.

Into a cylinder exhausted of air and provided with a piston there is introduced just enough water to saturate the space at 20° C. Describe what happens under the following conditions:—

(*a*) The volume of the space is increased by pulling out the piston.

(*b*) The volume is diminished by pushing the piston down.

(*c*) The volume remaining as at first, the temperature is increased to 30° C.

(*d*) The temperature falls to 10° C.

8. Point out the various ways in which a hot body may lose its heat. What methods would you adopt to reduce the rate at which heat is lost in each of these ways ?

9. Explain, with diagrams, the method of propagation of sound in air. Define the terms wave-length and vibration-frequency.

10. State the laws of the transverse vibration of strings, and describe any method of verifying them.

The string of a certain monochord vibrates 100 times a second. Its length is doubled and its tension altered until it makes 150 vibrations a second. What is the relation of the new tension to the original ?

SECOND PAPER.

Only SIX *questions are to be attempted.*

The questions carry equal marks.

1. State what is meant by the *candle-power* of a lamp, and explain how it can be determined by a shadow photometer.

In a darkened room, a lamp and a standard candle, distant 10 and 15 decimetres respectively from a vertical stick, cause shadows of equal intensity to fall side by side upon a screen 20 centimetres behind the stick. Find the candle-power of the lamp.

2. A right-angled isosceles glass prism is sometimes used in place of a plane mirror. Explain by the aid of a diagram how it can be so used. Is it more advantageous ? If so, why ?

3. A convex lens of 6 in. focal length is employed to read the graduations of a scale, and is held so as to magnify them three times. Find how far it is held from the scale, and give a diagram of the arrangement.

4. What is the colour of an object due to ? Why does a mixture of ordinary blue and yellow pigments appear green ?

Objects which appear variously coloured in white light are illuminated by sodium flame. Describe and explain the effect observed.

5. What is meant by a line of force in a magnetic field ? Make a sketch of lines of force due to a pair of bar magnets lying parallel to each other, and one-third of their length apart and with similar poles opposite each other.

How are the lines of force of a magnet altered if a piece of soft iron is placed in its field ?

6. Describe a gold leaf electroscope. Given an uncharged body A on an insulating stand and a body B charged negatively, how by means of B can you give A (*a*) a positive, (*b*) a negative charge ?

7. Describe the construction and action of a Leyden jar.

How do you explain the fact that if the Leyden jar is placed on an insulating stand, it will not take so large a charge as when uninsulated ?

8. What do you understand by polarisation in a voltaic cell ? Describe Leclanché's cell. What are the means taken to obviate the effects of polarisation in this cell ? How far is this object attained ? What properties make this cell a suitable one for electric bells ?

9. Explain the terms—electrolyte, electrodes, cathode, anode, ions.

A current is passed through three electrolytic cells, the first containing dilute sulphuric acid with platinum electrodes, the other two containing a saturated solution of copper sulphate with platinum electrodes in one cell and copper electrodes in the other. State what occurs at each electrode.

10. Describe a telephone, and explain its action with the help of a diagram.

BOMBAY UNIVERSITY.

(The black figures to the right indicate full marks.)

Not more than FOUR *question from* EACH *section are to be attempted.*

SECTION I,

1. State what is meant by S.H.M., and show that in the case of the simple pendulum the period $T=2\pi\sqrt{l/g}$.

If a pendulum 32 feet long oscillates and the amplitude is 4 inches, find the acceleration at its highest point. **13**

2. Define the terms—*force, pressure* and *co-efficient of friction.*

What force does a horse exert on the level when pulling a cart weighing one ton=2240 lb. if the co-efficient of friction is 0·2 and the diameters of the wheel and the axle are 3 feet and 3 inches respectively ?

What force would be required if there were no wheels, the coefficient of friction remaining the same ? **13**

3. Describe carefully any standard type of barometer and explain what precautions are necessary when taking readings.

Of what practical use is a barometer ? **12**

4. How does a noise differ from a musical note ? Distinguish between *intensity, pitch* and *quality* of a musical note.

How would you vary the pitch and intensity in a stringed instrument.

Describe in detail any method to measure the pitch of a sounding body. **12**

5. Explain clearly the terms—*calorie, specific heat, latent heat* and *water equivalent.* Describe a method by which the latent heat of vaporisation of water can be determined. What pre-

cautions and corrections are necessary to obtain an accurate result ?

How much ice at 0° C. would a kilogram of steam at 100° C. melt if the resulting water was at 0° C. ? (Latent heat of steam $=537$ calories per gram. Latent heat of water$=80$ calories per gram.) **12**

6. What is meant by the terms : *relative humidity, absolute humidity, dew-point* and *hygrometry* ? Give a short survey of the various methods by which the relative humidity of the atmosphere can be determined.

How would you proceed to determine the mass of aqueous vapour in a litre of the atmosphere ? **12**

Section II.

7. (*a*) Show that the minimum height of a plane mirror in which a man can see a full-sized image of himself is half the height of the man.

(*b*) Show that in the case of a beam of light incident on a plane mirror, if the mirror rotates through an angle θ, the reflected beam rotates through an angle 2θ.

In the case of a mirror galvanometer, calculate the angles of deflection when the spot of light moves through (i) 100 cm., (ii) 12·5 mm., the mirror and the scale being at the usual distance of 100 cm. **12**

8. Prove that the magnifying power of a telescope for a distant object is F/f.

A certain lens when held close to the eye magnifies 5 times. What must be the focal length and power of the lens if the eye can see distinctly at a distance of 30 cm. ? **13**

9. Explain the terms *magnetic moment* of a magnet and *intensity* of field.

A magnet placed at an angle of 30° with a uniform field of intensity ·32 experiences a couple whose moment is 8 ; calculate the magnetic moment of the magnet and, the length of the magnet being 5 cm., calculate also its pole strength. **12**

10. Define *electrical potential* and *capacity* of a conductor.

Show that the potential at a point in an electrical field due to a charge $+Q$ on a conductor X is Q/r, where r is the distance of the point from the conductor.

A metal sphere of 10 cm. radius carrying a positive charge of 100 units is brought into contact with another metal sphere of 5 cm. radius carrying a negative charge of 50 units. Find the charge on, and the potential of each sphere after contact. **12**

11. State the most important effects of an electric current, and describe experiments illustrating each of these effects.

The radius of a coil of a tangent galvanometer is 10 cm., and the coil has 5 turns of wire. The horizontal component of the

earth's magnetic field is 0·314 C.G.S. unit. Find how many coulombs of electricity will pass through the galvanometer per minute when the deflection is 27° (tan 27°=0·51). Prove the formula you use. **13**

12. Explain clearly how induced currents are obtained. Describe experiments illustrating your answer; and show how you would deduce the direction of the induced currents in the experiments you describe.

How does an induced current differ from a current obtained from an electric battery? **12**

PUNJAB UNIVERSITY.

(The black figures on right indicate full marks.)

PART I.

1. What definition for force would you give? What is momentum? What do you understand by conservation of momentum?

A train of mass 175 tons has its velocity reduced from 40 miles per hour to zero in 5 minutes. Calculate the value of the retarding force, assuming that it is uniform. What has been the change in momentum? **5**

Or,

An automobile weighing 3000 lb. climbs a hill that rises 8 feet in 44 feet of its length at the rate of 30 miles per hour. What is the minimum horse-power developed by the engine? **5**

2. State the conditions of equilibrium of a freely floating body. A glass tube, 30 mm. long and $\frac{1}{2}$ sq. cm. in cross section, is closed at one end. Its weight is 4 gm., and 10 gm. of mercury are poured into it. What will be the specific gravity of a liquid in which it floats vertically with 2 cm. length of its stem above the surface? **5**

3. Give Newton's formula for velocity of sound. What correction has been suggested by Laplace, and why?

Discuss the effect of change of pressure and temperature on the velocity of sound. **5**

4. What do you understand by saturated and unsaturated vapours? Explain by means of diagrams how, of the three quantities P, V, T of a vapour, any two are related to each other when the third is kept constant. **5**

5. What is meant by the water equivalent of a calorimeter? What part does it play in calorimetric experiments?

How many grammes of ice can be melted by 40 grammes of steam at 100° C.? **5**

PART II.

6. Explain the apparent raising of a picture stuck on the bottom of a cube of glass when viewed perpendicularly from the top. If the refractive index is 1·6, and the thickness of the cube is 5 cm., by how much will the picture appear to be raised ? **5**

7. How will you graphically determine the position of an image formed by (*a*) reflection at a convex surface, (*b*) refraction through a convex lens ?

Give a diagrammatic representation of an astronomical telescope. **5**

8. What is meant by the moment of a magnet ?

Define strength of a magnetic field, and calculate its value at a point on the prolongation of the axis of a bar magnet. **5**

9. State the law of force between two point charges of electricity, and explain how it is applied to define unit quantity of electricity.

Two small equal metal spheres are placed 5 cm. apart in air. What will be the force between them if one has a charge of +5 units and the other −10 units ? The spheres are connected momentarily by an insulating tong. What is the force now ? **5**

10. Define a volt and a coulomb.

A flat-iron weighing 3 kilograms uses 4·5 amperes of current when operating on a 110-volt circuit. How much time will it take to heat the flat-iron from 20° C. to 200° C. if there is no loss of heat by radiation ? The specific heat of iron is 0·113, and one calorie is equal to 4·2 joules. **5**

Or,

(*a*) Explain the principle on which an induction coil works. Give a diagrammatic sketch of such a coil, with explanation.

(*b*) Describe any three good cells. Explain the action, and advantages and disadvantages of each. **5**

UNITED PROVINCES (UNIVERSITY OF ALLAHABAD).

FIRST PAPER.

Only EIGHT *questions are to be attempted.*

The questions are of equal value.

1. Explain how the thermal expansion of air may be utilised for measuring temperature.

2. Describe any method of determining the mechanical equivalent of heat.

3. The following meteorological observations were recorded at Lucknow :—

Maximum temperature (in shade) of past 24 hours, 95·5.
Minimum temperature (in shade) of past 24 hours, 78·2.
Humidity, 95.

Explain what you understand by the numbers given above, and describe a method for measuring each of them. What inference do you draw from the above figures regarding the season of the year during which those observations were taken?

4. Account for the difference between the specific heat of a gas at constant volume and that at constant pressure.

Describe a method for measuring one of them.

5. What is meant by the 'apparent coefficient of expansion of a liquid'? How will you proceed to determine the apparent coefficient with the help of the weight thermometer?

6. What do you understand by 'parallax'? Describe how you will use the method of parallax to determine the focal length of a convex lens, being given a pin, a plane mirror and a scale.

7. You are given a block of glass, a piece of paper with a pencil mark, some lycopodium powder, and a microscope capable of vertical motion along a scale. Explain clearly how you would find out the refractive index of glass.

8. Describe a method for determining the velocity of light.

9. Describe the Bunsen grease spot photometer, and explain its action.

It was found in an experiment that a lamp with a dirty chimney when placed at a distance of 10 cm. from the grease spot balanced a candle. On cleaning the chimney it was found that the lamp had to be moved 2 cm. nearer for balancing the same candle, which was not moved during the experiment.

Calculate the percentage of light which was being absorbed by the dirty chimney.

10. Describe the spectrometer, and explain clearly the function of the collimator. How will you use this instrument to determine the angle of a prism?

11. Describe, with diagram, any form of astronomical telescope.

Second Paper.

Not more than Eight *questions are to be attempted.*

All questions are of equal value.

1. What effect is produced on the frequency and quality of the note given by an open organ pipe if the top is suddenly closed? If the frequencies of the first overtones of the two notes so obtained differ by 440, what was the original frequency?

2. What do you understand by pitch and intensity of sound? How would you determine the pitch of a tuning fork with a sonometer?

3. How would you determine experimentally the velocity of sound in (*a*) air, (*b*) a solid?

4. What is magnetic induction ? A magnet is placed horizontally in the magnetic meridian due south of a compass needle. How will its action on the latter be affected if (*a*) a thick plate of quite soft iron be interposed vertically between the two, and (*b*) a rod of soft iron be placed along the line joining the two ?

5. What reasons are there for stating that the earth is magnetized ? State what you know of the distribution of this magnetism.

6. Describe some form of electrical condenser. Explain what is meant by the capacity of a condenser, and point out the factors on which it depends.

7. Describe carefully the construction of an electrophorus, and indicate the source of electrical energy produced.

8. A copper wire and an iron wire are connected to an accumulator first in series and then in parallel. In the first case the iron wire gets red hot, and in the second case the copper wire.

Explain these facts, and show how to compare the resistances from the rates at which the heat is developed in each case.

9. What is an ammeter ? How is it used ? How does it differ from a voltmeter ?

10. Describe the construction and explain the action of an induction coil.

11. Describe the construction and action of either a telephone or of an electric bell.

MADRAS UNIVERSITY.

SECTION I.

1. Give a concise account of what you understand by (*a*) a unit, (*b*) a standard, of length, of mass and of time.

2. State Newton's Second Law of Motion, and deduce therefrom the relation between the force (f) applied to a mass (m) of matter and the acceleration (a) which results.

3. Prove that the pressure (p) at a depth (h) below the surface of a fluid of density (d) is given by the expression $p=hgd$, where g is the acceleration due to gravity.

Find the value in dynes per sq. cm. of one atmosphere pressure, taking the height of the barometer to be 30 inches of mercury.

4. Explain :

(*a*) Why it is of advantage to paint the roof of a house white in the hot weather ;

(*b*) the principle of the thermos-flask.

5. Find in gallons the amount of boiling water (at 100° C.) which must be added to 2 gallons of water at 30° C. (contained

in a cylindrical brass vessel also initially at 30° C.) in order to produce a mixture of temperature 50° C.

(Neglect loss of heat due to radiation.)

One gallon of water weighs 4536 grammes.

Specific heat of brass, 0·094.

Dimensions of the brass vessel:

Height, 25 cm.; Diameter, 20 cm.
Thickness of wall, 1 millimetre.
Density of brass, 8 gm. per c.c.

SECTION II.

1. Give a geometrical construction for finding the positions of object and image in the case of a convex mirror.

A pin is placed at a distance of 20 cm. from a convex mirror of radius 20 cm. Find the position of the image and its magnification.

2. A small white object is placed at the bottom of a beaker. The beaker is now filled with water up to a height of 20 cm. from the bottom. The refractive index of water is 1·33. Find graphically the apparent position of the object.

3. Define 'unit magnetic pole.' How would you compare the strength of different bar magnets with the help of an ordinary compass?

4. Explain 'electrification by induction.' How would you demonstrate experimentally that the induced charge is equal and opposite to the inducing charge?

5. Explain the metre bridge method of comparing resistances. With a resistance of 10 ohms in one arm of a metre bridge, balance is obtained at 20 cm. in a bridge wire of length 100 cm. The 10-ohm resistance is now replaced by an unknown resistance and the new balance point is obtained at the middle of the bridge wire. Calculate the value of the unknown resistance.

6. Explain the use of a resonance tube for finding the velocity of sound in air.

ANSWERS TO NUMERICAL EXERCISES

Chapter I. (p. 16.)

1. 1093 yd. 1 ft. 10 in. **2.** 4808·2 m. **3.** 24 ; 0·6 m.
4. 29·92 in. **5.** $\frac{1}{25 \cdot 4}$; $\frac{1}{3 \cdot 047}$, $\frac{1}{2 \cdot 54}$.

Chapter II. (p. 24.)

1. 9000 sq. cm. ; 150 sq. in. ; 8·976 sq. m. **2.** 6 metres ; 15 ft.
3. 96 sq. cm. ; 24 sq. cm. **4.** 78°·7, 57°, 44°·4 ; 14·7 sq. cm.
5. 450 sq. ft. **6.** 24854·8 miles. **7.** $47\frac{3}{4}$. **8.** 79.
9. 706·9 lb. **10.** 346·9 sq. yd. **11.** 744 sq. ft.
12. 1·82 sq. ft. **13.** 1018 sq. in. **14.** 21913 sq. ft.

Chapter III. (p. 32.)

1. 28·316. **2.** 11·0 ; 219·9. **3.** 64969·0.
4. 13 c. ft. ; $81\frac{1}{4}$; 812 lb. **5.** 2262·0 c.c. **6.** 9352·0 c. ft.
7. 500 sq. ft. ; 645·3 c. ft. **8.** 117290 c. ft. **9.** 532 c. in.
10. 648,000. **11.** 59·55 c.c. **12.** 1 : 2 : 3.
16. $\frac{2}{3}$.

Chapter IV. (p. 48c.)

1. 708 lb. per c. ft. **2.** 112·3 cm. **3.** 0·7055 gm. per c.c.
4. 4·5 cm. **5.** 27 : 10. **6.** 68·48 gm. **7.** 2 mm.
13. 486·9 lb. per c. ft. **14.** 77·2 gm.

Chapter VI. (p. 74.)

1. 100 gm. per sq. cm. ; 1360 gm. per sq. cm. **2.** $4 \cdot 273 \times 10^4$ gm.
3. 4100 gm. per sq. cm. ; 3203 lb. per sq. ft. **4.** 138·2 ft.

5. 49·1 ft. **8.** 11·54 ; 0·88. **9.** 1·06. **10.** 0·5 ; 93 lb.
13. 82 c. ft. ; 104·3 c. ft. **18.** 36·42 c.c. ; 7·55 gm. per c.c.
21. 16 gm. **22.** 0·91 ; 1·045. **24.** 10·5 c.c.
25. 3 lb. ; 2·67 ; 83 c. in. ; 4·25 lb. **26.** 6437·5 c. yd.
27. 7·59. **28.** 0·794.

Chapter VII. (p. 90.)

1. 34·64 in. **2.** 34 ft. ; 13·6 in. **3.** 1·053 kg. per sq. cm.
4. 14·01 lb. per sq. in. **5.** 306 c.c. **6.** 1·327 gm.
7. 33·124 gm. ; 18·004 gm. **8.** 76 cm. ; 561·7 cm.
9. 6·5 gm. per sq. cm.; 0·0925 lb. per sq. in. **18.** 3·675 lb.; 41,270 tons.

Chapter VIII. (p. 108.)

4. $10\sqrt{2}$ lb. ; direction, S.W. **7.** 156 lb. **8.** 10 lb.
12. 90°, 143°·1, 126°·9. **13.** 72°, 158°, 130°. **14.** $\sqrt{3}$ lb.
15. $10\sqrt{3}$ gm. **17.** 10 ft. per sec. **18.** 3870 lb. **19.** 10 sec.; 150 ft.

Chapter IX. (p. 119.)

2. 5600 ft. lb. **3.** 240,000 ft. lb. **4.** $1400\sqrt{3}$ ft. lb.
5. $9\frac{1}{11}$ h.p. **6.** 400 h.p. **11.** 12,000 ft. lb. ; $\frac{5}{12}$.
12. 200 ft. lb. **13.** 16 hours. **14.** 118,800 ft. lb.
15. 24×10^3 ft. lb. **16.** 250 ft. poundals. **17.** 300 ft. lb.

Chapter X. (p. 130.)

1. 4 lb. **4.** 33·3 cm. from fulcrum. **5.** 30 lb.
6. 2 cwt. ; $1\frac{1}{4}$ cwt.

Chapter XI. (p. 140.)

8. 4·5. **9.** 90·6%. **10.** 45° ; 0·084 kgm. metres.
11. 12,000 ft. lb. **12.** $\frac{2}{25}$.

Chapter XII. (p. 152.)

9. 21°·7 C. ; −17°·8 C. ; −40° C. **10.** 181°·4 F. ; 59° F. ; 23° F.
11. (*a*) −435°·1 F., −421°·6 F. ; (*b*) −38°·56 F., 674°·6 F.; (*c*) 207°·7 F. 1517° F.
14. +0°·7 C. ; −0°·7 C.

Chapter XIII. (p. 171.)

3. 2·0066 m.; 153°·6 C. **4.** 1·112 ft. **5.** 300·926 sq. cm.
6. 20·8 sec. **7.** 13·29 gm. per c.c. **8.** 0·000178.
9. 99·75 gm. approximately. **10.** 0·00015. **11.** 7·5 cm.
12. 0·000734. **13.** It will rise through 6·3 mm. **14.** 110·24 c.c.
15. 1 litre. **16.** 340·10 c.c. **17.** 0·00366. **19.** 496·8 kgm.

Chapter XIV. (p. 181.)

4. 77916 calories. **5.** 0·033. **6.** 0·055.
7. $14\frac{4}{9}$ gallons of cold water, $5\frac{5}{9}$ gallons of hot water. **8.** 704°·9 C.
10. 96°·3 C. **14.** 20° C. **15.** 0·089.

Chapter XV. (p. 202.)

2. 14·4 mm. **9.** 54·7%. **10.** 8·62 kgm.

Chapter XVI. (p. 213.)

1. 12·5 lb. **4.** 63°·6 C. **5.** Final temperature = 10° C.
6. 41·2 gm.; 5·87 gm. **8.** 522·5; 534·8. **12.** 2 h. 41 m.

Chapter XVII. (p. 230.)

7. 126 kgm.

Chapter XVIII. (p. 245.)

5. 7·8 ft. **6.** $(91{\cdot}7 \times 10^4)$ miles. **8.** 78·5 sq. ft.
9. 7·8 c.p. **10.** 16 : 9. **11.** 2·5 : 1.
12. 42 cm. from the candle; 70 cm. on the distant side of the candle.
13. The 100 c.p. lamp. **14.** 2·56%. **15.** 75%.

Chapter XIX. (p. 256.)

4. 60°.

Chapter XXI. (p. 287.)

3. 2 ft. from the mirror; relative size $=\frac{1}{3}$.
4. 15 cm. behind the mirror; 1·87 cm. long. **6.** Diameter = 0·03 ft.
17. 46·5 cm.; 28 cm.; 26 cm.

Chapter XXII. (p. 301.)

4. – 15 inches. **5.** – 13·3 inches. **8.** – 7·5 cm.
9. 7·5 cm. in front of lens ; 0·5 cm. long.
10. 22·5 cm. from lens , 52 in. × 38 in. **11.** 1 ft. or 2 ft. from candle.
13. + 16 cm. **14.** + 33·3 cm. **15.** 4·5 in. from lens.
17. 10 cm. ; 11·1 cm. ; + 10 dioptres. **18.** – 16 cm.
19. 11 ft. ; 11 in. **23.** Convex : 6·15 in. ; 10·56 ft. square.

Chapter XXIII. (p. 313.)

1. – 2·4 in. **2.** Concave ; + 5·14 in. **3.** 28·8 cm.
4. 120 cm. **8.** – 13·8 in. **9.** $f = +6$ in. ; 6 inches. **11.** 4 times.

Chapter XXV. (p. 335.)

2. 15 ft. ; 2·5 ft. per sec. **5.** 4 times. **8.** 1104 ft. per sec.

Chapter XXVI. (p. 341.)

3. The bullet ; 0·31 sec. **7.** (ii) 4 : 9.

Chapter XXVII. (p. 351.)

1. 292·6. **2.** 4 lb. **3.** $\frac{1}{4}$ of its length from the end. **4.** 259.
6. 46·8. **9.** 5. **10.** 600. **16.** 269·5 and 243·8.

Chapter XXVIII. (p. 360.)

1. 360, approximately. **2.** 558. **3.** 307·8. **5.** 560 ; 6 inches.

Chapter XXXIII. (p. 417.)

1. 15·59.
4. Either (*a*) the vertical intensity of the earth's field, or (*b*) the horizontal intensity and the angle of dip.
6. 0·00196 gauss. **7.** (i) 38·25 ; (ii) 13·96 cm.
8. 0·0593 gauss. **9.** 1 : 2·048. **11.** 0·444 gauss.
12. 0·49 : 1. **13.** 0·36 gauss , 14·14. **16.** 3·60 ; 2·77 ; 1.
18. (*a*) 9 : 16 ; (*b*) 41 : 16. **19.** 1·23 : 1.
22. 0·6 ; dyne per unit pole ; 2·12 sec.
23. Paris/Bombay = 1·367/1·00.

Chapter XXXIV. (p. 437.)

14. 8 : 1.

Chapter XXXVIII. (p 523.)

3. 1·2 amp. **4.** 122 m. 22 sec. **5.** 0·0095 mm.
6. 31·51 hours. **13.** Equal quantities of heat developed.
15. 35 ohms. **16.** 28°·67 C. **17.** 0·9165 amp.
18. 0·538 amp.; 0·081 ohm. **19.** 0°·10 C. **20.** 79·5 per cent.
21. 14·87 min. **22.** 1·22/1; 1·53/1. **23.** 44·5.
25. 0·182 amp.; 6·4 pence.
26. (*a*) 201·7 ohms, (*b*) 960 ohms; 14·4 pence.
27. 7·7 min. **28.** 0·0003298.

Chapter XXXIX. (p. 537.)

2. 0·0744 amp. **3.** 0·089 amp. **4.** 3 : 1.
5. 0·1005 gauss. **6.** 0·034 absolute units; 0·34 amp.
7. 0·94. **9.** 4·775 amp. **10.** 30°.
11. 234. **12.** 0·121; 9·56 ohms.

Chapter XL. (p. 558.)

1. 0·615 volt. **2.** 0·75 ohm; 0·628 volt.
3. 2·05 volts; 0·244 calorie. **4.** 1·75 ohm; 22 ohms.
5. 5. **6.** 10 ohms. **7.** Each is reduced by 75%.
8. $1{\cdot}622 \times 10^{-6}$ ohm. **9.** $94{\cdot}07 \times 10^{-6}$ ohm.
10. 472 cm. **11.** 40·23 ohms. **13.** 9 ohms and 6 ohms.
14. 820 ohms. **15.** 0·52 amp. **16.** 10 : 1.
17. $\frac{1}{3}$ amp. and $\frac{1}{4}$ amp.; 4 : 3; 2·58 volts.
18. Reduced to $\frac{1}{11}$. **19.** Use a shunt, of resistance 0·167 ohm.
20. 0·408 amp. and 0·511 amp.; 0·278 amp. and 0·348 amp.
21. 1 : 4.
22. $i_{100} = 0{\cdot}20$ amp., $i_{200} = 0{\cdot}10$ amp., $i_{300} = 0{\cdot}30$ amp.; $V_A - V_B = 20{\cdot}0$ volts, $V_B - V_C = 90{\cdot}0$ volts.
23. A shunt, of resistance $\frac{17}{199}$ ohm; 183 ohms in series.
24. 5 ohms, 1·5 volts. **25.** 110 ohms.
29. Two groups, each of two cells in series.

ANSWERS TO TYPICAL EXAMINATION QUESTIONS AND PAPERS

(WITH SUPPLEMENTARY NOTES).

SCHOOL LEAVING CERTIFICATE QUESTIONS. (p. 622.)

I. Mechanics and Hydrostatics.

1. 156·25 cm. **2.** (i) 980·5, (ii) 99·34 cm. **4.** 36 gm.
6. 6| lb. (The specific gravity of the fish does not affect the answer.)
7. 56·52. **8.** 0·92. **9.** The solid will just float.
10. (*a*) 46,320 c.c., (*b*) 46·32 kg.
11. $P(\frac{3}{4})^3$, where P is atmospheric pressure.
12. 126 cm. of mercury. **13.** 75 cm. of mercury. **14.** 15 c.c.
15. 7·1 lb., due South. **16.** 30 gm.; 20 cm. from fulcrum.
17. 1154·7 gm.; 577·4 gm.
18. Angle between 4 and 5, 98°; between 5 and 6, 139°; between 6 and 4, 123°.
19. 467·5.

II. Heat.

2. 0·000163. **5.** 100 gm. **6.** 0·093.
7. $-5°·55$ C. **8.** $-35°·55$ C.; $-28°·44$ R.
9. 330 gm. **11.** (ii) 4500 calories. **14.** 16·57 gm.

III. Light.

5. 60 cm. from mirror, and on same side as object. **8.** -16 cm.

IV. Magnetism and Electricity.

3. Attraction, of 61·11 dynes. **4.** 101·33 units. **11.** 0·708 volt.
12. 1·28 volt. **13.** 0·009 volt. **14.** 5°·71 C.

INTERMEDIATE QUESTIONS. (p. 630.)

I. Mechanics and Hydrostatics.

1. 8·97 mm. **2.** 18·23 gm. per c.c. **3.** 51·875 gm. per cm.^{2}
4. 103 3. tons. **6.** 0 247 **7.** 1/9·32. **8.** 8·53; 0·217.

10. Let the initial velocity of a body be denoted by u, and let the acceleration be denoted by f. Then, at the end of the first second, the velocity will be $(u+f)$; at the end of the next second, it will be $(u+2f)$; and, at the end of t seconds, it will be $(u+tf)$. This last expression may be termed the final velocity v; and the equation is written

$$v = u + ft. \quad \text{.......(i)}$$

The space (s) traversed by a body, moving with uniform acceleration, is equal to the product (*average velocity × number of seconds occupied*); or

$$s = \frac{u + (u + ft)}{2} \times t$$

$$= ut + \tfrac{1}{2}ft^2. \quad \text{.......(ii)}$$

It is often convenient to have an equation in which a period of time (t) does not appear. Such an equation may be derived by eliminating t from the above equations. Thus, from equation (i),

$$t = (v - u)/f;$$

substitute this value of t in equation (ii), then

$$s = \frac{u(v-u)}{f} + \frac{f}{2} \cdot \frac{(v-u)^2}{f^2}$$

$$= \frac{2uv - 2u^2 + (v^2 - 2uv + u^2)}{2f}$$

$$= \frac{v^2 - u^2}{2f};$$

or

$$v^2 = u^2 + 2fs. \quad \text{.......(iii)}$$

Problem.—(i) 400 ft.; (ii) 5·70 sec., (iii) 143·1 ft./sec.

11. 37·64 kg. and 0·588 kg. per cm.^{2}, 17·64 kg. and 0·744 kg. per cm.^{2}

12. Young's Modulus of Elasticity is an application of *Hooke's Law*, which states that, in any case of the pulling or pushing of a rod, *the strain produced is proportional to the stress applied*. Hence, assuming that the stress applied is not so great as to deform the solid beyond the limits of *perfect elasticity*, the ratio **stress/strain** is a constant quantity; and this ratio is termed the **modulus of elasticity.** When the modulus refers to a deformation due to a steady pull (or push), and not to a shearing stress, it is known as *Young's Modulus of Elasticity.*

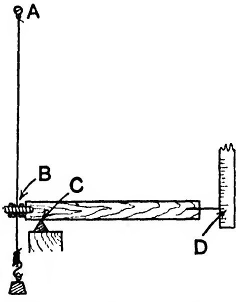


FIG. 421.—Determination of Young's Modulus.

Young's Modulus of a given metal, *e.g.* steel, is determined by suspending a measured length of a wire of the metal from a fixed point, and observing the amount of extension due to a known applied force. The **stress** is expressed as the applied force per unit cross-section, or $F/\pi r^2$, where r is the radius of the wire; and the **strain** is expressed as the elongation per unit length, or l/L, where l is the observed extension, and L is the total initial length. Hence

$$\text{Young's Modulus} = \frac{F}{\pi r^2} \Big/ \frac{l}{L} = FL/\pi r^2 \times l.$$

The diagram (Fig. 421) represents a simple arrangement for the experiment · AB is the length L of wire under test, the extension of which is measured by means of the wooden lath pivoted at C. A small load is attached to the wire sufficient to ensure that it is quite straight; and the reading of the needle-point D, on the vertical scale, is noted. An additional known weight F is then added to the load, and the reading of D is again noted. The movement of D is CD/CB times as great as the elongation l. By this means, l is determined with considerable accuracy. The radius of the wire is measured by means of a micrometer screw-gauge.

13. 69·42. **14.** 30,250 foot-tons; 5250 foot-tons.

15. (*a*) 3·75 lb. and 3·25 lb., (*b*) 2·08 lb. and 4·92 lb.

II. Heat.

1. Let V_0 and V_t be the volume of the bulb at 0° C. and t° C. respectively, then

$$V_t = V_0(1 + at).$$

Let D_0 and D_t be the density of the liquid at 0° C. and t° C. respectively, then (see p. 159)

$$D_0 = D_t(1 + \beta t)$$

or

$$D_t = D_0/(1 + \beta t) = D_0(1 - \beta t).$$

If w_0 and w are the weight of liquid displaced at 0° C. and t° C. respectively, then, since *weight = volume × density*,

$$w_0 = V_0 D_0$$

and

$$w = V_t D_t = V_0(1 + at) \times D_0(1 - \beta t).$$

Hence

$$\begin{aligned} w/w_0 &= V_0(1 + at) \times D_0(1 - \beta t)/V_0 D_0 \\ &= (1 + at)(1 - \beta t) \\ &= 1 + (a - \beta)t. \end{aligned}$$

2. (i) Errors in the readings obtained with mercury-in-glass thermometers may arise from the following causes:

(*a*) The bore of the tube may not be absolutely uniform, and a scale which is uniformly graduated will not give a correct reading at all temperatures.

(*b*) When the bulb only is exposed to the temperature which is to be measured, the observed reading will be too low. The whole of the mercury, *including the thread*, must be exposed to the temperature.

(*c*) The volume of the glass bulb is affected by changes in external pressure. Thus, when the bulb is placed in a partial vacuum, it expands slightly, and the reading will be too low.

Similarly, when the thread is long, hydrostatic pressure within the mercury itself will cause the reading to depend upon whether the stem is in a vertical position or in a horizontal position.

(*d*) After being raised to a high temperature, and then quickly cooled, the glass does not return at once to its original volume: it nearly does so, but not quite. The original volume is resumed only after a considerable rest. For this reason, when verifying the accuracy of the 'fixed points,' the lower fixed-point usually is taken first.

(ii) Let V_0 and V_t be the volume of the bulb, at 0° C. and t° C. respectively; then

$$V_t = V_0\{1 + (3 \times 0{\cdot}083 \times 10^{-4})100\} = V_0(1 + 0{\cdot}00249).$$

Let D_0 and D_t be the density of mercury at 0° C. and t° C. respectively; then

$$D_t = D_0\{1 - (0{\cdot}18 \times 10^{-3})100\} = D_0(1 - 0{\cdot}018).$$

Since the weight of mercury in the bulb at 0° C. is 950 gm.,

$$950 = V_0 D_0;$$

and the weight of mercury in the bulb at 100° C. is

$$\begin{aligned} V_t D_t &= V_0(1 + 0{\cdot}00249) \times D_0(1 - 0{\cdot}018) \\ &= V_0 D_0(1 - 0{\cdot}0155) \\ &= 950(1 - 0{\cdot}0155) \\ &= 935{\cdot}275 \text{ gm.} \end{aligned}$$

Hence, weight of mercury expelled $= 950 - 935{\cdot}275$
$= 14{\cdot}725$ gm.

3. A convenient form of **weight thermometer** (W, Fig. 422) can be made from thin-walled glass tubing, about 1·5 cm. diameter. After weighing the empty vessel, it is supported vertically with the open end dipping into the liquid, the expansion of which is to be measured. A loosely fitting jacket G, of copper gauze, which is carried by wires from the ring of a retort-stand, is a suitable method of supporting the vessel. The vessel is warmed for a few moments by means of a Bunsen flame, and some of the contained air is expelled. On removing the flame, the remaining air contracts, and some of the liquid is drawn into the vessel. This process of alternately heating and cooling the vessel is continued until it is completely filled with the liquid. While the open end is still submerged, the main body of the vessel is immersed in a large beaker of water, the temperature of which is kept constant, for at least 5 minutes, at the *lower* of the two temperatures between which the coefficient of expansion is to be determined. The vessel is removed from its jacket, the outside carefully dried, and then weighed. The vessel now is replaced in its jacket, the water bath is warmed to the *higher* temperature, and kept constant for at least 5 minutes: it is then removed, dried and again weighed.

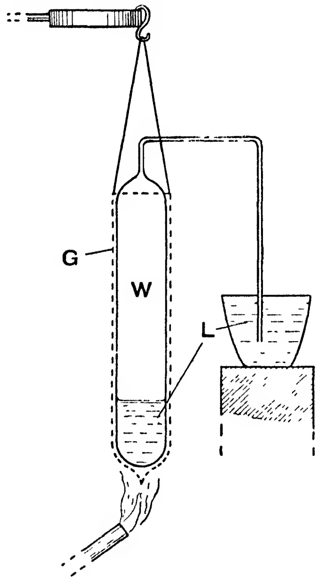


FIG. 422.—A weight thermometer

Let w_1 = weight of liquid in the vessel at t_1° C.,
w_2 = ,, ,, ,, ,, t_2° C.,
β = absolute coefficient of expansion of the liquid,
α = coefficient of cubical expansion of the material of the vessel

Also, let V_{t_1} and V_{t_2} be the volume of the vessel at t_1° C. and t_2° C. respectively, and let D_{t_1} and D_{t_2} be the density of the liquid at t_1° C. and t_2° C. respectively. Then

$$V_{t_2} = V_{t_1}\{1 + \alpha(t_2 - t_1)\}$$

or

$$V_{t_1} = V_{t_2}\{1 - \alpha(t_2 - t_1)\};$$

also

$$D_{t_1} = D_{t_2}\{1 + \beta(t_2 - t_1)\}.$$

Since weight = volume × density,

$$w_1 = V_{t_1} \times D_{t_1} = V_{t_2}\{1 - \alpha(t_2 - t_1)\} \times D_{t_2}\{1 + \beta(t_2 - t_1)\}$$

$$= V_{t_2}D_{t_2}\{1 + (\beta - \alpha)(t_2 - t_1)\}, \quad \text{.........................(i)}$$

and

$$w_2 = V_{t_2}D_{t_2}. \quad \text{...(ii)}$$

Hence, from (i) and (ii),

$$w_1/w_2 = 1 + (\beta - \alpha)(t_2 - t_1)$$

or

$$(\beta - \alpha) = \left(\frac{w_1}{w_2} - 1\right)\Big/(t_2 - t_1)$$

or

$$\beta = \frac{w_1 - w_2}{w_2(t_2 - t_1)} + \alpha.$$

The first part of the expression on the right-hand side of this equation is the '*apparent* coefficient of expansion' of the liquid; and it is evident that the '*absolute* coefficient' is obtained by adding to the 'apparent' coefficient the coefficient of cubical expansion of the material of the containing vessel.

Problem: Absolute coefficient = 0·000303.

4. 0·932.

5. The fact that 1090 c.c. of ice at 0° C. contract, when melted, to form 1000 c.c. of water at 0° C. is the basis of *Bunsen's ice calorimeter.*

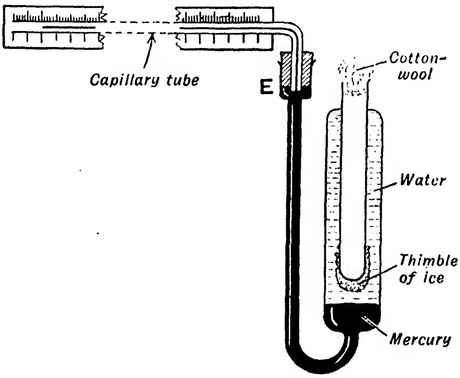


FIG. 423.—Bunsen's ice calorimeter.

A contraction of 90 c.c., due to the imparting of heat, indicates therefore that (1000 × 80) = 80,000 calories have been given up. The calorimeter (Fig. 423) consists of a glass test-tube to which is fused a wider glass jacket, joined below to a narrow glass U-tube which terminates above

in an expansion, to which is connected, by means of a rubber stopper, a long length of capillary tubing, the diameter of which is known accurately. A long millimetre-scale is attached to this tube. The upper part of the jacket contains boiled distilled water; and the remainder of the jacket and U-tube is occupied by pure mercury. The whole apparatus, except the capillary tube, is maintained at 0° C. by surrounding it with clean snow, or broken pure ice. A thimble of ice is formed round the base of the test-tube by pouring in a small quantity of ether, and blowing through this a steady stream of air. The capillary tube is inserted in the rubber stopper, and adjusted so that the mercury thread extends nearly to the distant end. A small quantity of ice-cold water is poured into the test-tube, and its mouth is closed with a plug of cotton wool. The position of the mercury surface on the scale is noted. A weighed specimen of the substance, the specific heat of which is to be determined, is heated to an observed high temperature, and quickly dropped into the test-tube. The heat liberated melts more or less of the ice thimble, and the mercury surface recedes along the tube: the extent of this movement is accurately noted. From this contraction is calculated the number of calories given up by the hot substance.

Problem : specific heat = 0·100.

7. 69°·56 C. **8.** 9 kg. **9.** 754 gm.

10. The 'thermal conductivity' of a poor conductor may be determined by means of the apparatus shown in Fig. 424. This consists of a flat cylindrical box A through which steam can be passed; and the bottom of the box is a brass plate B, more than 1 cm. thick. A radial hole is bored in B, for the insertion of a thermometer. The vessel rests on a circular slab C of the material under test; and this rests on a brass disc D, of the same dimensions as the bottom of the steam box. The disc D also has a radial hole, for the insertion of a thermometer.

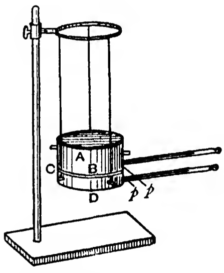


FIG. 424.—Determination of thermal conductivity of a solid.

In order to measure the mean thickness of the disc C, three pins project from equidistant points near to the bottom of the steam box, and under each of these a corresponding pin projects from the disc D By measuring the distance between each pair of pins, before and after inserting C, the mean thickness of the disc is determined. One pair of these pins is indicated at p and p

The whole apparatus is suspended above the bench by means of three strings. When steam is passed through the box, heat will pass through the disc C, and the temperature of D will gradually rise. In due course, a 'steady' stage is reached when the rate of loss of heat from the surface of D is equal to the rate at which heat is supplied through the disc C. The quantity of heat (Q) flowing through the disc, *in each second*, is equal to

$$kA \times \frac{t_2 - t_1}{d},$$

where A and d are the area and thickness of the disc; t_2 and t_1 are the thermometer readings, and k is the conductivity of the material.

The quantity of heat Q may be determined by removing the steam box, and warming the disc D, by means of a Bunsen flame, until its temperature is about 10° above that recorded during the steady state (viz. t_1), and then observing its *rate of cooling* until its temperature has fallen to about 10° below t_1. The temperature is noted at half-minute intervals, and these are plotted on squared paper. By drawing a tangent to the graph, the rate of cooling at the temperature t_1° is obtained. The *quantity* of heat lost per second is equal to the product

$$w \times s \times \text{' rate of cooling,'}$$

where w is the weight of the disc and s is the specific heat of brass. By equating the above expressions the value of k is calculated.

Problem. 473,000 calories.

12. $4{\cdot}09 \times 10^7$ ergs. **13.** $3{\cdot}022 \times 10^{11}$ ergs. **14.** 74°·4 C.

III. Light.

1. 210,000 miles ; 10 miles. **3.** 6·58 cm. from upper surface.

4. 6 cm. **5.** 1/3 inch within the glass. **7.** 1·631. **8.** 0·6 cm.

9. 30 cm. in front of mirror ; 0·6 cm. **13.** 106·8 cm. , the object.

14. (i) Let A (Fig. 425) be one position of the lens which gives a well-defined image; then, another position B for the lens can be found

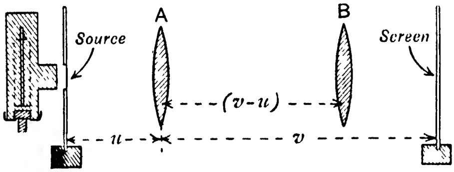


FIG 425.—A method of determining the focal length of a converging lens.

which also gives a well-defined image, its distance from the screen being equal to the distance of A from the source. it is an example of *conjugate foci*.

Let d be the measured distance between source and screen, and
b ,, ,, ,, ,, A and B ;

then $\quad d = v + u \quad$ and $\quad b = v - u.$

Hence $\quad d^2 - b^2 = 4uv \quad$ or $\quad uv = (d^2 - b^2)/4.$

But $\quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{u-v}{uv} \quad$ or $\quad f = \frac{uv}{-(v-u)}.$

Hence $\quad f = (d^2 - b^2)/-4b.$

(This is an excellent method of finding the focal length of a converging lens.)

(ii) Let L be the size of the luminous source; then, in position A,

$L/a = u/v$; (See Fig. 205, p. 296.)

and, in position B, $\quad L/b = v/u.$

Or $\left.\begin{array}{l} L = a \times u/v, \\ L = b \times v/u. \end{array}\right\}$ Hence $L^2 = ab$, or $L = \sqrt{ab}.$

IV. Sound.

1. (i) At a distance from one of the cliffs equal to one-sixth of the total distance. (ii) Seven seconds after firing the gun.

2. Place the bridges at the readings 54·55 cm. and 81·82 cm.

3. Reduce tension to 0·64 kg.

4. (i) 8481 gm. (ii) Halve the length; increase the tension four-fold.

5. 1/9. **5.** 432. **6.** 23·2. **7.** 46·79. **10.** 0·75 cm.

V. Magnetism and Electricity.

3. 0·125. **4.** Moment of couple = MH sin θ. **5.** 0·25.

6. 489·7. **10.** Attraction = 1·33 dyne.

13. The *dielectric constant* (or the *specific inductive capacity*) of a material is *the ratio of the capacity of a condenser with the material as dielectric to its capacity when the dielectric is air.*

16. 2·31 ohms. **17.** 0·316 mm. **18.** 60.

19. Five batteries in parallel, each containing six cells.

20. 0·563 ohm. **21.** Under-lighted, to the extent of 7·1%.

22. 0·25 volt. **25.** 150×10^3 joules, or 150×10^{10} ergs.

26. 14·3 calories.

TYPICAL INTERMEDIATE EXAMINATION PAPERS.

CALCUTTA UNIVERSITY. (p. 641.)

First Paper.

2. 20. **3.** 2 cm.

5. The boiling-point of water at each station is observed by means of a *hypsometer*, the principle of which is shown in Fig. 106 (p. 149). From these observations the atmospheric pressure at each station is found by reference to a Physical Table such as that given on p. 614. At the same time, the temperature of the external air at each station is observed by means of a second thermometer. *The difference in pressure is the weight of a vertical column of air one square centimetre in area between the two stations.* This weight, divided by the density of air at the mean temperature and under the mean pressure, will give the length of the air column—or, the difference in height of the two stations.

In practice, it is more usual to use Babinet's formula:

$$\text{Height (in metres)} = \frac{H_1 - H_2}{H_1 + H_2} \times 32(500 + t_1 + t_2),$$

where H_1 and H_2 are the barometer readings at the lower and higher stations respectively, and $t_1°$ C. and $t_2°$ C. are the air temperatures.

6. 3 gm.

7. When one or two drops of water are introduced into a large air-tight vessel which is initially entirely exhausted, *all the water evaporates*, and the vessel is filled with water-vapour. This vapour, under such conditions, has the properties of a gas, and obeys Boyle's Law: *i.e.* when the volume of the space occupied is diminished (within certain limits)

or increased, the pressure exerted by the vapour will increase or diminish in an inverse ratio. The water-vapour now is in an **unsaturated** condition.

When, however, more water is introduced, until the vessel contains more than will evaporate, the space above the water is occupied by the **saturated** vapour of water. This vapour exerts on the walls of the vessel a definite pressure which is independent of the volume of the vessel, and depends only upon the temperature. When, by some device, this volume is diminished, a portion of the vapour condenses to the liquid form, and in such quantity that the remaining vapour exerts the same pressure as before. On the other hand, if the volume of the vessel is increased, more water will evaporate, until the vapour-pressure again becomes the same as before. (See Expt. 137, p. 191.)

9. $F_1/F_2 = 1/9$.

SECOND PAPER.

1. 0·5 approximately. **3.** 4 inches.

BOMBAY UNIVERSITY. (p. 643.)

1. (i) **Circular Motion.**—Suppose a particle P (Fig. 426) to move at a uniform speed v along a circular path of radius r. Let PR be a small arc described in a time t. In the absence of any constraining force, the particle would traverse the path PQ; and QR represents the distance and direction through which P is pulled by the constraining force.

If a is the acceleration due to the force, then $QR = 1/2 \cdot at^2$ and $PQ = vt$.

But (by Euclid, III. 36), $QR \times QS = (QP)^2$

or $QR(2r + QR) = (QP)^2$

or $QR \times 2r = (QP)^2$ (since QR is small, $(QR)^2$ may be neglected)

or $1/2 \cdot at^2 \times 2r = v^2t^2$

or $a = v^2/r = \omega^2r^2/r = \omega^2 r$

(where ω is the 'angular velocity,' and equal to v/r).

The **period** (T) of the particle is the time occupied in describing one complete revolution. Hence, $T = 2\pi r/v = 2\pi/\omega$.

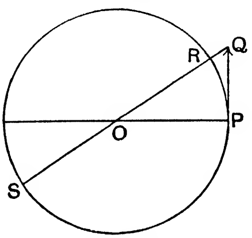


FIG. 426.

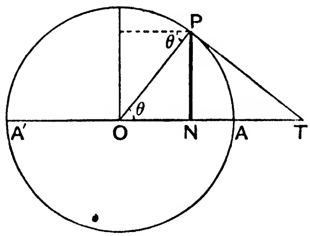


FIG. 427.

(ii) **Simple Harmonic Motion.** In Fig. 427 a particle P is moving with uniform speed round a circle. As P travels, the point N (the foot

of the perpendicular drawn from P to the diameter AOA′) travels to and fro along the diameter, and it is said to travel with *Simple Harmonic Motion*. The acceleration of P, at any instant, along OP may be resolved into horizontal and vertical components; and the acceleration of N, at the same instant, will be equal to the former component. Hence,

$$\text{acceleration of N} = \omega^2 r \cos\theta = \omega^2 r\,.\,\text{ON/OP} = \omega^2\text{ON}.$$

This leads to the following definition of Simple Harmonic Motion:—
When a point moves in a straight line so that its acceleration is always directed towards, and varies as its distance from, a fixed point in the straight line, the point is said to move in S.H.M.

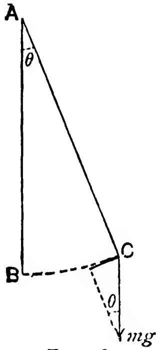


FIG. 428.

(iii) **The simple pendulum.**—Suppose that the simple pendulum AB (Fig. 428), consisting of a heavy particle of mass m suspended by a weightless thread of length l, be displaced through a small angle θ, to the position AC. The force acting on m, and towards B, is $mg\sin\theta$; and the acceleration towards B is $g\sin\theta$. But, as θ is small,

$$g\sin\theta = g\theta = g\,.\,\text{BC}/l.$$

Hence, *since the acceleration is proportional to the displacement* BC, *the pendulum oscillates* in S.H.M., and obeys the law, stated above, according to which the acceleration is equal to $\omega^2 \times$ *displacement*. Therefore

$$\omega^2\text{BC} = g\,.\,\text{BC}/l \quad \text{or} \quad \omega = \sqrt{g/l}.$$

But $\quad T = 2\pi/\omega;$

hence $\quad T = 2\pi\sqrt{l/g}.$

Problem: Acceleration at highest point $= 0{\cdot}33$ ft./sec.^{2}

2. The friction is equivalent to a force F (Fig. 429), where F/W $= 0{\cdot}2$; hence F $= 0{\cdot}2 \times 2240 = 448$ lb.

Let P represent the pull of the horse, and suppose that the cart be pulled forward through a distance equal to the circumference of the wheel; the work done by the horse will be $(P \times 3\pi)$ ft.-lb. Assuming that there is no 'rolling-friction' between the wheel and the road, the whole of this work will be spent in overcoming the friction between the axle and hub. The distance through which this resistance F is over-come is equal to the circumference of the axle; and the work done against F will be $(F \times 3\pi/12)$ ft.-lb. Hence

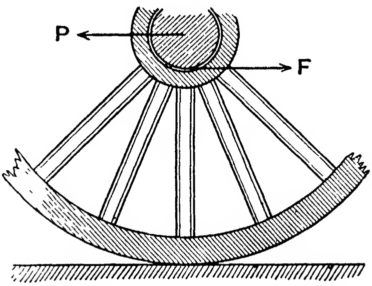


FIG. 429.—Friction in the bearing of a cart-wheel.

$$P \times 3\pi = F \times 3\pi/12$$

or $\quad P = F/12 = 448/12 = 37{\cdot}33$ lb.

(ii) In the absence of a wheel, P $= 448$ lb.

5. $7{\cdot}975$ kg. **7.** (*b*) (i) $22^\circ{\cdot}5$; (ii) $0^\circ{\cdot}358$.

8. (i) Suppose that the object viewed is situated at a distance u_1 from the object-glass, and that it forms an image at a distance v_1 on the other side of the object-glass. If the length of the object is L, the length of the image is Lv_1/u_1. If the eye-piece is at a distance u_2 beyond this image, and v_2 is the distance of the image from the eye-piece, the magnification produced by the eye-piece is v_2/u_2; and the length of the final image is

$$L\frac{v_1}{u_1} \times \frac{v_2}{u_2}.$$

The final image is practically at a distance v_2 from the eye, and the *visual angle* which it subtends at the eye is

$$L \cdot \frac{v_1}{u_1} \cdot \frac{v_2}{u_2} \Big/ v_2 = Lv_1/u_1u_2.$$

The object itself is practically at a distance u_1 from the eye, and *the visual angle* is equal to L/u_1. The magnification, which is the ratio of these two visual angles, is

$$Lv_1/u_1u_2 \times u_1/L = v_1/u_2.$$

Since the object is supposed to be at a great distance from the object glass, v_1 is practically equal to its focal length (F). And, if the final image is to be viewed by the eye without accommodation being required, it must be situated at a considerable distance from the eye, and the first image must be not appreciably distant from the principal focus of the eye-piece; hence u_2 is practically equal to f, the focal length of the eye-piece.

Hence, magnifying power $= v_1/u_2 = F/f$.

(ii) $-7{\cdot}5$ cm.; $13{\cdot}33$ dioptres.

9. Moment of magnet $= 50$; pole-strength $= 10$.

10. (i) **Potential at a point near to a charged conductor.**—Let a small conductor, charged with $+Q$ units, be situated at O (Fig. 430), and let

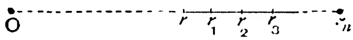


FIG. 430.—Electric potential in a field due to a point charge

r, r_1, etc., be points distant r, r_1, etc., centimetres from O, and very near together.

The force acting on one unit of positive electricity at r is Q/r^2, and the force ,, ,, ,, ,, ,, r_1 is Q/r_1^2.

The *average force*, between r and r_1,

$$= \frac{Q}{2}\left(\frac{1}{r^2} + \frac{1}{r_1^2}\right)$$

$$= \frac{Q}{2}\left\{\frac{(r+\delta)^2 + r^2}{r^2 r_1^2}\right\} \quad \text{(where } r_1 = r + \delta\text{)}$$

$$= \frac{Q}{2}\left\{\frac{2r^2 + 2r\delta + \delta^2}{r^2 r_1^2}\right\}.$$

But, since δ is small, δ^2 may be neglected, and

$$\text{average force} = Q\left\{\frac{r(r+\delta)}{r^2 r_1^2}\right\} = \frac{Q}{rr_1}.$$

The *work done* by the electric force, in moving one unit from r to r_1, is

$$\text{average force} \times \text{distance} = \frac{Q}{rr_1}(r_1 - r) = \frac{Q}{r} - \frac{Q}{r_1}.$$

Similarly, the work done, in moving one unit from r_1 to r_2,

$$= \frac{Q}{r_1 r_2}(r_2 - r_1) = \frac{Q}{r_1} - \frac{Q}{r_2};$$

and, the work done, in moving one unit from r_{n-1} to r_n,

$$= \frac{Q}{r_{n-1}} - \frac{Q}{r_n}.$$

Hence, by adding all these small portions of work, the total work done, in moving one unit from r to r_n, is $\frac{Q}{r} - \frac{Q}{r_n}$.

But the total work done is also equal to the potential-difference $(V - V_n)$ between r and r_n. Hence

$$V - V_n = \frac{Q}{r} - \frac{Q}{r_n} = Q\left(\frac{1}{r} - \frac{1}{r_n}\right).$$

When r_n is at an infinite distance, $1/r_n = 0$ and $V_n = 0$, hence $V = Q/r$.

(ii) **The electrical capacity of a sphere.**—*A charge on an insulated sphere acts at any external point as though the charge were concentrated at the centre.* If a sphere, of radius r cm., is charged with Q units, the potential v at a point distant d cm. from the sphere's centre is given by the equation $v = Q/d$. But, if $d = r$, then v will be the potential of any point on the sphere's surface. Hence $v = Q/r$.

The *capacity* (C) of any charged conductor is the ratio of the *charge* to the *potential* to which it is raised by that charge; hence

$$C = Q/v \quad \text{or} \quad v = Q/C.$$

But v is also equal to Q/r; hence $C = r$; or, in words, **the capacity of a sphere is equal to its radius, expressed in centimetres.**

In the problem, both spheres after contact have a potential of 3·33 units; and the charges are 33·3 and 16·65 units respectively.

11. 30·6 coulombs.

PUNJAB UNIVERSITY. (p. 645.)

1. (i) Retarding force = 1·072 tons weight; change in momentum = 10270 ton-feet/sec., (ii) Horse power = 43·64.

2. Specific gravity = 1. **5.** 310 gm. **6.** 1·87 cm.

9. Attraction = 2 dynes; repulsion = 0·25 dyne. **10.** 8 min. 38 sec.

UNITED PROVINCES (UNIVERSITY OF ALLAHABAD). (p. 646.)

FIRST PAPER.

4. Suppose that one gram of a given gas is enclosed within a rigid metal vessel (Fig. 431, i), and that heat is imparted to the gas sufficient to raise its temperature through 1° C. No appreciable change of volume of the gas will take place, and all the heat will be stored up in the gas as additional kinetic energy of the molecules. The quantity of heat required measures *the specific heat of the gas at constant volume*: this may be denoted by the symbol C_v.

Suppose, now, that the same weight of the same gas is enclosed in a cylinder which is fitted with a freely moving piston (Fig. 431, ii), upon the upper surface of which a constant pressure, of p dynes per sq. cm., is acting, and let the initial volume of the gas be v_1 c.c. When heat is imparted to the gas sufficient to raise its temperature through 1° C., part of the heat will be absorbed in warming the gas, and part will be utilised in raising the piston and, therefore, in doing work against the pressure p. If v_2 is the new volume of the gas, the work done against p is equal to $p(v_2 - v_1)$ ergs; and, expressed in terms of heat units, this will be $p(v_2 - v_1)/J$ calories (where J is the 'mechanical equivalent of heat'). Hence, the total heat required is

$$C_v + \frac{p(v_2 - v_1)}{J}.$$

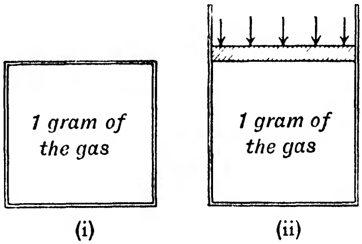


FIG. 431.

This total heat is *the specific heat of the gas at constant pressure*: it may be denoted by the symbol C_p. Hence, C_p is greater than C_v by an amount represented by $p(v_2 - v_1)/J$.

The experimental method, adopted by Regnault, of determining C_p of a gas consisted in allowing a steady stream of the gas to proceed from a large reservoir (maintained at a constant temperature), through a long spiral tube immersed in an oil-bath heated to a constant high temperature, and then through a spiral copper tube immersed in water contained in a calorimeter. The rise in temperature of the water and calorimeter gives the quantity of heat given up by the gas; and the quantity of gas used was determined by observing the initial and final pressures of the gas in the reservoir.

The value of C_v of a gas has been determined experimentally by means of *Joly's steam calorimeter*. From each end of the beam of a balance is suspended, by means of a long thin wire, a hollow copper sphere. The spheres are of the same size, exhausted of air, and adjusted to the same weight. The spheres are enclosed in a chamber which can be filled with dry steam. One of the spheres is filled with the gas under considerable pressure, and the weight of the gas is determined by means of the balance. Steam is then passed into the steam chamber: more steam will condense on the sphere containing the gas, since heat is absorbed in warming both sphere and gas, and the additional weight of steam is measured by the balance, and thus the quantity of heat absorbed by the gas is determined. From these data, *the specific heat of the gas at constant volume* is calculated.

The following values have been obtained:

	C_p	C_v
Air - - - - - -	0·237	0·172
Hydrogen - - - -	3·409	2·402

9. (There appears to be an error in the data of this question, as the lamp with the clean chimney would require to be moved further away from, and not nearer to, the grease-spot.)

In an experiment, a standard candle was placed 40 cm. from the grease-spot, and a balance was obtained when the flame of a paraffin lamp (with *clean* chimney) was 61·5 cm. from the grease-spot. The

top of the chimney was partially closed for nearly a minute, so as to make the flame smoke, and thus to dirty the chimney. It was then found that a balance was obtained when the lamp was moved to a distance of 52 cm. from the grease-spot.

Hence, with *clean* chimney,

$$\text{candle-power} = 1 \times (61{\cdot}5/40)^2 = 2{\cdot}36\ ;$$

and, with *dirty* chimney,

$$\text{candle-power} = 1 \times (52/40)^2 = 1{\cdot}69.$$

Percentage of light absorbed by dirty chimney $= (67 \times 100)/236 = 28{\cdot}4$.

10. (i) The **collimator** of a spectrometer is the tube through which the light passes before it enters the prism. The end of this tube which is near to the prism is closed by a converging lens; and the position of the narrow vertical slit at the distant end of the tube is adjusted so that it is situated at the focus of the lens. Thus, light passing through the slit is converted by the lens into a parallel (or 'collimated') beam of light. Since parallel rays of the same wave-length are diverted to the same extent by the prism, each wave-length will emerge from the prism as a parallel beam; but the beams will be deviated by the prism to different degrees, depending on the wave-length. Each of these beams will be brought to a focus on the focal plane of the converging lens in the telescope tube; and this image is viewed through the eye-piece of the telescope.

(ii) A method of determining the angle of a prism is shown in Fig. 432, which represents a horizontal section through the prism ABC, of which the angle B is to be measured. The prism is supported on a circular table which can be adjusted by three levelling screws (H, K and L) so that the faces of the prism are vertical. A parallel beam of light DB, from the collimator, falls partly on the face AB and partly on the face BC; and these faces reflect the parallel beams BF and BG respectively. *The angle between these two reflected beams is equal to twice the angle* B *of the prism.* The angle between the beams is measured by rotating the telescope until the image of the slit formed by the beam BF coincides with the centre of the field of view of the eye-piece; and the position of the telescope is observed on the circular horizontal scale by means of the vernier attached to the telescope. The telescope is then rotated until the image due to BG is in the centre of the field of view, and the position of the telescope is again noted. The difference between the two readings gives the angle between the beams. The relationship between this angle and that of the prism is proved in the following manner:—Produce DB in the direction E; then

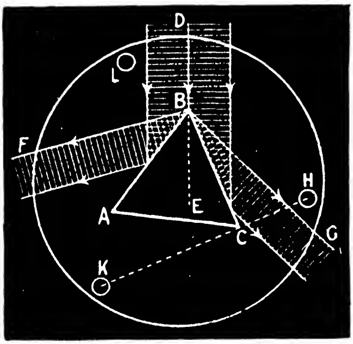


FIG. 432.—A method of measuring the angle of a glass prism.

$$\angle CBE = \angle CBG \quad \text{and} \quad \angle ABE = \angle ABF.$$

Hence

$$\angle FBG = \angle ABF + \angle ABE + \angle CBE + \angle CBG$$
$$= 2(\angle ABE + \angle CBE) = 2\,\angle ABC.$$

SECOND PAPER.

1. When stationary air-waves are set up in any organ pipe, whether open or closed, *an open end must be an antinode.* Hence, in an open pipe (Fig. 433, i) the simplest stationary wave has a node at its middle point, and an antinode at each end. the pitch of the note is its 'fundamental,' and the frequency is expressed by the equation $n = V/2l$, where l is the length of the pipe, and V is the velocity of sound in air. The next simplest distribution of nodes and antinodes is shown in Fig. 433, ii, in which case the wave-length is equal to l and $n = V/l$. This is the 'first overtone.'

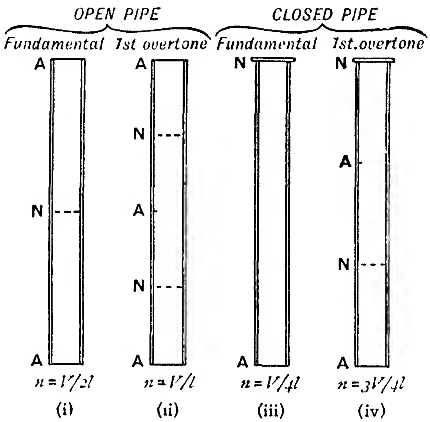


FIG 433.—Distribution of nodes and antinodes in open, and in closed, pipes

When a pipe of the same length is closed at one end, the 'fundamental' note (Fig. 433, iii) has a wave-length equal to $4l$ and $n = V/4l$. The next simplest distribution of nodes and antinodes is shown in Fig. 433, iv, where the wave-length is equal to $4l/3$ and $n = V\Big/\frac{4l}{3} = 3V/4l$. This is the 'first overtone.'

In the problem, $(V/l) - (3V/4l) = 440$

or $$V/4l = 440$$

or $$V/l = 1760.$$

The original frequency of the pipe, open at both ends, is

$$V/2l = 1760/2 = 880.$$

7. The energy which is evident when an earth-connected conductor, such as the finger, is brought near to the charged disc of an electrophorus is the equivalent of the *work* done in stretching the lines of force which connect the opposite charges on the lower surface of the disc and on the surface of the wax respectively. The work done in lifting the *charged* disc is greater, by this amount, than the work required to lift the disc when the apparatus is devoid of any electrical charge.

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